THEORETICAL PREDICTION OF THE THERMODYNAMIC BEHAVIOR OF AQUEOUS ELECTROLYTES AT HIGH PRESSURES AND TEMPERATURES: IV. CALCULATION OF ACTIVITY COEFFICIENTS, OSMOTIC COEFFICIENTS, AND APPARENT MOLAL AND STANDARD AND RELATIVE PARTIAL MOLAL PROPERTIES TO 600°C AND 5 KB

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ABSTRACT. Extrapolation to infinite dilution of the apparent molal properties of aqueous electrolytes reported in the literature with the aid of a one-parameter extension of the Debye-Hückel equation corresponding in form to that proposed by Hückel (1925) affords a comprehensive and internally consistent set of standard partial molal heat capacities ($C_p^*$), volumes ($\bar{V}^*$), and compressibilities ($\bar{\kappa}^*$), as well as the extended term parameters and their partial derivatives at temperatures $\leq 200°C$ and pressures corresponding to liquid-vapor equilibrium. The extrapolation equations, which yield close approximation of the thermodynamic properties of concentrated electrolytes, take into account both long- and short-range ionic interaction, electrostriction solvation of aqueous species, the concentration dependence of the dielectric constant of electrolyte solutions, and the effect of ion association on ionic strength. The Debye-Hückel term incorporates values of $\lambda$ computed independently from the effective electrostatic radii of ions generated by Helgeson and Kirkham (1976) from consideration of ion-solvent interaction at infinite dilution. Contrary to assumptions commonly made in the literature, the reciprocal of the dielectric constant of electrolyte solutions can be regarded as a linear function of (1) ionic strength at constant pressure and temperature, and (2) the reciprocal of the dielectric constant of the solvent as a function of temperature and pressure at constant ionic strength. Regression of the extended term parameters for $C_p^*$, $\bar{V}^*$, and $\bar{\kappa}^*$, as well as the extrapolated values of $C_p^*$, $\bar{V}^*$, and $\bar{\kappa}^*$, using equations derived from a theoretical model providing simultaneously for the intrinsic properties of aqueous species and the extent to which ion solvation, cavity formation, and collapse (or expansion) of the local solvent structure in the vicinity of the species affect their thermodynamic behavior permits prediction of ionic activity coefficients and the standard partial molal properties of aqueous species at high pressures and temperatures. Taking account of dissociation constants ($K$) and their partial derivatives affords corresponding calculation of osmotic and stoichiometric mean activity coefficients, as well as the apparent molal and relative partial molal properties of aqueous electrolytes. Calculations of this kind indicate that activity coefficients of aqueous electrolytes become monotonic functions of ionic strength at high temperatures and low pressures, where ion association contributes significantly to the thermodynamic behavior of electrolyte solutions. The extrema in $C_p^*$, $\bar{V}^*$, $\bar{\kappa}^*$, and their extended term analogs as a function of temperature at low pressures move toward higher temperatures with increasing pressure and disappear at pressures above 1 to 3 kb, depending on the property and the electrolyte. The extrema are an additive consequence of opposing contributions.

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by an asymptotic function of temperature describing the calorimetric consequences of short-range ionic interaction and/or solvent collapse (or expansion) and an exponential function representing the electrostatic influence of ion solvation on the thermodynamic behavior of electrolytes. Owing to dramatic changes in these relative contributions to the standard partial molal volumes of aqueous electrolytes with increasing temperature, \( \mathcal{C}^\ast \), and its extended term analog also exhibit extrema with increasing pressure at (constant) high temperatures. The strong dependence of the thermodynamic properties of the solute on the electrostatic properties of the solvent causes the standard partial molal heat capacities, entropies, enthalpies, and volumes of aqueous electrolytes to approach negative infinity at the critical point of H\(_2\)O. In contrast, the extended term analogs of these properties approach infinity at the critical point. The thermodynamic properties of aqueous electrolytes are thus highly sensitive to slight changes in pressure, temperature, and concentration in the critical region, where they may exhibit extrema and/or range from large negative values to large positive values at high ionic strengths. As a consequence, the heats of solution of electrolytes at high temperatures and low pressures may be more than a million cal mole\(^{-1}\). Experimental dissociation constants and calculated values of the activity coefficient of molecular NaCl\(_{aq}\) as a function of temperature at low pressures suggests the occurrence of maxima in the degrees of association of electrolytes as a function of concentration at high temperatures and pressures.

INTRODUCTION

Calculation of the thermodynamic properties of aqueous species at high pressures and temperatures is a requisite for quantitative description of chemical equilibrium in hydrothermal systems, as well as prediction of the extent to which components are redistributed among minerals and aqueous solutions in geochemical processes. The purpose of the present communication is threefold: (1) to derive from theoretical considerations and experimental data a comprehensive and internally consistent set of equations and coefficients for such calculations, (2) to facilitate application of the calculations to geochemical problems by tabulating and depicting activity coefficients and partial molal properties of aqueous species at temperatures and pressures to 600\(^\circ\)C and 5 kb, and (3) to examine and critique the causes and consequences of the thermodynamic behavior of aqueous electrolytes at high pressures and temperatures.

Aqueous solutions involved in geochemical processes range from nearly pure H\(_2\)O/CO\(_2\) fluids to concentrated electrolyte solutions. The thermodynamic components of aqueous electrolytes in geologic systems consist primarily of alkali and alkaline earth chlorides with lesser concentrations of bicarbonates and sulfates or bisulfates, together with minor H\(_2\)S, SiO\(_2\), AlCl\(_3\), FeCl\(_3\), FeCl\(_2\), ZnCl\(_2\), PbCl\(_2\), CuCl, AgCl, et cetera. These components are distributed among a host of ionic and neutral species, most of which associate to an increasing but differential degree as the density and dielectric constant of the solvent decrease with increasing temperature and/or decreasing pressure. At a given pressure and temperature, the distribution of species may vary from simple solvated ions at infinite dilution to polyligand complexes at high ionic strengths. Because the temperatures and pressures at which reactions among minerals and aqueous solutions take place in geochemical processes range from 0\(^\circ\)C and 1 bar at the surface of the Earth to the melting temperatures of rocks at pressures ranging up to 30 kb or more (Delany and Helgeson, 1978), all such species must be taken into account in computing the thermodynamic properties of these solutions.
Definitive equilibrium and mass transfer calculations for geochemical processes require a general set of predictive equations for the thermodynamic properties of aqueous species which are applicable to the wide variety of temperatures, pressures, and chemical perturbations encountered in geologic systems. Although the approach must be based on sound theoretical concepts, it must also be practical, comprehensive in scope, and yield internally consistent results for both the solvent and solute that afford close approximations of geologic reality. Because the thermodynamic properties of the solvent control to a large extent the thermodynamic behavior of the solute, and because H₂O is the only common denominator for all electrolyte solutions encountered in nature, the first two papers in this series (Helgeson and Kirkham, 1974a and b) were devoted to calculation of the thermodynamic/electrostatic properties of H₂O at high pressures and temperatures. Certain of these properties were used in the third contribution (Helgeson and Kirkham, 1976) to formulate and evaluate an equation of state for aqueous electrolytes at infinite dilution. The present communication extends the theoretical approach to include standard partial molal heat capacities (and thus all standard partial molal properties), activity and osmotic coefficients, and the apparent molal and relative partial molal properties of aqueous electrolytes as a function of pressure, temperature, and ionic strength to 5 kb and 600°C.

CONVENTIONS, UNITS, AND NOTATION

The standard state for aqueous species (other than H₂O) adopted in the present study is one of unit activity of the species in a hypothetical one molal solution referenced to infinite dilution at any pressure and temperature. The standard state for the solvent calls for unit activity of pure H₂O at any pressure and temperature. Accordingly, regardless of

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1 Typographical errors in a number of the equations given by Helgeson and Kirkham (1974a and b, 1976) are corrected in footnote 1 of Helgeson and Kirkham (1976), footnotes 1 and 2 of Walther and Helgeson (1977), and footnotes 1 of Delany and Helgeson (1978). In addition, the heading of table 35 in Helgeson and Kirkham (1974a) should read bars instead of kb, and Cₓ(ln T)r in footnote 1 of Delany and Helgeson (1978) should read Cₓ(ln T)/r. Footnote 1 of table 1 in Walther and Helgeson (1977) also contains a typographical error which should be corrected. Instead of 49,003 and 31,208 cal mol⁻¹ (K)⁻¹, the values shown in the footnote for the standard molal entropies of SiO₂ and O₂ are, should read 4.5 and 49,003 cal mol⁻¹ (K)⁻¹, respectively. In addition, eq (10) in the sentences preceding eqs (15) and (25) should read eq (13), the second left parenthesis in the third term on the right side of eq (25) should be deleted, and the units of a₂ and e₁ on page 1330 should be corrected to read cal mol⁻¹ bar⁻¹ and cal mol⁻¹ (K)⁻¹, respectively. The value given for the electronic charge (e) on page 1204 of Helgeson and Kirkham (1974b) and pages 99 and 119 of Helgeson and Kirkham (1976) should read 4.80298 × 10⁻¹⁹ esu, rather than 4.80298 esu. Also, the value of Y given in footnote n of table 7 on page 151 of Helgeson and Kirkham (1976) should appear as −5.8021 × 10⁻⁷ (K)⁻¹, rather than 5.8021 × 10⁻⁶ (K)⁻¹, and the first two terms on the right side of the second identity in eq (126) should be multiplied by Z₂. The following corrections should be made in table 18 of Helgeson and Kirkham (1974b); the multipliers shown at the top of columns 3, 7, and 8 should read 10⁻⁴, 10⁻⁴, and 10⁻⁴, respectively, and footnote e in the table should appear as kg⁻¹ cm⁻¹ mole⁻¹/², rather than kg⁻¹ cm⁻¹ mole⁻¹/². In addition, σ₀bL/2 in the line preceding eq (66) of Helgeson and Kirkham (1974b) should be positive, 2(2.303)RT in the equation should be replaced with a minus sign, and 2(2.303)RT should be deleted from the right side of the first equality in eq (80). Finally, the T immediately preceding the second integral sign in eq (18) of Helgeson and others (1978) should be deleted.
the pressure and temperature, the activity coefficients of all solute species approach unity at infinite dilution, as does the activity of the solvent.

Standard Gibbs free energies and enthalpies of formation from the elements at high pressures and temperatures are expressed below as apparent molal \( (\Delta G^o \text{ or } \Delta H^o) \) or apparent partial molal \( (\Delta G^o \text{ or } \Delta \Pi^o) \) quantities, which are defined by (Benson, 1968; Helgeson, 1969; Helgeson and Kirkham, 1974a, 1976)

\[
\Delta G^o_{j,P,T} = \Delta G^o_{j,P,T} = \Delta G^o_{f,j} + (\Pi^o_{j,P,T} - \Pi^o_{f,j})
\]

and

\[
\Delta H^o_{j,P,T} = \Delta \Pi^o_{j,P,T} = \Delta \Pi^o_{f,j} + (\Pi^o_{j,P,T} - \Pi^o_{f,j})
\]

where \( \Delta G^o_{j,P,T}, \Delta H^o_{j,P,T}, \Delta G^o_{j,P,T}, \text{ and } \Delta \Pi^o_{j,P,T} \) stand for the conventional apparent standard molal and partial molal Gibbs free energy and enthalpy of formation of the \( j \)th aqueous species at the subscripted pressure \( (P) \) and temperature \( (T) \), \( \Delta G^o_{f,j} \) and \( \Delta \Pi^o_{f,j} \) denote the conventional standard partial molal Gibbs free energy and enthalpy of formation from the elements of the species at a reference pressure \( (P_r) \) and temperature \( (T_r) \) of 1 bar and 298.15K, and the two parenthetical terms correspond, respectively, to the difference in the conventional standard partial molal Gibbs free energy and enthalpy of the species at the pressure and temperature of interest and those at \( P_r, T_r \). The apparent molal Gibbs free energy and enthalpy of formation of an electrolyte solution at a given pressure and temperature (\( \Delta G_{P,T} \) and \( \Delta H_{P,T} \), respectively) are then given by

\[
\Delta G_{P,T} = \Delta G^o_{P,T} + 55.51 \left( \Pi_{w,P,T} - \Pi^o_{w,P,T} \right) + \sum_k m_k \left( \Pi_{k,P,T} - \Pi^o_{k,P,T} \right)
\]

and

\[
\Delta H_{P,T} = \Delta H^o_{P,T} + 55.51 \left( \Pi_{w,P,T} - \Pi^o_{w,P,T} \right) + \sum_k m_k \left( \Pi_{k,P,T} - \Pi^o_{k,P,T} \right)
\]

which are consistent with

\[
G_{\text{solution},P,T} - G^o_{\text{solution},P,T} = \Delta G_{P,T} - \Delta G^o_{P,T}
\]

and

\[
H_{\text{solution},P,T} - H^o_{\text{solution},P,T} = \Delta H_{P,T} - \Delta H^o_{P,T}
\]

where \( G^o_{\text{solution},P,T}, H^o_{\text{solution},P,T}, G_{\text{solution},P,T}, \text{ and } H_{\text{solution},P,T} \) denote the molal Gibbs free energy and enthalpy of the solution in the standard state and the state of interest, respectively, 55.51² corresponds to the number of moles of \( H_2O \) (kg \( H_2O \))⁻¹, \( m_k \) stands for the molality of the \( k \)th thermodynamic component of the solute,³ the parenthetical quantities

²For the sake of brevity, 55.51 is used in eq (3) and subsequent equations to represent 55.50837, which is the value used in the calculations summarized below.

³The term component is used in the present communication in its strict thermodynamic sense; that is, the term refers to a chemical formula unit representing one of the minimum number of independent compositional variables in the system.
represent the relative partial molal Gibbs free energy and enthalpy of the solvent and solute components, which are designated by the subscripts \( w \) and \( k \), respectively, and

\[
\Delta G^\circ_{P,T} = 55.51 \Delta G^\circ_{w,P,T} + \sum_k m_k \Delta G^\circ_{k,P,T} \tag{7}
\]

and

\[
\Delta H^\circ_{P,T} = 55.51 \Delta H^\circ_{w,P,T} + \sum_k m_k \Delta H^\circ_{k,P,T} \tag{8}
\]

where \( \Delta G^\circ_{w,P,T} \) and \( \Delta H^\circ_{w,P,T} \) are defined by appropriate statements of eqs (1) and (2) with \( j \) representing \( H_2O \) and

\[
\Delta G^\circ_{k,P,T} = \Delta G^\circ_{j,P,T} = \sum_j \nu_{j,k} \Delta G^\circ_{j,P,T} \tag{9}
\]

and

\[
\Delta H^\circ_{k,P,T} = \Delta H^\circ_{j,P,T} = \sum_j \nu_{j,k} \Delta H^\circ_{j,P,T} \tag{10}
\]

where \( \nu_{j,k} \) refers to the number of moles of the \( j \)th aqueous species (mole of the \( k \)th solute component)\(^{-1}\). Eqs (1) through (10) are consistent with

\[
\Delta G_{P,T} = \Delta G_{solvent,P,T} + \Delta G_{solute,P,T} \tag{11}
\]

and

\[
\Delta H_{P,T} = \Delta H_{solvent,P,T} + \Delta H_{solute,P,T} \tag{12}
\]

where

\[
\Delta G_{solvent,P,T} = 55.51 \Delta G^\circ_{w,P,T} + 55.51 (\overline{G}_{w,P,T} - \overline{G}^\circ_{w,P,T}), \tag{13}
\]

\[
\Delta H_{solvent,P,T} = 55.51 \Delta H^\circ_{w,P,T} + 55.51 (\overline{H}_{w,P,T} - \overline{H}^\circ_{w,P,T}), \tag{14}
\]

\[
\Delta G_{solute,P,T} = \sum_k m_k \Delta G^\circ_{k,P,T} + \sum_k m_k (\overline{G}_{k,P,T} - \overline{G}^\circ_{k,P,T}), \tag{15}
\]

and

\[
\Delta H_{solute,P,T} = \sum_k m_k \Delta H^\circ_{k,P,T} + \sum_k m_k (\overline{H}_{k,P,T} - \overline{H}^\circ_{k,P,T}). \tag{16}
\]

Because few completely dissociated two or three-component electrolyte solutions exist in nature, the major part of the discussion below is devoted to thermodynamic relations and predictive equations which apply to individual aqueous species. Similarly, to afford maximum flexibility in geochemical calculations and provide adequately for the diversity and highly variable extent of ion association in natural electrolyte solutions, all relative partial molal properties are expressed in terms of the concentrations of these species, rather than the stoichiometric concentrations of the thermodynamic components of the solutions. The standard state thermodynamic properties of individual aqueous species correspond to conventional properties consistent with

\[
\Xi^\circ_j = \Xi^\circ_j^{abs} - Z_j \Xi_{H^+,j}^{abs} = \Xi^\circ_j = \Xi^\circ_j^{abs} - Z_j \Xi_{H^+,j}^{abs} \tag{17}
\]
\[
\Delta \Xi^o_j = \Delta \Xi^o_j^{\text{abs}} - Z_j \Delta \Xi^o_{\text{H}, \text{H}}^{\text{abs}} = \Delta \Xi^o_j = \Delta \Xi^o_j^{\text{abs}} - Z_j \Delta \Xi^o_{\text{H}, \text{H}}^{\text{abs}}
\]  

where \(\Xi^o_j\) stands for the conventional standard molal entropy, heat capacity, volume, expansibility, or compressibility of the \(j\)th aqueous species, \(\Delta \Xi^o_j\) represents the conventional apparent standard molal Gibbs free energy or enthalpy of the subscripted species, \(\Xi^o_j^{\text{abs}}\) and \(\Delta \Xi^o_j^{\text{abs}}\) denote the absolute counterparts of \(\Xi^o_j\) and \(\Delta \Xi^o_j\), and \(\Xi^o_{\text{H}, \text{H}}\), \(\Delta \Xi^o_{\text{H}, \text{H}}\), \(\Delta \Xi^o_j\), and \(\Delta \Xi^o_j^{\text{abs}}\) correspond to the partial molal equivalents of \(\Xi^o_j\), \(\Xi^o_j^{\text{abs}}\), \(\Delta \Xi^o_j\), and \(\Delta \Xi^o_j^{\text{abs}}\). It thus follows that \(\Xi^o_{\text{H}, \text{H}} = \Xi^o_{\text{H}, \text{H}} = \Delta \Xi^o_{\text{H}, \text{H}} = \Delta \Xi^o_{\text{H}, \text{H}} = 0\), and the standard partial molal and apparent standard partial molal thermodynamic properties of acid solutes designated by \(H_\nu L\) (where \(\nu_H\) represents the number of moles of \(H^+\) (mole of \(H_\nu L\))^{-1} are equal to \(\Xi^o_L\) and \(\Delta \Xi^o_L\), respectively. Although eqs (17) and (18) are written for the \(j\)th ion, analogous expressions can be written for charged complexes. All symbols representing standard thermodynamic properties of individual aqueous species in the pages that follow refer to the conventional properties unless indicated otherwise by the superscript \(\text{abs}\) or \(\text{absolute}\). In contrast, all symbols representing nonstandard state properties refer to the absolute properties of the species.

Enthalpies and Gibbs free energies are expressed below in thermochemical calories (4.184 joules) or kilocalories mole\(^{-1}\) (cal mole\(^{-1}\) or kcal mole\(^{-1}\)), entropies and heat capacities in cal mole\(^{-1}\) (°K)\(^{-1}\), and volumes in cm\(^3\) mole\(^{-1}\) (which can be converted to cal mole\(^{-1}\) bar\(^{-1}\) by multiplying by 0.0239 cal cm\(^{-3}\) bar\(^{-1}\)). The activity of the \(i\)th species (\(a_i\)) and equilibrium constants (K) are dimensionless, which requires activity coefficients to have reciprocal units of concentration. All thermodynamic properties of \(H_2O\) are consistent with those given by Helgeson and Kirkham (1974a) for a molecular weight of 18.0153 gm mole\(^{-1}\). Pressure is expressed in bars (b) or kilobars (kb) and temperature in °C or °K.

**GLOSSARY**

\(a\) — coefficient in eqs (151) and (152).

\(\hat{a}, \hat{a}_k\) — ion size parameters (eqs 124 and 125).

\(a_1, a_2, a_3, a_4, \theta, \omega\) — equation of state coefficients for computing the state thermodynamic properties of aqueous species at high pressures and temperatures (eqs 253 and 273-276).

\(\hat{a}_1, \hat{a}_2, \hat{a}_3, \hat{a}_4, \hat{a}_5, \theta, \omega\) — equation of state coefficients for computing extended term parameters for aqueous electrolytes at high pressures and temperatures (eqs 288-295).

\(\hat{a}_{5}, \hat{a}_{6}\) — electrostatic parameters for predicting values of \(\hat{b}_k\) and the dielectric constant of electrolyte solutions at high temperatures and pressures (eqs 286 and 287).
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$a_i, a_j, a_k, a_l, a_q, a_n$ — activity of the subscripted aqueous species.

$a, b$ — coefficients in eqs (182) through (184), (281), and (282).

$a_i, b_1$ — coefficients in eq (245).

$a_2, b_2$ — coefficients in eq (246).

$a_3, b_3$ — coefficients in eq (247).

**abs, absolute** — superscripts designating absolute thermodynamic properties of aqueous species.

**anions,** — subscripts or superscripts designating anions.

**aq, aqueous** — subscripts designating the aqueous state.

**aqueous disordered** — subscript designating a hypothetical unsolvated aqueous state in schematic diagram (126).

**aqueous ordered** — subscript designating a solvated aqueous state in schematic diagram (126).

$a_w$ — activity of $H_2O$.

A+ — representation of a monovalent cation.

$A_y, A_G, A_H, A_J, A_V, A_k, A_E$ — electrostatic Debye-Hückel parameters defined by eqs (C-1) and (C-8) through (C-8) in app. C.

$b_y, \lambda, b_y, n$ — extended term parameters (eqs 174 and 176) for computing mean ionic activity coefficients of the $k$th "completely" dissociated electrolyte and the $n$th neutral aqueous species from eqs (173) and (175).

$b_{H,k}, b_{J,l}, b_{V,k}, b_{k,k},$ $b_{E,x,k}, b_{H,q}, b_{J,q}, b_{V,q}$ — extended term parameters for the $k$th "completely" dissociated electrolyte and $q$th aqueous complex defined by eqs (C-26) through (C-30) in app. C and (214K) through (214M).

$b_{y,NaCl}, b_{y,NaCl^+}$ — extended term parameters for computing activity coefficients of dissociated and associated $NaCl_{(aq)}$, respectively.

$b_{ij}, b_{ji}, b_{HI,ij}, b_{H,ij},$ $b_{J,ij}, b_{V,ij}, b_{V,ij}$ — short-range interaction parameters for the $i$th, $j$th, $l$th, and $q$th aqueous species in eqs (196) through (199) and (214K) through (214M) defined by equations analogous to eqs (C-19) through (C-22) in app. C.

$b_{il}, b_{ul}, b_{nl}, b_{j}$ — short-range interaction parameters for computing activity coefficients of the $i$th, $l$th, $n$th, $j$th, and $j$th aqueous species defined by eqs (170) through (172) and (C-19) in app. C.

$b_{il}, b_{ul}, b_{nl}, b_{j}$ — short-range interaction parameters for computing relative partial molal Gibbs free energies of the $i$th, $l$th, $n$th, $j$th, and $j$th aqueous species.

$b_{ij}, b_{HI,ij}, b_{H,ij}, b_{V,ij},$ $b_{k,j}, b_{V,ij}$ — short-range interaction parameters for the $j$th and $i$th aqueous species defined by eqs (C-19) through (C-24) in app. C.

$b_{k}, b_{H,k}, b_{J,k}, b_{V,k}$ — electrostatic solvation parameters for the $k$th
$b_{k,k}$, $b_{E_x,k}$ — electrolyte defined by eqs (149) and (C-14) through (C-18a) in app. C.

$b_k$ — electrostatic parameter for computing the dielectric constant of the $k$th electrolyte from eq (143).

$B_{y}, B_{I}, B_{j}, B_{V}, B_{v}$, $B_{E_x}$ — electrostatic Debye-Hückel parameters defined by eqs (C-2) and (C-9) through (C-13) in app. C.

$\theta$, $\omega$ — heat capacity coefficients for computing thermodynamic properties of aqueous species at infinite dilution from eqs (273) through (276).

$\theta_s$, $\theta$, $\omega$ — heat capacity coefficients for computing extended term parameters for aqueous electrolytes from eqs (288) through (295).

cations, + — subscripts or superscripts designating cations.

$C_P$ — isobaric heat capacity.

$C^o_P$ — standard molal isobaric heat capacity.

$C_{P,j}^o$, $C_{P,j}^{o,a,b}$ — conventional and absolute standard molal isobaric heat capacity of the $j$th aqueous species.

$C_P$ — partial molal isobaric heat capacity.

$C_P - C^o_P$ — relative partial molal isobaric heat capacity.

$C^o_P$ — standard partial molal isobaric heat capacity.

$C_{P,j}^o$, $C_{P,j}^{o,a,b}$ — conventional and absolute standard partial molal isobaric heat capacity of the $j$th aqueous species.

$C_{P,j}^o$, $C_{P,j}^{o,a,b}$ — conventional standard partial molal intrinsic heat capacity of the $j$th aqueous species at the pressures specified by $P$ and $P_r$, respectively (eq 271).

$C_{P,r,j}$ — conventional standard partial molal heat capacity of the $j$th aqueous species at the reference pressure specified by $P_r$ (eq 270).

$\Delta C_{P,r,j}^o$, $\Delta C_{P,r,c,j}^o$ — contribution by collapse of the local solvent structure to the conventional standard partial molal heat capacity of the $j$th aqueous species at the pressures specified by $P$ and $P_r$, respectively (eq 268).

$\Delta C_{P,r,n,j}^o$, $\Delta C_{P,r,n,j}^o$ — sum of the intrinsic and collapse contributions to the conventional standard partial molal heat capacity of the $j$th aqueous species ($\Delta C_{P,r,n,j}^o = C_{P,r,i,j}^o + \Delta C_{P,r,c,j}^o$) at the pressures specified by $P$ and $P_r$, respectively (eq 269).

$\Delta C_{P,r,n,k}$ — sum of the intrinsic and collapse contributions to the standard partial molal isobaric heat capacity of the $k$th electrolyte.

$\Delta C_{P,r,q}^o$ — standard partial molal isobaric heat capacity of overall dissociation for the $q$th aqueous complex.
$\Delta C_{P,s,j}$ — conventional standard partial molal isobaric heat capacity of solvation of the $j$th aqueous species (eq 264).

$C_{sat,k}$, $C^o_{sat,k}$ — partial molal and standard partial molal heat capacity of the $k$th electrolyte along the liquid-vapor equilibrium curve for the electrolyte solution and H$_2$O, respectively.

e — electronic charge ($4.80298 \times 10^{-10}$ esu).

$E_x$ — isobaric expansibility ($E_x = (\partial V / \partial T)_p$).

$E^o_x$ — standard molal isobaric expansibility ($E^o_x = (\partial V^{o}_x / \partial T)_p$).

$E^o_{x,j}$ — conventional standard molal isobaric expansibility of the $j$th aqueous species ($E^o_{x,j} = (\partial V^{o}_{x,j} / \partial T)_p$).

$E^o_{x,j}^{abs}$ — absolute standard molal isobaric expansibility of the $j$th aqueous species ($E^o_{x,j}^{abs} = (\partial V^{o}_{x,j}^{abs} / \partial T)_p$).

$E_x$ — partial molal isobaric expansibility ($E_x = (\partial V / \partial T)_p$).

$E_x - E^o_x$ — relative partial molal isobaric expansibility ($E_x - E^o_x = (\partial V / \partial T)_p - (\partial V^{o}_x / \partial T)_p$).

$E^o_x$ — standard partial molal isobaric expansibility ($E^o_x = (\partial V^{o}_x / \partial T)_p$).

$E^o_{x,i,j}$ — conventional standard intrinsic partial molal expansibility of the $j$th aqueous species ($E^o_{x,i,j} = (\partial V^{o}_{x,i,j} / \partial T)_p$).

$E^o_{x,j}$ — conventional standard partial molal isobaric expansibility of the $j$th aqueous species ($E^o_{x,j} = (\partial V^{o}_{x,j} / \partial T)_p$).

$E^o_{x,j}^{abs}$ — absolute standard partial molal isobaric expansibility of the $j$th aqueous species ($E^o_{x,j}^{abs} = (\partial V^{o}_{x,j}^{abs} / \partial T)_p$).

$\Delta E^o_{x,c,j}$ — contribution by collapse of the local solvent structure to the conventional standard partial molal isobaric expansibility of the $j$th aqueous species ($\Delta E^o_{x,c,j} = (\partial \Delta V^{o}_{x,c,j} / \partial T)_p$).

$\Delta E^o_{x,n,j}$ — sum of the intrinsic and collapse contributions to the conventional standard partial molal isobaric expansibility of the $j$th aqueous species ($\Delta E^o_{x,n,j} = \Delta E^o_{x,i,j} + \Delta E^o_{x,c,j} = (\partial \Delta V^{o}_{x,n,j} / \partial T)_p$).

$\Delta E^o_{x,s,j}$ — conventional standard partial molal isobaric expansibility of solvation of the $j$th aqueous species ($\Delta E^o_{x,s,j} = (\partial \Delta V^{o}_{x,s,j} / \partial T)_p$).
\( f \) — subscript designating final state in eqs (236) and (237).

\( G \) — Gibbs free energy.

\( G^\circ \) — standard molal Gibbs free energy.

\( \mathcal{G} \) — partial molal Gibbs free energy.

\( \mathcal{G}_{d,j} \) — long-range ionic interaction contribution to the relative partial molal Gibbs free energy of the \( j \)th aqueous species (eq 120).

\( G_{\text{excess}} \) — excess molal Gibbs free energy of mixing.

\( \mathcal{G}_{\text{ideal}} \) — ideal partial molal Gibbs free energy of mixing.

\( \mathcal{G}_{r,i}, \mathcal{G}_{r,j}, \mathcal{G}_{r,l}, \mathcal{G}_{r,n} \) — sum of the intrinsic, collapse, and short-range interaction contributions to the relative partial molal Gibbs free energies of the \( i \)th, \( j \)th, \( l \)th and \( n \)th aqueous species (eqs 156-158 and footnote 14).

\( \mathcal{G} - \mathcal{G}^\circ \) — relative partial molal Gibbs free energy.

\( \mathcal{G}^\circ \) — standard partial molal Gibbs free energy.

\( \mathcal{G}^\circ_{i,j} \) — conventional standard partial molal intrinsic Gibbs free energy of the \( j \)th aqueous species.

\( \mathcal{G}^\circ_p, \mathcal{G}^\circ_{j,abs} \) — conventional and absolute standard partial molal Gibbs free energy of the \( j \)th aqueous species.

\( \mathcal{G}^\circ_{(X)} \) — rational standard partial molal Gibbs free energy defined for the \( i \)th species by eq (164A).

\( \mathcal{G}^\circ_{\text{ideal}}(X) \) — ideal rational partial molal Gibbs free energy of mixing defined for the \( i \)th species by eq (164B).

\( \Delta G \) — apparent molal Gibbs free energy of formation defined by eq (3).

\( \Delta G^\circ \) — apparent standard molal Gibbs free energy of formation defined by eq (1).

\( \Delta G_{r, \Delta G_a} \) — Gibbs free energy change accompanying transfer of unsolvated and solvated ions to and from a vacuum as shown in schematic diagram (126).

\( \Delta G^\circ_{j, \Delta G_{j,abs}} \) — conventional and absolute standard molal Gibbs free energy of formation of the \( j \)th aqueous species.

\( \Delta G_s \) — Gibbs free energy of solvation within an aqueous phase (eq 127).

\( \Delta \mathcal{G}_k \) — apparent partial molal Gibbs free energy of formation of the \( k \)th electrolyte (\( \Delta \mathcal{G}_k = \Delta \mathcal{G}^\circ_k + RT \ln a_k \)).

\( \Delta \mathcal{G}_{h,i}, \Delta \mathcal{G}_{h,l}, \Delta \mathcal{G}_{h,n} \) — solvation contributions to the relative partial molal Gibbs free energy of the \( i \)th, \( l \)th, \( n \)th, and \( n \)th aqueous species (eqs 153-155).
\(\Delta G_{n,j}\) — sum of the intrinsic and collapse contributions to the relative partial molal Gibbs free energy of the \(j\)th aqueous species.

\(\Delta G_{s,j}, \Delta G_{s,k}\) — partial molal Gibbs free energy of solvation of the \(j\)th aqueous species and the \(k\)th electrolyte, respectively (eqs 128 and 131).

\(\Delta G^\circ\) — apparent standard partial molal Gibbs free energy of formation defined by eq (1).

\(\Delta G^\circ_{c,j}\) — contribution by collapse of the local solvent structure to the conventional standard partial molal Gibbs free energy of the \(j\)th aqueous species.

\(\Delta G^\circ_f\) — standard partial molal Gibbs free energy of formation from the elements in their stable form at 298.15\(^\circ\)K and 1 bar.

\(\Delta G^\circ_{f,j}, \Delta G^\circ_{f,j}^{abs}\) — conventional and absolute standard partial molal Gibbs free energy of formation of the \(j\)th aqueous species from its elements in their stable form at 298.15\(^\circ\)K and 1 bar.

\(\Delta G^\circ_{p,j}, \Delta G^\circ_{j}^{obs}\) — conventional and absolute apparent standard partial molal Gibbs free energy of formation of the \(j\)th aqueous species.

\(\Delta G^\circ_{n,j}\) — sum of the intrinsic and collapse contributions to the conventional standard partial molal Gibbs free energy of the \(j\)th aqueous species (\(\Delta G^\circ_{n,j} = \Delta G^\circ_{i,j} + \Delta G^\circ_{c,j}\)).

\(\Delta G^\circ_r\) — standard partial molal Gibbs free energy of reaction.

\(\Delta G^\circ_{s,j}, \Delta G^\circ_{s,j}^{obs}, \Delta G^\circ_{s,n}, \Delta G^\circ_{s,n}^{obs}\) — conventional and absolute standard partial molal Gibbs free energies of solvation of the \(j\)th and \(n\)th aqueous species.

\(\Delta G^\circ_{s,k}\) — standard partial molal Gibbs free energy of solvation of the \(k\)th electrolyte (eq 134).

\(H\) — enthalpy.

\(H^\circ\) — standard molal enthalpy.

\(\bar{H}\) — partial molal enthalpy.

\(\bar{H} - \bar{H}^\circ\) — relative partial molal enthalpy.

\(\bar{H}^\circ\) — standard partial molal enthalpy.

\(\bar{H}^\circ_{j}, \bar{H}^\circ_{j}^{abs}\) — conventional and absolute standard partial molal enthalpy of the \(j\)th aqueous species.

\(\Delta H\) — apparent molal enthalpy of formation defined by eq (4).

\(\Delta H^\circ\) — apparent standard molal enthalpy of formation defined by eq (2).

\(\Delta H_d\) — heat of dilution (eq 236).
\( \Delta H^o_j, \Delta H^o_{j,abs} \) — conventional and absolute standard molal enthalpy of formation of the \( j \)th aqueous species.

\( \Delta H \) — heat of solution.

\( \Delta \tilde{\Pi}_k \) — apparent partial molal enthalpy of formation of the \( k \)th electrolyte.

\( \Delta \tilde{\Pi}^o \) — apparent standard partial molal enthalpy of formation defined by eq (2).

\( \Delta \tilde{\Pi}^o_j \) — standard partial molal enthalpy of formation from the elements in their stable form at 298.15\(^\circ\)K and 1 bar.

\( \Delta \tilde{\Pi}^o_{f,j}, \Delta \tilde{\Pi}^o_{f,j,abs} \) — conventional and absolute standard partial molal enthalpy of formation of the \( j \)th aqueous species from its elements in their stable form at 298.15\(^\circ\)K and 1 bar.

\( \Delta \tilde{\Pi}^o_j, \Delta \tilde{\Pi}^o_{j,abs} \) — conventional and absolute apparent standard partial molal enthalpy of formation of the \( j \)th aqueous species.

\( \Delta \tilde{\Pi}^o_{r,q} \) — standard partial molal enthalpy of overall dissociation for the \( q \)th aqueous complex.

\( i \) — index designating aqueous cations \( (i = 1, 2, \ldots) \) or a subscript designating an intrinsic property or an initial state.

\( ideal \) — subscript designating ideal contributions to the thermodynamic properties of mixing.

\( I \) — stoichiometric ionic strength of an electrolyte solution defined by eq (85).

\( I \) — "true" ionic strength of an electrolyte solution defined by eq (83).

\( I', I''', I'_T \) — ionic strength derivative functions defined by eqs (203) through (205).

\( j \) — index designating aqueous ions \( (j = 1, 2, \ldots) \).

\( J \) — relative partial molal heat capacity.

\( k \) — Boltzmann's constant \((1.38054 \times 10^{-16} \text{ erg} \cdot \text{K}^{-1})\).

\( k \) — subscript representing a single electrolyte or an index designating the thermodynamic components of the solute of a mixed electrolyte solution \((k = 1, 2, \ldots)\).

\( k \) — proportionality constant in eq (284) defined by eq (285).

\( K_{NaCl^*}, K_{NaCl} \) — standard molal dissociation constant for the NaCl molecule.

\( K_q, K_n \) — stepwise standard molal dissociation constant for the \( q \)th aqueous complex and \( n \)th neutral species.

\( l \) — index designating aqueous anions \( (l = 1, 2, \ldots) \).
\( \bar{L} \) — relative partial molal enthalpy.
\( \text{LiCl}^\circ \) — designation of the aqueous LiCl molecule.
\( m \) — molality.
\( m^\ast \) — sum of the molalities of all solute species in an electrolyte solution (eq 107).
\( m'_{j,p}, m'_{q,p}, m'_{i,p} \) — molality derivative functions defined by eq (207) and its analog for the \( q \)th aqueous complex and \( i \)th and \( l \)th cation and anion, respectively.
\( m''_{i,p}, m''_{q,p}, m''_{i,p} \) — molality derivative functions defined by eq (208) and its analog for the \( q \)th aqueous complex and \( i \)th and \( l \)th cation and anion, respectively.
\( m'_{j,T}, m'_{q,T}, m'_{i,T} \) — molality derivative functions defined by eq (206) and its analog for the \( q \)th aqueous complex and \( i \)th and \( l \)th cation and anion, respectively.
\( \hat{m}(k), \hat{m}(j,q) \) — subscript constraint stipulating that the number of moles of \( \text{H}_2\text{O} \) and each thermodynamic component indexed by \( k \) or the number of moles of \( \text{H}_2\text{O} \) and each aqueous species indexed by \( j \) and \( q \) are held constant.
\( \hat{m}_o, \hat{m}_j, \hat{m}_k \) — subscript constraints stipulating constant molalities of all aqueous species or thermodynamic components other than the \( q \)th, \( j \)th, or \( k \)th.
\( m_{t,j} \) — total molality of the \( j \)th ion defined by eq (23).
\( M \) — molality.
\( \ddot{M} \) — molecular weight.
\( n \) — index designating neutral aqueous species (\( n = 1, 2, \ldots \hat{n} \)) or a subscript designating the sum of the intrinsic and collapse properties of aqueous species.
\( n_{\text{sc}} \) — number of moles of \( \text{H}_2\text{O} \) in solution.
\( n^\ast_{\text{sc}} \) — number of moles of \( \text{H}_2\text{O} \) liter\(^{-1} \) of solution (eq 243).
\( \hat{n}(k) \) — subscript constraint stipulating that the number of moles of each solute component is held constant.
\( N \) — electrostatic Born function defined by eq (260).
\( N^\circ \) — Avogadro’s number (6.02252 \( \times \) 10\(^{23} \) molecules mole\(^{-1} \)).
\( \text{NaCl}^\circ \) — designation of the aqueous NaCl molecule.
\( P \) — pressure in bars or kilobars.
\( P_r \) — reference pressure (1 bar).
\( q \) — index designating aqueous complexes (\( q = 1, 2, \ldots \hat{q} \)).
\( Q \) — electrostatic Born function defined by eq (256).
\( r_{e,j}, r_{e,n} \) — effective electrostatic radius of the \( j \)th and \( n \)th aqueous species.
\( r_{i,j} \) — intrinsic radius of the \( j \)th aqueous species.
\( r_{x,j} \)  — crystallographic radius of the \( j \)th aqueous species.

\( R \)  — gas constant (1.9872 cal mole\(^{-1}\)(°K)\(^{-1}\)).

\( S \)  — entropy.

\( S_{\text{excess}} \)  — excess molal entropy of mixing.

\( S^o \)  — standard molal entropy.

\( S^o_{j}, S^o_{j,\text{abs}} \)  — conventional and absolute standard molal entropy of the \( j \)th aqueous species.

\( \bar{S}_{r,j} \)  — partial molal entropy.

\( \bar{S}_{r,j} \)  — sum of the intrinsic, collapse, and short-range interaction contributions to the relative partial molal entropy of the \( j \)th aqueous ion.

\( \bar{S} - S^o \)  — relative partial molal entropy.

\( S^o_{s,j} \)  — conventional standard partial molal intrinsic entropy of the \( j \)th aqueous species \((-\partial C^o_{s,j}/\partial T)_p\).

\( S^o_{j}, S^o_{j,\text{abs}} \)  — conventional and absolute standard partial molal entropy of the \( j \)th aqueous species.

\( \Delta S^o_{c,j} \)  — contribution by collapse of the local solvent structure to the conventional standard partial molal entropy of the \( j \)th aqueous species \((\Delta S^o_{c,j} = - (\partial \Delta C^o_{c,j}/\partial T)_p)\).

\( \Delta S^o_{n,j} \)  — sum of the intrinsic and collapse contributions to the conventional standard partial molal entropy of the \( j \)th aqueous species \((\Delta S^o_{n,j} = S^o_{i,j} + \Delta S^o_{c,j} = - (\partial \Delta C^o_{n,j}/\partial T)_p)\).

\( \Delta S^o_{s,j}, \Delta S^o_{s,j,\text{abs}} \)  — conventional and absolute standard partial molal entropy of solvation of the \( j \)th aqueous species.

\( \text{SAT, sat, saturation} \)  — designations of liquid-vapor equilibrium for aqueous systems.

\( \text{SAT (H}_2\text{O), Sat (H}_2\text{O), Saturation (H}_2\text{O) solution} \)  — designation of the liquid-vapor equilibrium curve for \( \text{H}_2\text{O}\).

\( \text{solute} \)  — subscript designating the solute of an electrolyte solution.

\( \text{solution} \)  — subscript designating an electrolyte solution.

\( \text{solvent} \)  — subscript designating the solvent of an electrolyte solution.

\( T \)  — temperature in °C or °K.

\( T_r \)  — reference temperature (298.15°K).

\( U \)  — electrostatic Born function defined by eq (259).

\( u^* \)  — ionic strength parameter defined by eq (192).

\( V \)  — volume.

\( V^o \)  — standard molal volume.

\( V^o_{j}, V^o_{j,\text{abs}} \)  — conventional and absolute standard molal volume of the \( j \)th aqueous species.

\( \bar{V} \)  — partial molal volume.

\( \bar{V} - V^o \)  — relative partial molal volume.
\( \nabla^0 \) — standard partial molal volume.
\( \nabla^0_{i,j} \) — conventional standard intrinsic partial molal volume of the \( j \)th aqueous species.
\( \nabla^0_{j, \text{abs}} \) — conventional and absolute standard partial molal volume of the \( j \)th aqueous species.
\( \nabla^0_w \) — standard partial molal volume of \( \text{H}_2\text{O} \).
\( \Delta \nabla^0_{c,j} \) — contribution by collapse of the local solvent structure to the conventional standard partial molal volume of the \( j \)th aqueous species.
\( \Delta \nabla^0_{n,j} \) — sum of the intrinsic and collapse contributions to the conventional standard partial molal volume of the \( j \)th aqueous species (\( \Delta \nabla^0_{n,j} = \nabla^0_{i,j} + \Delta \nabla^0_{c,j} = (\partial \Delta \Gamma^0_{n,j}/\partial P)_w \)).
\( \Delta \nabla^0_{r,q} \) — standard partial molal volume of overall dissociation for the \( q \)th aqueous complex.
\( \Delta \nabla^0_{s,j, \text{abs}} \) — conventional and absolute standard partial molal volume of solvation of the \( j \)th aqueous species.

\( \text{vacuum} \) — subscript designating a vacuum.
\( w \) — subscript designating \( \text{H}_2\text{O} \).
\( X \) — mole fraction.
\( X \) — electrostatic Born function defined by eq (266).
\( \gamma \) — subscript constraint stipulating constant \( \gamma_k \) for all values of \( k \).
\( \gamma_j, \gamma_k \) — stoichiometric ionic strength fractions defined by eqs (86) and (89).
\( \tilde{\gamma}_j, \tilde{\gamma}_k, \tilde{\gamma}_q \) — “true” ionic strength fractions defined by eqs (81), (88), and the analog of eq (81) for the \( q \)th aqueous complex.
\( \tilde{\gamma}_{i,j} \) — total “true” ionic strength fraction defined by eq (93).
\( Y \) — electrostatic Born function defined by eq (265).
\( Z \) — ionic charge.
\( Z_{c,n} \) — effective local charge on a neutral polar molecule.
\( Z_{v,T}, Z_{P,T} \) — negative reciprocal of the dielectric constant of \( \text{H}_2\text{O} \) (at the subscripted pressures and temperatures) in eqs (276), (277), and (293).
\( \alpha \) — coefficient of isobaric thermal expansion (\( \alpha = (\partial \ln V/\partial T)_P \)).
\( \alpha_j \) — degree of formation of the \( j \)th aqueous species defined by eq (70A).
\( \alpha^* \) — Debye-Hückel function defined by eq (232).
\( \beta \) — coefficient of isothermal compressibility (\( \beta = (\partial \ln \rho/\partial P)_T \)).
\( \beta_q \) — overall standard molal dissociation constant for the \( q \)th aqueous complex.
\( \bar{\beta}(\hat{\Delta}B, I^{1/2}) \) — Debye-Hückel ionic strength function defined by eq (233).

\( \beta^o_w \) — coefficient of isothermal compressibility of \( \text{H}_2\text{O} \) \( (\beta^o_w = (\partial \ln \rho^o_w / \partial P)_T) \).

\( \gamma \pm, k \) — stoichiometric mean activity coefficient of the \( k \)th electrolyte.

\( \gamma_i, \gamma_j, \gamma_l \) — stoichiometric activity coefficients of the \( i \)th, \( j \)th, and \( l \)th ions.

\( \bar{\gamma} \pm, k \) — mean ionic activity coefficient of the \( k \)th electrolyte.

\( \bar{\gamma}_i, \bar{\gamma}_j, \bar{\gamma}_l \) — individual ion activity coefficients of the subscripted species.

\( \bar{\gamma}_n \) — activity coefficient of the \( n \)th neutral aqueous species.

\( \bar{\gamma}_q \) — activity coefficient of the \( q \)th aqueous complex.

\( \bar{\gamma} \pm, k, \bar{\gamma}^o \pm, k \) — mean ionic activity coefficients of the \( k \)th electrolyte (eq 188).

\( \Gamma, \Gamma_\gamma, \Gamma_H, \Gamma_J, \Gamma_V \) — mole fraction/molality conversion functions defined by eqs (122), (169), and (209) through (211).

\( \delta_k, \delta^o_k \) — activity coefficient difference functions defined by eqs (188) and (189).

\( \delta_{\phi, k}, \delta_{C_p, \text{solution}} \) — finite difference uncertainties in eq (239).

\( \epsilon, \epsilon^o \) — dielectric constant of an electrolyte solution and \( \text{H}_2\text{O} \), respectively.

\( \bar{\xi}(\hat{\Delta}B, I^{1/2}) \) — Debye-Hückel ionic strength function defined by eq (234).

\( \eta \) — electrostatic parameter defined by eq (129).

\( \theta \) — “structural” temperature parameter.

\( \kappa \) — isothermal compressibility \((\kappa = - (\partial V / \partial P)_T)\).

\( \kappa^o \) — standard molal isothermal compressibility \( (\kappa^o = - (\partial V^o / \partial P)_T) \).

\( \kappa^o_j \) — conventional standard molal isothermal compressibility of the \( j \)th aqueous species \( (\kappa^o_j = - (\partial V^o_j / \partial P)_T) \).

\( \kappa^o_j^{\text{abs}} \) — absolute standard molal isothermal compressibility of the \( j \)th aqueous species \( (\kappa^o_j^{\text{abs}} = - (\partial V^o_j^{\text{abs}} / \partial P)_T) \).

\( \bar{\kappa} \) — partial molal isothermal compressibility \( (\bar{\kappa} = - (\partial V / \partial P)_T) \).

\( \bar{\kappa} - \bar{\kappa}^o \) — relative partial molal isothermal compressibility \((\bar{\kappa} - \bar{\kappa}^o = - ((\partial V / \partial P)_T - (\partial V^o / \partial P)_T)) \).

\( \bar{\kappa}^o \) — standard partial molal isothermal compressibility \( (\kappa^o = - (\partial V^o / \partial P)_T) \).

\( \bar{\kappa}^o_{i,j} \) — conventional standard intrinsic partial molal isothermal compressibility of the \( j \)th aqueous species \( (\bar{\kappa}^o_{i,j} = - (\partial V^o_{i,j} / \partial P)_T) \).
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\[ \bar{R}^0_j \] — conventional standard partial molal isothermal compressibility of the \( j \)th aqueous species \( (\bar{R}^0_j = -(\partial \bar{V}^0_j / \partial P)_T) \).

\[ \bar{R}^0_{j,\text{abs}} \] — absolute standard partial molal isothermal compressibility of the \( j \)th aqueous species \( (\bar{R}^0_{j,\text{abs}} = -(\partial \bar{V}^0_{j,\text{abs}} / \partial P)_T) \).

\[ \Delta \bar{R}^0_{c,j} \] — contribution by collapse of the local solvent structure to the conventional standard partial molal isothermal compressibility of the \( j \)th aqueous species \( (\Delta \bar{R}^0_{c,j} = -(\partial \Delta \bar{V}^0_{c,j} / \partial P)_T) \).

\[ \Delta \bar{R}^0_{\text{n},j} \] — sum of the intrinsic and collapse contributions to the standard partial molal isothermal compressibility of the \( j \)th aqueous species.

\[ \Delta \bar{R}^0_{s,j} \] — conventional standard partial molal isothermal compressibility of solvation of the \( j \)th aqueous species \( (\Delta \bar{R}^0_{s,j} = -(\partial \Delta \bar{V}^0_{s,j} / \partial P)_T) \).

\[ \Lambda, \Lambda_k \] — Debye-Hückel function defined by eq (121) and its analog for the \( k \)th electrolyte.

\[ \Lambda', \Lambda''', \Lambda'''_T \] — Debye-Hückel derivative functions defined by eqs (200) through (202).

\[ \mu, \mu_j, \mu_k, \mu_i, \mu_u, \mu_q \] — chemical potential of the subscripted species.

\[ \nu \] — stoichiometric number of moles of ions (mole of solute)\(^{-1}\) in an electrolyte solution.

\[ \nu_{i,k}, \nu_{j,k}, \nu_{j,k}, \nu_{l,k} \] — stoichiometric number of moles of the subscripted ion (mole of the \( k \)th component of an electrolyte solution)\(^{-1}\).

\[ \nu_{j,q}, \nu_{j,n} \] — stoichiometric number of moles of the \( j \)th ion in one mole of the \( q \)th or \( n \)th aqueous complex.

\[ \nu_k \] — stoichiometric number of moles of ions in one mole of the \( k \)th thermodynamic component of an electrolyte solution (eq 58).

\[ \bar{m} \] — designation of an extensive or molal property.

\[ \bar{m}^0 \] — designation of a standard molal property.

\[ \bar{m}^0_j, \bar{m}^0_{j,\text{abs}} \] — conventional and absolute standard molal properties of the \( j \)th aqueous species.

\[ \bar{m}^0_{\text{sc}} \] — standard molal property of \( \text{H}_2\text{O} \).

\[ \bar{m}_p \] — designation of a partial molal property.

\[ \bar{m}_q \] — long-range ionic interaction (Debye-Hückel) contribution to a relative partial molal property.

\[ \bar{m}_{\text{ideal}} \] — ideal mixing contribution to a relative partial molal property.

\[ \bar{m}_j \] — partial molal property of the \( j \)th aqueous species (eq 21).

\[ \bar{m}_q \] — partial molal property of the \( q \)th aqueous complex (eq 20).
\( \Xi_r \) — sum of the intrinsic, collapse, and short-range interaction contributions to a relative partial molal property.

\( \Xi_w \) — partial molal property of H\(_2\)O defined by eq (25).

\( \Xi - \Xi^\circ, \Delta \Xi \) — relative partial molal property.

\( \Xi^\circ \) — designation of a standard partial molal property.

\( \Xi^\circ_{i,j} \) — conventional standard intrinsic partial molal property of the \( j \)th aqueous species.

\( \Xi^\circ_{i,j}, \Xi^{abs}_{i,j} \) — conventional and absolute standard partial molal property of the \( j \)th aqueous species.

\( \Delta \Xi_h \) — solvation contribution to a relative partial molal property (eq 119).

\( \Delta \Xi^\circ_{c,j} \) — contribution by local collapse of the solvent structure to a conventional standard partial molal property of the \( j \)th aqueous species.

\( \Delta \Xi^\circ_{e,j} \) — electrostriction contribution to a conventional standard partial molal property of the \( j \)th aqueous species (eq 114).

\( \Delta \Xi^\circ_{f,j}, \Delta \Xi^{abs}_{f,j} \) — conventional and absolute standard partial molal property of formation of the \( j \)th aqueous species from its elements in their stable form at 298.15\(^\circ\)K and 1 bar.

\( \Delta \Xi^\circ_{j}, \Delta \Xi^{abs}_{j} \) — conventional and absolute apparent standard partial molal property of formation of the \( j \)th aqueous species.

\( \Delta \Xi^\circ_{n,j} \) — sum of the intrinsic and collapse contributions to a conventional standard partial molal property of the \( j \)th aqueous species (eq 115).

\( \Delta \Xi^\circ_{s,j} \) — conventional standard partial molal property of solvation of the \( j \)th aqueous species.

\( \rho \) — density.

\( \rho^\circ_w \) — density of H\(_2\)O.

\( \rho^*_{\gamma} \) — activity coefficient difference function defined by eq (177).

\( \rho^*_{H} \) — apparent molal enthalpy difference function defined by eq (235).

\( \rho^*_{\Delta H_d} \) — heat of dilution difference function defined by eq (237).

\( \rho^*_{\Delta H_s} \) — heat of solution difference function plotted in figures 60 through 64.

\( \rho^*_{j} \) — apparent molal heat capacity difference function defined by eq (238).

\( \rho^*_{e} \) — apparent molal compressibility difference function defined by eq (241).
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\[ \rho^* \phi \]  
— osmotic coefficient difference function defined by eq (194).

\[ \rho^* V \]  
— apparent molal volume difference function defined by eq (240).

\[ \sigma(\alpha B, I^{1/2}) \]  
— Debye-Hückel ionic strength function defined by eq (191).

\[ \phi \]  
— osmotic coefficient defined by eq (106).

\[ \phi_{CP} \]  
— apparent molal heat capacity.

\[ \phi_{E_x} \]  
— apparent molal expansibility.

\[ \phi_G \]  
— apparent molal Gibbs free energy.

\[ \phi_H \]  
— apparent molal enthalpy.

\[ \phi_k \]  
— apparent molal compressibility.

\[ \phi_Z \]  
— apparent molal property defined by eq (77).

\[ \phi_V \]  
— apparent molal volume.

\[ \psi_j, \psi_{n}, \psi_k \]  
— ionic charge parameters defined by eqs (82), (84), and (87).

\[ \omega_j, \omega_{n}^{abs}, \omega_{n}, \omega_{n}^{abs} \]  
— conventional and absolute Born coefficients for the jth and nth aqueous species (eqs 130, 135, 138, and \( \omega_n = \omega_n^{abs} \)).

\[ \omega_k \]  
— Born coefficient for the kth electrolyte (eqs 132 and 136).

RESUMÉ OF PREVIOUS WORK

Experimental observations of the thermodynamic behavior of aqueous electrolytes at high pressures and temperatures have increased exponentially since the turn of the century when Noyes (1907) pioneered systematic conductance measurements of electrolyte solutions at temperatures ranging up to 305°C. With a few notable exceptions (Noyes, Kato, and Sosman, 1910; Spillner, 1940; Swinnerton and Owen, 1948; Fogo, Benson, and Copeland, 1954) little was done over the next 50 years to augment substantially these high-temperature measurements. Attention was focused instead on extending early experimental studies of the conductance of electrolytes as a function of pressure at low temperatures (Colladon and Sturm, 1827; Herwig, 1877; Fink, 1885; Fanjung, 1894) to higher pressures (Adams, 1931; Adams and Hall, 1931; Zisman, 1932; Buchanan and Hamann, 1953; Hamann and Strauss, 1955, 1956). The relative hiatus in high-temperature conductance measurements in the first half of the century was brought to an end by Franck’s (1956a, b, and c) outstanding advances in experimental technology, which sparked a myriad of systematic and comprehensive high pressure/temperature conductance studies. As a result, extensive conductance data (and in many cases derived dissociation constants) are now available for H₂O (for example, see Marshall and Franck, 1981) and a large number of electrolytes at temperatures and pressures to 1000°C and 12 kb.¹ The systems studied include aqueous solutions of HCl, HF, HBr, NaCl, NaI, NaBr, NaF, KCl,

¹ In the case of H₂O, conductance measurements and derived values of \( K_w \), the activity product of H₂O, have been published for temperatures and pressures to 1000°C and > 130 kb (Holzapfel and Franck, 1966; Hamann, 1974).

The conductance measurements summarized above have been complemented over the past 40 yrs by numerous high pressure/temperature equilibrium studies of aqueous CO₂, KI, LiF, KCl, NaCl, PbCl₂, Li₂SO₄, and K₂CO₃ solutions, as well as a large number of other aqueous systems (Kracek, 1931; Gibson, 1934; Benedict, 1939; Keevil, 1942; Booth and Bidwell, 1950; Morey and Hesselgesser, 1951, 1952; Morey and Chen, 1956; Fedotyev and Shlepov, 1958; Tödheide and Franck, 1963; Takeiouchi and Kennedy, 1964; Gehrig, Lentz, and Franck, 1979). In addition, over the past decade or so systematic and extensive solubility measurements have been carried out in systems involving H₂O and NaCl, KCl, HCl, CO₂, CaCO₃, NaOH, PbCl₂, CaSO₄, CaSO₄·2H₂O, MgSO₄, MgSO₄·H₂O, BaSO₄, AgCl and NaCl, AgSO₄, Fe₂O₃, UO₂, SiO₂, copper and iron sulfides, calcium and magnesian silicates, and/or other components, many of which are definitive with respect to ion association at high pressures and/or temperatures (Sourirajan and Kennedy, 1962; Urosova and Ravick, 1971; Ellis, 1959a; Ellis and Golding, 1963; Barnes and Ernst, 1963; Nriagu and Anderson, 1971; Marshall, 1967, 1980a and b; Marshall, Slusher, and Jones, 1964; Marshall and Jones, 1966; Marshall and Slusher, 1966, 1968, 1973, 1975a and b; Yeatts and Marshall, 1972b; Kalyanaraman, Yeatts, and Marshall, 1973a and b; Stoughton and Lietzke, 1960; Lietzke and Stoughton, 1959a, b, c, d, and e, 1962a, b, and c, 1963; Templeton, 1960; Templeton and Rodgers, 1967; Sweeton and Baes, 1970; Chou and

With few exceptions, high-temperature osmotic and activity coefficients, as well as relative partial molal enthalpies and heat capacities derived from vapor pressure measurements, emf studies, isopiestic data, or mineral solubilities are available only at temperatures \( \leq 300^\circ C \) at pressures corresponding to those along the vapor-liquid or vapor-liquid-solid equilibrium curves for aqueous electrolyte systems. A large number of electrolytes have been considered in these studies, including HCl, NaCl, HCl-NaCl, CaCl₂-NaCl, HBr, DCl, KCl-HCl, HBr-NaBr, LiCl, CsCl, KCl, RbCl, NiCl₂, CoCl₂, LiBr, LiNO₃, LiNO₂-KNO₃, LiNO₂-CsNO₃, Ca(NO₃)₂, Ca(NO₃)₂-CsNO₃, CaSO₄-Na₂SO₄-NaNO₂, MgCl₂, CaCl₂, BaCl₂, MgSO₄, Na₂SO₄, Ca(OH)₂, HCl-NaCl, and sea salt solutions (Bates and Bower, 1954; Harned, 1959, 1960; Harned and Geary, 1937; Harned and Paxton, 1953; Patterson, Gilpatrick, and Soldano, 1960; Soldano and Patterson, 1962; Humphries, Kohrt, and Patterson, 1968; Moore, Humphries, and Patterson, 1972; Soldano and Meek, 1963; Soldano and Bien, 1966; Greeley and others, 1960a and b; Towns, Greeley, and Lietzke, 1960; Gardner, Jones, and De Nordwall, 1963; Gardner, 1969; Faita, Mussini, and Oggioni, 1966; Fabuss and Korosi, 1966; Cerquetti, Longhi, and Mussini, 1968; Yeatts and Marshall, 1967, 1969; Liu and Lindsay, 1970, 1971, 1972; Lindsay and Liu, 1968, 1971; Braunstein and Braunstein, 1971; Gibbard and Scatchard, 1972, 1973; Urusova, 1971, 1974; Gibbard and others, 1974; Lietzke, Hupf, and Stoughton, 1965; Stoughton and Lietzke, 1965, 1967; Lietzke and Stoughton, 1962a, b, and c, 1963b, 1964a and b, 1974; Mashovets, Zarembo, and Federov, 1973; Holmes and Mesmer, 1981; Holmes, Baes, and Mesmer, 1978, 1979, 1981). Emphasis in calorimetric investigation of electrolyte solutions has been placed on heat capacities and heats of mixing, dilution, and solution, but for the most part only at temperatures \( \leq 300^\circ C \) along the vapor-liquid equilibrium curves for the various solutions. Data of this kind are available for H₂O, NaCl, LiBr, CaCl₂, SrCl₂, MgCl₂, HCl, NaOH, NaF, NaI, NaBr, LiCl, KCl, KF, CsCl, CsI, KBr, KI, LiI, LiF, RbCl, NH₄Cl,
$\text{NH}_2\text{NO}_3$, RbF, Rbl, RbBr, CsBr, CsF, BaCl$_2$, CuCl$_2$, CoCl$_2$, FeCl$_2$, Cd(NO$_3$)$_2$, NiCl$_2$, Cu(ClO$_4$)$_2$, Mg(ClO$_4$)$_2$, Mn(ClO$_4$)$_2$, H$_3$PO$_4$, NaClO$_4$, H$_2$SO$_4$, K$_2$SO$_4$, Li$_2$SO$_4$, MgSO$_4$, Na$_2$SO$_4$, Na$_2$CO$_3$, NaHSO$_4$, NaHCO$_3$, NaReO$_4$, HReO$_4$, CsI, GdCl$_3$, K$_2$SO$_4$, KHS, Na$_2$SiO$_3$, Na$_2$S$_2$O$_3$, K$_2$S$_2$O$_8$, K$_2$Cr$_2$O$_7$, Na$_2$MoO$_4$, Na$_2$WO$_4$, NaClO$_3$, NaNO$_3$, NaBrO$_3$, NaIO$_3$, KClO$_3$, KBrO$_3$, KIO$_3$, NH$_2$ClO$_4$, AgNO$_3$, AgClO$_4$, KMnO$_4$, MnCl$_2$, CH$_3$COOH, and sea salt solutions (Eigen and Wicke, 1951, 1954; Wicke, Eigen, and Ackermann, 1954; Ackermann, 1958; Ackermann and Schreiner, 1958; Egan, Luff, and Wakefield, 1958; Wakefield, Luff, and Reed, 1972; Mastroianni, ms; Readnour and Cobble, 1969; Ahluwalia and Cobble, 1964a and b; Mitchell and Cobble, 1964; Jekel, Criss, and Cobble, 1964; Cobble and others, 1972; Criss and Cobble, 1961; Gardner, Mitchell, and Cobble, 1969a and b; Stephens and Cobble, 1971; Gardner, Jekel, and Cobble, 1969; Sen, Murray, and Cobble, ms; Cobble and Murray, 1977; Rütgerjans and others, 1969; Leung and Grunwald, 1970; Likke, ms; Likke and Bromley, 1973; Bromley, 1968, 1972; Bromley and others, 1970; Singh and Bromley, 1973; Ensor and Anderson, 1973; Snipes, Manly, and Ensor, 1975; Millero, 1973a; Millero, Hansen, and Hoff, 1973; Millero, Perron, and Desnoyers, 1973; Fortier, Leduc, and Desnoyers, 1974; Fortier, Philip, and Desnoyers, 1974; Leung and Millero, 1975a and b; Borodenko and Galinker, 1975, 1976; Puchkov, Styazhkin, and Feodorov, 1976, 1978; Puchkov and Zarembo, 1978; Messikomer and Wood, 1975; Desnoyers and others, 1976; Perron, Fortier, and Desnoyers, 1975; Perron, Desnoyers, and Millero, 1974, 1975; Olofsson, G., 1975; Olofsson, I. V., 1979; Olofsson and Hepler, 1975; Olofsson and Olofsson, 1973, 1977; Olofsson and Sunner, 1979; Olofsson, Spitzer, and Hepler, 1978; Singh and others, 1976, 1977, 1978; Roux and others, 1978; Liphard, Jost, and Schneider, 1977; Spitzer and others, 1978a and b, 1979a and b; Tanner and Lamb, 1978; Bernarducci, Morss, and Mikszial, 1979; Kasper, Holloway, and Navrotsky, 1979; Rogers, ms; Allred and Wooley, 1981a and b). Densities, sound speeds, compressibilities, and expansibilities of electrolyte solutions have also been measured at temperatures $\leq 200^\circ$C along vapor-liquid equilibrium curves or at 20 bars, but relatively few such studies have been carried out at higher temperatures and/or pressures. Data of this kind are available for aqueous solutions of NaCl, NaBr, NaF, HCl, LiCl, KCl, CsCl, RbCl, NH$_4$Cl, NaBr, NH$_4$Br, LiBr, KBr, KF, LiI, NaI, NH$_4$I, KI, NaHCO$_3$, KNO$_3$, AgNO$_3$, LiNO$_3$, NaNO$_3$, NH$_4$NO$_3$, NaHS, NH$_4$ClO$_4$, H$_2$O, D$_2$O, MgCl$_2$, CaCl$_2$, SrCl$_2$, BaCl$_2$, Na$_2$SO$_4$, MgSO$_4$, K$_2$SO$_4$, NaOH, and Na$_2$SO$_4$ (Adams, 1981; Adams and Hall, 1931; Gibson, 1934, 1935, 1938; Gibson and Loeffler, 1941; Copeland, Silverman, and Benson, 1953; Allam, ms; Allam and Lee, 1964, 1966; Ellis, 1966, 1967, 1968; Ellis and McFadden, 1968, 1972; Desnoyers and others, 1969; Rowe and Chou, 1970; Haas, 1970; Bell, Helton, and Rogers, 1970; Dunn, 1966, 1968, 1974; Millero, 1968, 1970, 1972a and b, 1973b and c, 1977; Millero, ms; Millero, Hoff, and Kahn, 1972; Millero, Knox, and Emmet, 1972; Millero and Drost-Hansen, 1968a and b; Millero and Lepple, 1973; Millero and Masterton, 1974; Millero and Kubinski, 1975; Millero, Gon-

Perhaps because of the phenomenal increase over the past two decades in the incidence, diversity, scope, and reliability of experimental investigations in the field of high pressure/temperature solution chemistry, theoretical interpretation of the myriad of data resulting from these studies has yet to be integrated into a unified, rigorous, and comprehensive frame of reference. Most equations advanced to account for observed changes in one or another thermodynamic property of electrolytes are based to a large extent on extrathermodynamic assumptions and/or semiempirical arguments, many of which fail to take account of related changes in other properties that are inconsistent with the equations. For example, the Guggenheim (1935) and Redlich-Meyer (Redlich and Meyer, 1964) equations have been used extensively to extrapolate experimental calorimetric and volumetric data to infinite dilution, despite the fact that the two equations are incompatible with one another. Few of the equations so far proposed provide adequately for both electrostatic and nonelectrostatic contributions to the thermodynamic behavior of electrolytes as a function of temperature, pressure, and composition. As a consequence, most such equations are restricted to a given property and/or region of pressure/temperature/composition space.

Activity coefficients, osmotic coefficients, and the relative partial molal properties of aqueous electrolytes at low pressures and temperatures have been represented over the past 50 yrs by a myriad of empirical and theoretical modifications and extensions of Debye-Hückel theory. Among these, the first-degree extension of the Debye-Hückel equation proposed originally by Hückel (1925), the Guntelberg (1926), Guggenheim (Guggenheim, 1935; Guggenheim and Turgeon, 1955), Davies (1938), and Scatchard (1936) equations (which are modifications of the equation proposed by Hückel), and the empirical power functions of stoichiometric ionic strength adopted by Lietzke and Stoughton (1962c, 1974) have received the most attention. These equations, as well as various modifications proposed by Meissner and Tester (1972), Meissner, Kusik, and Tester (1972), Meissner and Kusik (1972), and Bromley (1972, 1973) have been used extensively to represent activity and osmotic coefficients in solutions of various ionic strengths at temperatures to 300°C (Robinson and Stokes, 1959; Harned and Owen, 1958; Lewis and Randall, 1961; Lietzke and Stoughton, 1959a, b, c, d, and e, 1962a, b, and c, 1974; Greeley and others, 1960a and b; Towns, Greeley, and Lietzke, 1960; Marshall, Slusher, and
Jones, 1964; Marshall and Slusher, 1966; Marshall, 1967; Mesmer, Baes, and Sweeton, 1970; Guggenheim and Stokes, 1969; Helgeson and James, 1968; Helgeson, 1969; Wood, 1972, 1975, 1976). Owing to differences in the concentration and temperature dependence of ion association, solvation, and long- and short-range interaction in different electrolytes, various truncations of the Lietzke-Stoughton power function were used in many of these studies. The fallibility of this approach is manifested by the fact that in certain cases, one, or more, of the empirical parameters in the truncated expressions varies with concentration and/or temperature, but not in others.

Solvation models such as those adopted by Stokes and Robinson (1948, 1973) and Gluekauf (1955) afford close approximation of activity coefficients in concentrated electrolytes, but the equations are not generally applicable, nor are they as easy to use as their empirical counterparts. This is especially true of many of the more recent theoretical equations that have been advanced, most of which are comprehensive and accurate, but also intricate, complicated, and unwieldy. The more elaborate of these models include the statistical mechanical approaches taken by Friedman (1962, 1972a, b, and c), Justice and Justice (1976, 1977), Justice (1978), and Justice and Ebeling (1979), the series of equations proposed by Scatchard (1961, 1968, 1969, 1976), Wu, Rush, and Scatchard (1968, 1969), and Scatchard, Rush, and Johnson (1970), who include third and fourth virial coefficients to account for short-range interaction among all ions in solution, and those adopted by Reilly and Wood (1969), Reilly, Wood, and Robinson (1971), Robinson, Wood, and Reilly (1971), Pitzer (1973, 1975, 1977, 1979), Pitzer and Mayorga (1973, 1974), Pitzer and Kim (1974), Pitzer, Roy, and Silvester (1977), Pitzer and Silvester (1976), Silvester and Pitzer (1977, 1978), Pitzer, Peterson, and Silvester (1978), and Pitzer and others (1979). Of these, the equations derived by Pitzer and his co-workers, which include modification of the second virial coefficient to provide for changes in short-range interaction as a function of ionic strength, are the simplest and most practical. Nevertheless, even these equations impose a considerable computational burden, involve two or more adjustable parameters which must be obtained by regression of experimental data, and (like most other empirical and theoretical extensions and/or revisions of Debye-Hückel theory) fail to take explicit account of ionic association. As a consequence, they are not suitable for extrapolation of low-temperature/pressure data to higher pressures and temperatures, which restricts their practical application to reproducing and interpolating experimental data.

Many semiempirical equations have been proposed to describe the temperature and pressure dependence of the thermodynamic properties of aqueous species and dissociational reactions in the standard state. One of the more prominent of these takes advantage of the principle of corresponding states (Criss and Cobble, 1964a and b; Cobble, 1964, 1966a and b; Khodakovskiy, 1969; Ellis, 1968; Helgeson, 1967, 1969) by representing the absolute standard partial molal entropies, volumes, and heat
capacities of aqueous species at a given temperature and pressure as linear functions of the corresponding property at another temperature and pressure. Despite its success and obvious generality, the requirement for extensive experimental data at the temperature and pressure of interest to establish values of correlation coefficients restricts severely the utilitarian value of the corresponding states approach.

The disadvantages of the corresponding states algorithm have to some extent been overcome by application of semiempirical electrostatic/nonelectrostatic models and continuum theory to describe standard partial molal properties of ionization and hydration at high temperatures (Helgeson, 1967; Goldman and Bates, 1972; Goldman and Morss, 1975; Tremaine and Goldman, 1978; Cobble and Murray, 1977; Sen, Murray, and Cobble, ms). However, provision in these models for nonelectrostatic contributions to the thermodynamic properties of aqueous species is largely empirical and commonly inadequate and inconsistent with experimental data. For example, Cobble and Murray (1977) and Sen, Murray, and Cobble (ms) consider the nonelectrostatic statistical and structural contributions to the standard partial molal entropy of ion hydration to be independent of temperature, which requires the standard partial molal heat capacities of aqueous species to be entirely electrostatic. Although experimental heat capacity data indicate clearly that this is not the case, the model nevertheless affords reasonably close approximations of apparent standard partial molal Gibbs free energies of formation at temperatures $\leq 200^\circ$C. Similar observations apply to the other models adduced above.

Despite their shortcomings, approaches of the kind proposed by Cobble and Murray (1977) and Sen, Murray, and Cobble (ms), as well as other models which take explicit account of electrostatic, structural, and statistical contributions to the thermodynamic properties of aqueous electrolytes offer considerable improvement over the consequences of many assumptions commonly made in high temperature solution chemistry. One of the more widely adopted of these is that the standard partial molal heat capacities of reactions involving aqueous species can be regarded as zero or taken to be independent of temperature, which in more cases than not leads to serious error in thermodynamic predictions (Helgeson, 1964, 1967, 1969; Naumov, Ryzhenko, and Khodakovskiy, 1968; Ryzhenko, 1981). Khodakovskiy, Ryzhenko, and Naumov (1968) attempted to improve on these assumptions by assuming instead that the conventional standard partial molal heat capacities of aqueous ions are proportional to temperature, which contravenes electrostatic theory and is clearly inconsistent with experimental heat capacity data. Errors in apparent standard partial molal Gibbs free energies of formation computed on the basis of this assumption are claimed by Khodakovskiy, Ryzhenko, and Naumov to be $\leq 1$ kcal mole$^{-1}$ at temperatures $\leq 300^\circ$C. It is interesting to note in this regard that the standard partial molal Gibbs free energies of reaction for many equilibria in hydrothermal systems are also of this order of magnitude.
As a rule, least squares fits of Gibbs free energies are insensitive to the form of the temperature function used to represent the heat capacity, which corresponds to the negative product of temperature and the second partial derivative of the regression equation with respect to temperature at constant pressure and composition. As a consequence, many models with fundamental flaws suffice to represent Gibbs free energy data. In a relatively recent publication, Naumov, Ryzhenko, and Khodakovskiy (1971) adopted for aqueous species the standard partial molal heat capacity power function employed by Maier and Kelley (1932) for solids. Although the Maier-Kelley power function is empirical, it closely approximates the temperature dependence of the heat capacities of many minerals at temperatures > 25°C, and its success can be justified at least in part on theoretical grounds. In contrast, there is no theoretical justification for application of the Maier-Kelley power function to aqueous species, and the form of the function is glaringly inconsistent with experimental observations of the standard partial molal heat capacities of aqueous electrolytes as a function of temperature.

A number of theoretical, empirical, and semiempirical equations have been proposed to represent the pressure dependence of various thermodynamic properties of aqueous electrolytes. One of the oldest and most successful of these is the Gibson-Tait equation (Gibson, 1934, 1935, 1938), which is based on Tamman’s internal pressure theory (Tamman, 1893, 1895) and Tait’s empirical observations on the H.M.S. Challenger (Tait, 1889). Owen and Brinkley (1941) employed the Gibson-Tait approach to formulate an equation representing the pressure dependence of dissociation constants. Various modifications of the Tait (1889) equation have been used extensively to represent pressure-volume-temperature data for aqueous solutions (for example, Li, 1967; Hayward, 1967; Wang and Millero, 1973; Emmet and Millero, 1974; Chen and Millero, 1976). An alternative to the Gibson-Tait/Owen-Brinkley equation, which is unwieldy, has been proposed by Lown, Thirsk, and Lord Wynne-Jones (1968, 1970) and adopted by Millero, Hoff, and Kahn (1972), Ward and Millero (1974b) and others. Lown, Thirsk, and Lord Wynne-Jones assume the standard partial molal compressibility of dissociation to be independent of pressure, which, at least in the case of acetic acid, introduces only slight errors in computed equilibrium constants at high pressures and low temperatures. Another approach, and one that has received considerable attention in recent years, has been taken by Quist and Marshall (1968a and e), Marshall (1968, 1969, 1970, 1972, 1975), Marshall and Mesmer (1981), Dunn and Marshall (1969), North (1973), and others who advocate the concept of a “complete” dissociation constant, which takes explicit account of ion solvation. The model is based on Franck’s (1956a, b, and c, 1961) early observation of the isothermal quasilinear relation between the logarithm of the density of H₂O and the logarithm of the dissociation constants of aqueous species at high pressures and temperatures. Like the Gibson-Tait/Owen-Brinkley equation and the model adopted by Lown, Thirsk, and Lord Wynne-Jones, the “complete” equi-
librium constant approach advocated by Marshall and his co-workers reproduces experimental dissociation constant data over reasonably wide ranges of pressure and temperature. However, all three approaches suffer from extrathermodynamic assumptions which contravene electrostatic theory and cannot be generalized (see discussions by Hamann, 1974; Matheson, 1969; Gilkerson, 1970; Helgeson and Kirkham, 1976).

Strictly electrostatic models of ion dissociation fail to represent adequately the thermodynamics of dissociational reactions over wide ranges of pressure and temperature. Nevertheless, with the aid of judicious assumptions they can be used to approximate the apparent standard partial molal Gibbs free energies of formation of aqueous electrolytes over restricted temperature ranges. Various attempts (for example, Ryzhenko, ms and 1981) have been made to fit high pressure/temperature standard partial molal Gibbs free energies of dissociation to the Born (1920) equation (as corrected by Bjerrum, 1929), which affords satisfactory representation of dissociation constants only at high temperatures and low pressures where the dielectric constant of H₂O is small. In other regions of pressure/temperature space, nonelectrostatic statistical and structural contributions to the ionization process lead to serious discrepancies between standard partial molal Gibbs free energies of dissociation computed from the Born-Bjerrum equation and those obtained experimentally (Helgeson and Kirkham, 1976).


REVIEW OF THERMODYNAMIC RELATIONS

The extensive thermodynamic properties of electrolyte solutions are expressed in classical thermodynamics as the sum of the products of the concentrations and corresponding partial molal properties of the thermodynamic components of the solution (see footnote 3). The classical ap-
proach thus provides implicitly rather than explicitly for the thermodynamic consequences of ion association.

Experimental data for dissociational equilibria in electrolyte solutions suggest that cations and anions in these solutions associate to an increasing degree with increasing temperature and decreasing pressure. Although most electrolytes are almost completely dissociated at 25°C and 1 bar, they may thus become highly associated under supercritical conditions. As a consequence, unless explicit provision is made for ion association, regression parameters obtained by fitting theoretical equations to low-pressure/temperature data cannot be used with confidence to predict the thermodynamic behavior of electrolyte solutions at high pressures and temperatures. Such provision can be incorporated in the framework of classical solution chemistry by first expressing the Gibbs free energies of electrolytes in terms of the chemical potentials of the cations and anions in solution.

Partial molal properties of aqueous species.—Homogeneous equilibrium in an electrolyte solution at constant pressure and temperature requires

$$
\tilde{\Xi}_q = \sum_j n_{j,q} \tilde{\Xi}_j
$$

where \( n_{j,q} \) refers to the number of moles of the \( j \)th cation or anion in one mole of the \( q \)th complex in solution, and \( \tilde{\Xi}_q \) and \( \tilde{\Xi}_j \) represent a given partial molal property of the subscripted complex and ion, respectively, which are defined by

$$
\tilde{\Xi}_q = \left( \frac{\partial \Xi}{\partial m_q} \right)_{P,T,\tilde{\mu}_q,n_w}
$$

and

$$
\tilde{\Xi}_j = \left( \frac{\partial \Xi}{\partial m_j} \right)_{P,T,\tilde{\mu}_j,n_w}
$$

where \( m_q \) and \( m_j \) stand for the molalities of the \( q \)th and \( j \)th aqueous species, the subscripts \( \tilde{\mu}_q \) and \( \tilde{\mu}_j \) designate constant molalities of all aqueous species other than the \( q \)th or \( j \)th, respectively, \( n_w \) refers to the number of moles of \( \text{H}_2\text{O} \) in solution, and \( \Xi \) corresponds to the extensive analog for the solution of the partial molal property represented by \( \tilde{\Xi}_q \) and \( \tilde{\Xi}_j \). It thus follows that

$$
\sum_j m_j \tilde{\Xi}_j + \sum_q m_q \tilde{\Xi}_q = \sum_j m_{t,j} \tilde{\Xi}_j
$$

where \( m_{t,j} \) stands for the total molality of the \( j \)th ion, which is given by

$$
m_{t,j} = m_j + \sum_q n_{j,q} m_q = \sum_k n_{j,k} m_k
$$

Note that specifying \( \tilde{\mu}_j, \tilde{\mu}_q, \) or \( \tilde{\mu}_w \) does not necessarily require \( n_w \) to be constant, and that the subscripts \( j \) and \( q \) refer to solvated species.
where $\nu_{j,k}$ refers to the number of moles of the $j$th cation or anion in one mole of the $k$th solute component, and $m_k$ represents the molality of the component in solution. Any extensive thermodynamic property of an electrolyte solution containing 1 kg of H$_2$O can thus be expressed as

$$\Xi = 55.51 \bar{\Xi}_w + \sum_j m_{t,j} \bar{\Xi}_j = 55.51 \bar{\Xi}_w + \sum_j \sum_k \nu_{j,k} m_k \bar{\Xi}_j$$

(24)

where 55.51 corresponds to the number of moles of H$_2$O (kg H$_2$O)$^{-1}$ in solution, and $\bar{\Xi}_w$ denotes the partial molal counterpart of $\Xi$ for the solvent; that is,

$$\bar{\Xi}_w = \left( \frac{\partial \Xi}{\partial n_w} \right)_{P,T,\bar{n}(k)},$$

(25)

where $n_w$ again refers to the number of moles of H$_2$O in solution, and the subscript $\bar{n}(k)$ signifies that the number of moles of each of the thermodynamic components of the solute is held constant. The classical analog of eq (24) can be written as

$$\Xi = 55.51 \bar{\Xi}_w + \sum_k m_k \bar{\Xi}_k$$

(26)

where

$$\bar{\Xi}_k = \left( \frac{\partial \Xi}{\partial m_k} \right)_{P,T,\bar{m}_k,n_w}$$

(27)

where the subscript $\bar{m}_k$ refers to constant molalities of all components of the solute other than the $k$th. Eqs (24) and (26) can now be combined to give

$$\sum_k m_k \bar{\Xi}_k = \sum_j \sum_k \nu_{j,k} m_k \bar{\Xi}_j = \sum_j m_{t,j} \bar{\Xi}_j$$

(28)

which relates explicitly the partial molal properties of thermodynamic components to those of their dissociated counterparts. Note that because eq (28) is independent of the number of components, it also follows that

$$\bar{\Xi}_k = \sum_j \nu_{j,k} \bar{\Xi}_j$$

(29)

and

$$d\bar{\Xi}_k = \sum_j \nu_{j,k} d\bar{\Xi}_j.$$
\[ dG = - S dT + V dP + \sum_k \varpi_k d\tilde{n}_k \] (31)

where \( G, S, \) and \( V \) stand for the Gibbs free energy, entropy, and volume of the solution, and \( \varpi_k \) represents the partial molal Gibbs free energy (chemical potential) of the \( k \)th solute component, which is defined by an appropriate statement of eq (27). Because eq (31) is an exact differential, it follows that eq (27) can also be written for \( \bar{V}_k \) and \( \bar{S}_k \) as

\[ \bar{V}_k = \left( \frac{\partial V}{\partial \tilde{n}_k} \right)_{P, T, \tilde{m}_k, n_w} = \left( \frac{\partial \varpi_k}{\partial P} \right)_{T, \tilde{m}_k} \] (32)

and

\[ \bar{S}_k = \left( \frac{\partial S}{\partial \tilde{n}_k} \right)_{P, T, \tilde{m}_k, n_w} = - \left( \frac{\partial \varpi_k}{\partial T} \right)_{T, \tilde{m}_k} \] (33)

where \( \bar{V}_k \) and \( \bar{S}_k \) stand for the partial molal volume and entropy of the \( k \)th solute component and the subscript \( \tilde{m}_k \) stipulates that the number of moles of \( H_2O \) and each of the thermodynamic components of the solution is held constant. It follows from eq (29) that

\[ \varpi_k = \sum_j \nu_{j,k} \varpi_j, \] (34)

which can be differentiated to give

\[ \left( \frac{\partial \varpi_k}{\partial P} \right)_{T, \tilde{m}_k} = \sum_j \nu_{j,k} \left( \frac{\partial \varpi_j}{\partial P} \right)_{T, \tilde{m}_k} \] (35)

and

\[ \left( \frac{\partial \varpi_k}{\partial T} \right)_{P, \tilde{m}_k} = \sum_j \nu_{j,k} \left( \frac{\partial \varpi_j}{\partial T} \right)_{P, \tilde{m}_k} \] . (36)

Combining eqs (32) and (33), respectively, with eqs (35) and (36) then leads to

\[ \bar{V}_k = \sum_j \nu_{j,k} \left( \frac{\partial \varpi_j}{\partial P} \right)_{T, \tilde{m}_k} \] (37)

and

\[ \bar{S}_k = - \sum_j \nu_{j,k} \left( \frac{\partial \varpi_j}{\partial T} \right)_{P, \tilde{m}_k} \] . (38)

If we now write statements of eq (29) as

\[ \bar{V}_k = \sum_j \nu_{j,k} \bar{V}_j \] (39)

and

\[ \bar{S}_k = \sum_j \nu_{j,k} \bar{S}_j \] , (40)
it follows from eqs (21) and (37) through (40), together with the fact that all these equations are independent of the number of aqueous species present in solution that

\[ V_j = \left( \frac{\partial V}{\partial m_j} \right)_{p,T,n_j,n_w} = \left( \frac{\partial \mathcal{G}_j}{\partial P} \right)_{p,T,n_j(k)} \]  

(41)

and

\[ S_j = \left( \frac{\partial S}{\partial m_j} \right)_{p,T,n_j,n_w} = -\left( \frac{\partial \mathcal{G}_j}{\partial T} \right)_{p,T,n_j(k)}, \]  

(42)

which describe the relation between the partial molal volume and entropy of the jth aqueous ion and the partial derivatives of the partial molal Gibbs free energy of the ion with respect to pressure and temperature, respectively, at constant composition. Note that by taking account of eqs (19), (20), (41), and (42) we can also write

\[ V_q = \sum_j \nu_{j,q} \left( \frac{\partial \mathcal{G}_j}{\partial P} \right)_{T,n_j(k)} \]  

(43)

and

\[ S_q = \sum_j \nu_{j,q} \left( \frac{\partial \mathcal{G}_j}{\partial T} \right)_{p,T,n_j(k)}. \]  

(44)

These equations, together with many other relations among partial molal properties of individual aqueous species and thermodynamic components are summarized in app. A. All the equations given in app. A can be generated from the expressions derived above.

The stipulation represented by the subscript \( n_j(k) \) in eqs (41) through (44) arises from constraints imposed by homogeneous equilibrium. It should perhaps be emphasized that these constraints are not inherent in

\[ dG = -SdT + VdP + \sum_j \mathcal{C}_j dm_j + \sum_q \mathcal{C}_q dm_q, \]  

(45)

which expresses the change in the Gibbs free energy of the solution at constant \( n_w \) as an explicit function of the changes in the molalities of all the aqueous species in solution. In contrast to eq (31), eq (45) is an exact differential only if homogeneous equilibrium is not maintained. Otherwise, \( m_j \) and \( m_q \) are not independent variables, and the cross partial derivatives of eq (45) are not necessarily equal. For example, if the jth ion occurs in the qth complex, and homogeneous equilibrium is maintained, it follows that

\[ S_j = \left( \frac{\partial S}{\partial m_j} \right)_{p,T,n_j,n_w} = -\left( \frac{\partial \mathcal{G}_j}{\partial T} \right)_{p,T,n_j(q)} = 0 \]  

(46)

and

\[ V_j = \left( \frac{\partial V}{\partial m_j} \right)_{p,T,n_j,n_w} \neq \left( \frac{\partial \mathcal{G}_j}{\partial P} \right)_{T,n_j(q)} = 0 \]  

(47)
where the subscript \( \overset{*}{m}(j,q) \) stipulates that the number of moles of \( \text{H}_2\text{O} \) and each of the \( j \)th and \( q \)th aqueous species are held constant.\(^6\) The inequalities in these expressions arise from the fact that \((\partial \xi_j / \partial T)_{P,\overset{*}{m}}(k)\) and \((\partial \xi_j / \partial P)_{T,\overset{*}{m}}(k)\) are not necessarily equal to \((\partial \xi_j / \partial T)_{P,\overset{*}{m}(j,q)}\) and \((\partial \xi_j / \partial P)_{T,\overset{*}{m}(j,q)}\), respectively. However, it follows from eq (19) that we can write

\[
\sum_j \xi_j \, dm_j = \sum_j \sum_q \xi_j \nu_{j,q} \, dm_q .
\]

Hence

\[
\sum_j \xi_j \, dm_j + \sum_q \xi_q \, dm_q = \sum_j \xi_{j,q} \, dm_{j,q} .
\]

where \( dm_{j,q} \) is given by the derivative of eq (23), which can be written as

\[
dm_{j,q} = dm_j + \sum_q \nu_{j,q} \, dm_q = \nu_{j,k} \, dm_k .
\]

Consequently, eq (45) can be expressed as

\[
dG = -SdT + VdP + \sum_j \xi_j \, dm_{j,q}
\]

or

\[
dG = -SdT + VdP + \sum_j \sum_k \nu_{j,k} \xi_j \, dm_k ,
\]

which can be combined with an appropriate statement of eq (29) to give eq (31). Hence, in contrast to the effect of homogeneous equilibrium on eq (45), eq (51) and (52) are exact differentials under all conditions.

Eqs (51) and (52), which correspond to analogs of eq (31), take explicit account of constraints imposed by homogeneous equilibrium. Note that the three equations can be combined with eq (45) to give

\[
\sum_k \xi_k \, dm_k = \sum_j \xi_j \, dm_{j,q} = \sum_j \sum_k \nu_{j,k} \xi_j \, dm_k
\]

\[
= \sum_j \xi_j \, dm_j + \sum_q \xi_q \, dm_q
\]

in which the right side of the last identity is inexact if homogeneous equilibrium is maintained.

Activity/concentration relations.—The activity of the \( k \)th solute component \((a_k)\) of an electrolyte solution is related to the chemical potential of the component \((\xi_k)\) by

\[
\xi_k = \xi^\circ_k + RT \ln a_k
\]

\(^*\)Note that stipulating \( \overset{*}{m}(j,q) \) also imposes implicitly the constraint represented by \( \overset{*}{m}(k) \) but not vice versa.
where \( R \) stands for the gas constant and \( T \) the temperature in \(^\circ\)K. However, we can also write

\[
\varphi_j = \varphi_j^{\text{obs}} + RT \ln a_j
\]  

(55)

where \( a_j \) represents the activity of the \( j \)th ion in solution, which is related to the total concentration of the ion \((m_{i,j})\) by

\[
a_j = \gamma_j m_{i,j} = \gamma_j \sum_k \nu_{j,k} m_k
\]  

(56)

where \( \gamma_j \) stands for the stoichiometric activity coefficient of the subscripted ion. The stoichiometric mean activity coefficient of the \( k \)th component \((\gamma_{\pm,k})\) is defined by

\[
\gamma_{\pm,k} = \left( \prod_j \gamma_j^\nu_{j,k} \right)^{1/\nu_k}
\]  

(57)

where

\[
\nu_k = \sum_j \nu_{j,k}.
\]  

(58)

Combining eqs (54) through (57) with alternate statements of eq (29) for \( \varphi_k \) and \( \varphi_k^{\text{obs}} \) leads to

\[
a_k = \prod_j a_j^\nu_{j,k} = \prod_j (\gamma_j m_{i,j})^\nu_{j,k} = \prod_j \gamma_j^\nu_{j,k} \left( \sum_k \nu_{j,k} m_k \right)^{\nu_{j,k}}
\]

\[
= \gamma_{\pm,k} \nu_k \prod_j m_{i,j}^\nu_{j,k} = \gamma_{\pm,k} \nu_k \prod_j \left( \sum_k \nu_{j,k} m_k \right)^{\nu_{j,k}}.
\]  

(59)

Hence, eqs (54) and (55) can also be written as

\[
\varphi_k = \varphi_k^{\text{obs}} + RT \left( \nu_k \ln \gamma_{\pm,k} + \sum_j \nu_{j,k} \ln \left( \sum_k \nu_{j,k} m_k \right) \right)
\]  

(60)

and

\[
\varphi_j = \varphi_j^{\text{obs}} + RT (\ln \gamma_j + \ln m_{i,j}).
\]  

(61)

For single electrolytes, or mixed electrolytes in which none of the solute components shares a common ion, eqs (59) and (60) reduce to

\[
a_k = (\gamma_{\pm,k} m_k)^{\nu_k} \prod_i \nu_{j,k}^{\nu_{j,k}}
\]  

(61A)

and

\[
\varphi_k = \varphi_k^{\text{obs}} + RT \left( \nu_k \ln (\gamma_{\pm,k} m_k) + \sum_i \nu_{j,k} \ln \nu_{j,k} \right).
\]  

(62)
The nonstoichiometric analogs of eqs (56) and (57) appear as

$$a_j = \bar{\gamma}_j m_j$$  \hspace{1cm} (63)

and

$$\bar{\gamma}_{\pm,k} = \left( \prod_j \bar{\gamma}_j^{\nu_j,k} \right)^{1/\nu_k}$$  \hspace{1cm} (64)

where $\bar{\gamma}_j$ stands for the individual ion activity coefficient of the $j$th ion and $\bar{\gamma}_{\pm,k}$ represents the mean ionic activity coefficient of the $k$th component. The overbar appears in these symbols to distinguish the individual and mean ionic activity coefficients from their stoichiometric counterparts. The activity of the $q$th complex in solution can be expressed as

$$a_q = \bar{\gamma}_q m_q$$  \hspace{1cm} (65)

where $\bar{\gamma}_q$ refers to the activity coefficient of the complex.

Eq (63) permits the partial molal Gibbs free energy of the $j$th ion to be expressed as

$$\bar{\mathcal{G}}_j = \mathcal{G}_j^{\text{o, abs}} + RT \ln (\bar{\gamma}_j m_j)$$  \hspace{1cm} (66)

which can be combined with eq (61) to give

$$\bar{\gamma}_j m_j = \gamma_j m_{t,j} = \gamma_j \sum_k \nu_{j,k} m_k$$  \hspace{1cm} (67)

Note that it also follows from eqs (57), (59), (64), and (67) that

$$\gamma_{\pm,k}^{\nu_k} = \prod_j (\bar{\gamma}_j \alpha_j)^{\nu_j,k} = \bar{\gamma}_{\pm,k}^{\nu_k} \prod_j \alpha_j^{\nu_j,k}$$  \hspace{1cm} (68)

and

$$a_k = \prod_j a_j^{\nu_j,k} = \prod_j (\bar{\gamma}_j m_j)^{\nu_j,k} = \bar{\gamma}_{\pm,k}^{\nu_k} \prod_j m_j^{\nu_j,k}$$

$$= \bar{\gamma}_{\pm,k}^{\nu_k} \prod_j \left( \alpha_j \sum_k \nu_{j,k} m_k \right)^{\nu_j,k}$$  \hspace{1cm} (69)

which permit eq (60) to be written as

$$\bar{\mathcal{G}}_k = \mathcal{G}_k^{\text{o}} + RT \left( \nu_k \ln \bar{\gamma}_{\pm,k} + \sum_j \nu_{j,k} \ln m_j \right)$$

$$= \mathcal{G}_k^{\text{o}} + RT \left( \nu_k \ln \bar{\gamma}_{\pm,k} + \sum_j \nu_{j,k} \left( \ln \alpha_j + \ln \left( \sum_k \nu_{j,k} m_k \right) \right) \right)$$  \hspace{1cm} (70)

where $\alpha_j$ (the degree of formation of the $j$th ion) is given by

$$\alpha_j \equiv m_j / m_{t,j}$$  \hspace{1cm} (70A)
Hence, as \( \alpha_j \to 1 \), \( \gamma_j \to \gamma \), which requires \( \gamma_{\pm,k} \to \gamma_{\pm,k} \) if \( \alpha_j \to 1 \) for all finite values of \( \nu_{j,k} \). Note that eqs (69) and (70) can be written for single electrolytes or mixed electrolytes in which none of the solute components shares a common ion as

\[
a_k = \gamma_{\pm,k}^{\nu_k} m_k^{\nu_k} \prod_j \left( \nu_{j,k} \alpha_j \right)^{\nu_{j,k}}
\]

and

\[
\mathcal{T}_k = \mathcal{G}_k^{a_k} + \nu_k RT \ln(\gamma_{\pm,k} m_k) + RT \sum_j \nu_{j,k} \ln(\nu_{j,k} \alpha_j).
\]

**Relative partial molal properties.**—The equations summarized above permit calculation of the relative partial molal Gibbs free energies and stoichiometric mean activity coefficients of the thermodynamic components of electrolyte solutions from expressions of \( \gamma_j \) as a function of the concentrations of the aqueous species in solution. All other partial molal properties of the components can be computed by taking the partial derivatives of \( \ln \gamma_j \) or \( \ln \gamma_j \) with respect to temperature or pressure, holding \( n_w \) and all \( m_k \) constant. Note that it follows from eq (50) that:

\[
\left( \frac{\partial m_j}{\partial P} \right)_{T,\hat{n}(k)} = -\sum_q \nu_{j,q} \left( \frac{\partial m_q}{\partial P} \right)_{T,\hat{n}(k)}
\]

and

\[
\left( \frac{\partial m_j}{\partial T} \right)_{P,\hat{n}(k)} = -\sum_q \nu_{j,q} \left( \frac{\partial m_q}{\partial T} \right)_{P,\hat{n}(k)}.
\]

Similarly, taking account of eq (49) we can write

\[
\sum_j \xi_j \left( \frac{\partial m_j}{\partial P} \right)_{T,\hat{n}(k)} = -\sum_q \xi_{q} \left( \frac{\partial m_q}{\partial P} \right)_{T,\hat{n}(k)}
\]

and

\[
\sum_j \xi_j \left( \frac{\partial m_j}{\partial T} \right)_{P,\hat{n}(k)} = -\sum_q \xi_{q} \left( \frac{\partial m_q}{\partial T} \right)_{P,\hat{n}(k)}.
\]

Equations relating the relative partial molal properties of thermodynamic components and ions to changes in activity coefficients and the distribution of species in solution are summarized in app. B. Calculation of the partial derivatives of \( m_j \) in these equations requires dissociation constants and standard partial molal properties of dissociation for the complexes in solution. Partial derivatives of the law of mass action, material balance, and activity coefficient equations can then be combined to compute (\( \partial \ln m_j/\partial P \))\(_{T,\hat{n}(k)}\), (\( \partial^2 \ln m_j/\partial P^2 \))\(_{T,\hat{n}(k)}\), (\( \partial (\partial \ln m_j/\partial P) \))\(_{T,\hat{n}(k)}\), (\( \partial^2 (\partial \ln m_j/\partial P) \))\(_{T,\hat{n}(k)}\), (\( \partial (\partial \ln m_j/\partial P) \))\(_{T,\hat{n}(k)}\), and (\( \partial^2 (\partial \ln m_j/\partial P) \))\(_{T,\hat{n}(k)}\) (see below).

*Note that the constraint imposed by specifying \( n_w \) and all \( m_k \) constant (\( \hat{n}(k) \)) requires all \( m_{-j} \) to be constant but not necessarily all \( m_j \) and \( m_q \).*
Apparent molal properties and the partial molal properties of the solvent.—Apparent molal properties of electrolytes are given by

$$\phi_z = \frac{\Xi - 55.51 \bar{\Xi}^\circ_w}{\sum_k m_k}$$  \hspace{1cm} (77)

where $\phi_z$ stands for a given apparent molal property, and $\bar{\Xi}^\circ_w$ represents the corresponding standard partial molal property of the solvent. Combining eqs (24), (26), and (77) leads to

$$55.51 (\Xi_w - \bar{\Xi}^\circ_w) = \phi_z \sum_k m_k - \sum_k m_k \bar{\Xi}_k = \phi_z \sum_k m_k$$

$$- \sum_j \sum_k \nu_{j,k} m_k \bar{\Xi}_j = \phi_z \sum_k m_k - \sum_j m_{i,j} \bar{\Xi}_j , \hspace{1cm} (78)$$

which relates the relative partial molal property of the solvent, the corresponding apparent molal property of the solute, and the partial molal properties of the components of the solute to those of the cations and anions in solution. Note that it also follows from eq (77) that eq (29) can be written as

$$\bar{\Xi}_k = \sum_j \nu_{j,k} \bar{\Xi}_j = \phi_z + \left( \frac{\partial \phi_z}{\partial m_k} \right)_{P,T,\hat{m}_k,\bar{m}_w} \sum_k m_k = \left( \frac{\partial (\phi_z \sum_k m_k)}{\partial m_k} \right)_{P,T,\hat{m}_k,\bar{m}_w} . \hspace{1cm} (79)$$

Hence, at constant pressure and temperature

$$\phi_z = \frac{1}{\sum_k m_k} \int_0^{m_k} \bar{\Xi}_k \, dm_k . \hspace{1cm} (80)$$

For convenience in later discussion, the "true" (or effective (Johnson and Pytkowicz, 1979)) ionic strength fraction of the $j$th ion ($\tilde{\gamma}_j$) can be defined as

$$\tilde{\gamma}_j = \frac{\psi_j m_j}{I} \hspace{1cm} (81)$$

where

$$\psi_j = \frac{Z_j^2}{2} \hspace{1cm} (82)$$

and

$$I = \sum_j \psi_j m_j + \sum_q \psi_q m_q \hspace{1cm} (83)$$
where $Z_j$ denotes the charge on the $j$th cation or anion and

$$\psi_q \equiv \frac{Z_q^2}{2} \quad (84)$$

where $Z_q$ stands for the charge on the $q$th complex in solution. The overbars on the symbols $I$ and $\bar{y}_j$ serve to distinguish "true" ionic strength and "true" ionic strength fractions from their stoichiometric counterparts ($I$ and $y_j$), which are given by

$$I = \sum_j \psi_j m_{1,j} = \sum_k \psi_k m_k \quad (85)$$

and

$$y_j = \frac{\psi_j m_{1,j}}{I} = \frac{\psi_j \sum_k \nu_{j,k} m_k}{I} \quad (86)$$

where

$$\psi_k \equiv \frac{\sum_j \nu_{j,k} Z_j^2}{2} = \sum_j \nu_{j,k} \psi_j \quad (87)$$

The "true" and stoichiometric ionic strength fractions of the $k$th solute component ($\bar{y}_k$ and $y_k$, respectively) are given by

$$\bar{y}_k = \frac{\sum_j \nu_{j,k} \psi_j m_k}{I} = \frac{\psi_k m_k}{I} \quad (88)$$

and

$$y_k = \frac{\psi_k m_k}{I} \quad (89)$$

It thus follows from eqs (86) and (89) that

$$y_j = \psi_j \sum_k \frac{\nu_{j,k} y_k}{\psi_k} \quad (90)$$

and we can write

$$\sum_j y_j = \sum_k y_k = \sum_j \bar{y}_j + \sum_q \bar{y}_q = 1 \quad (91)$$

where $\bar{y}_q$ is the analog of $\bar{y}_j$ for the $q$th complex in solution. In contrast,

$$\sum_k \bar{y}_k = \frac{1}{I} \quad (92)$$
Another convenient fraction, \( \hat{y}_{t,j} \), can be defined as
\[
\hat{y}_{t,j} = \frac{\psi_j m_{t,j}}{I} = \psi_j \frac{\sum_k \nu_{j,k} m_k}{I} = \psi_j \frac{\sum_k \nu_{j,k} \hat{y}_k}{\psi_k} = \frac{\gamma_j I}{I} = \gamma_j \sum_k \hat{y}_k
\]
so that
\[
\sum_j \hat{y}_{t,j} = \sum_k \hat{y}_k = \frac{I}{I}.
\]
(93)
(94)

Note that holding all \( \hat{y}_j \) and \( \hat{y}_q \) constant (indicated below by the subscript \( \hat{y} \)) implicitly requires all \( \hat{y}_{t,j} \) and \( \hat{y}_k \) to be constant as well, which is not true in reverse.

Taking account of the definitions summarized above, eqs (24), (77), and (78) can be expressed as
\[
\Xi = 55.51 \Xi_w + \sum_j \frac{\hat{y}_{t,j} \Xi_j I}{\psi_j} = 55.51 \Xi_w + \sum_j \sum_k \nu_{j,k} \hat{y}_k \frac{\Xi_j I}{\psi_k},
\]
(95)
\[
\phi_{\Xi} = \frac{\Xi - 55.51 \Xi^0_w}{\Xi^0_w},
\]
(96)

and
\[
55.51 (\Xi_w - \Xi^0_w) = \phi_{\Xi} \sum_k \frac{\hat{y}_k I}{\psi_k} - \sum_k \frac{\hat{y}_k \Xi_k I}{\psi_k} = \phi_{\Xi} \sum_k \frac{\hat{y}_k I}{\psi_k} - \sum_j \frac{\hat{y}_{t,j} \Xi_j I}{\psi_j}.
\]
(97)

Let us now employ the subscript \( \hat{y} \) to indicate constant \( \hat{y}_k \) for all values of \( k \) and differentiate eq (96) with respect to \( I \) at constant temperature, pressure, and \( \hat{y}_k \), which leads to
\[
\left( \frac{\partial \Xi}{\partial I} \right)_{P,T,\hat{y}} = \left( \phi_{\Xi} + I \left( \frac{\partial \phi_{\Xi}}{\partial I} \right)_{P,T,\hat{y}} \right) \sum_k \frac{\hat{y}_k}{\psi_k}.
\]
(98)

Note that eq (77) can also be written as
\[
\left( \frac{\partial \Xi}{\partial I} \right)_{P,T,\hat{y}} = \left( \frac{\partial (\phi_{\Xi} I)}{\partial I} \right)_{P,T,\hat{y}} \sum_k \frac{\hat{y}_k}{\psi_k}
\]
(99)

and expressed in terms of \( \Xi_j \) by taking account of the following isothermal-isobaric identity for constant \( n_w \):
\[
d\Xi = \sum_k \Xi_k dm_k = \sum_j \sum_k \nu_{j,k} \Xi_j dm_k = \sum_j \Xi_j dm_{t,j} = \sum_j \Xi_j dm_j + \sum_q \Xi_q dm_q
\]
(100)
Dividing eq (100) by dI and imposing constant pressure, temperature, and the constraint represented by $\gamma$ leads to

$$\left( \frac{\partial \Xi}{\partial I} \right)_{P,T,\gamma} = \sum_j \sum_k \nu_{j,k} \frac{\tilde{y}_k \tilde{z}_j}{\psi_k} = \sum_j \frac{\tilde{y}_{t,j} \tilde{z}_j}{\psi_j}$$

(101)

which can be combined with eq (99) to give

$$\left( \frac{\partial (\phi I)}{\partial I} \right)_{P,T,\gamma} = \frac{1}{\sum_k (\tilde{y}_k/\psi_k)} \sum_j \sum_k \nu_{j,k} \frac{\tilde{y}_k \tilde{z}_j}{\psi_k} = \frac{1}{\sum_k (\tilde{y}_k/\psi_k)} \sum_j \frac{\tilde{y}_{t,j} \tilde{z}_j}{\psi_j}$$

(102)

Integrating eq (102) at constant pressure, temperature, and $\tilde{y}_k$ for all values of $k$ then leads to

$$\phi = \frac{1}{I} \sum_k (\tilde{y}_k/\psi_k) \sum_j \int_0^I \frac{\nu_{j,k} \tilde{y}_k \tilde{z}_j}{\psi_k} \, dI = \frac{1}{I} \sum_k (\tilde{y}_k/\psi_k) \sum_j \int_0^I \frac{\tilde{y}_{t,j} \tilde{z}_j}{\psi_j} \, dI$$

(103)

If we now substitute eq (103) in eq (97) we can write

$$\xi_w - \xi^o_w = \frac{1}{55.51} \sum_j \sum_k \left( \left( \int_0^I \frac{\nu_{j,k} \tilde{y}_k \tilde{z}_j}{\psi_k} \, dI \right) - \frac{\nu_{j,k} \tilde{y}_k \tilde{z}_j I}{\psi_k} \right)$$

$$= \frac{1}{55.51} \sum_j \left( \left( \int_0^I \frac{\tilde{y}_{t,j} \tilde{z}_j}{\psi_j} \, dI \right) - \frac{\tilde{y}_{t,j} \tilde{z}_j I}{\psi_j} \right)$$

(104)

which corresponds to the integrated Gibbs-Duhem equation at constant pressure, temperature, and $\tilde{y}_k$. Eq (104) permits calculation of the relative partial molal properties of the solvent from those of the ions in solution, expressed as functions of “true” ionic strength. Note that if $y_k$ and I are substituted for $\tilde{y}_k$ and I in eq (104), the expression reduces to

$$\xi_w - \xi^o_w = \frac{1}{55.51} \sum_k \left( \left( \int_0^I \frac{y_k \tilde{z}_k}{\psi_k} \, dI \right) - \frac{y_k \tilde{z}_k I}{\psi_k} \right)$$

(105)

The osmotic coefficient of an electrolyte solution ($\phi$) is related to the activity of the solvent ($a_w$) by

$$\phi = - \frac{55.51 \ln a_w}{m^*}$$

(106)

where $m^*$ stands for the sum of the molalities of all the aqueous species in solution: that is,

$$m^* = \sum_j m_j + \sum_q m_q$$

(107)
For completely dissociated single electrolytes, eq (107) reduces to

\[ m^* = v_k m_k. \]  

(108)

Because,

\[ \Gamma_w - \Gamma^0_w = RT \ln a_w \]

(109)
it follows that

\[ \phi = -\frac{55.51 (\Gamma_w - \Gamma^0_w)}{RT m^*}. \]

(110)

Taking account of eqs (77), (97), (104), and (110), we can thus write

\[
\phi = -\frac{1}{RT m^*} \phi_G \sum_k \frac{\bar{y}_k I}{\psi_k} - \sum_j \sum_k \frac{\nu_{j,k} \bar{y}_k \Gamma_j I}{\psi_k}
\]

\[ = -\frac{1}{RT m^*} \phi_G \sum_k \frac{\bar{y}_k I}{\psi_k} - \sum_j \frac{\bar{y}_{t,j} \Gamma_j I}{\psi_j}
\]

\[ = -\frac{1}{RT m^*} \sum_j \sum_k \left( \int_{0}^{1} \frac{\nu_{j,k} \bar{y}_k \Gamma_j I}{\psi_k} \, dI \right) - \frac{\nu_{j,k} \bar{y}_k \Gamma_j I}{\psi_k}
\]

\[ = -\frac{1}{RT m^*} \sum_j \left( \int_{0}^{1} \frac{\bar{y}_{t,j} \Gamma_j I}{\psi_j} \, dI \right) - \frac{\bar{y}_{t,j} \Gamma_j I}{\psi_j}
\]

(111)

which defines the relation between the osmotic coefficient, the apparent molal Gibbs free energy of the solute ($\phi_G$), and the partial molal Gibbs free energies of the cations and anions in solution as a function of "true" ionic strength. Note that by taking account of eq (105) we can also write

\[
\phi = -\frac{1}{RT m^*} \sum_k \left( \int_{0}^{1} \frac{y_k \Gamma_k}{\psi_k} \, dI \right) - \frac{y_k \Gamma_k I}{\psi_k}
\]

(112)

By expressing $\bar{z}_j$ as a function of $I$ and computing values of $\bar{y}_k$ or $\bar{y}_{t,j}$ (see below), all apparent molal properties of the solute as well as the osmotic coefficient and the partial molal properties of the solvent can be computed from the equations summarized above.

**THEORETICAL CONCEPTS**

The thermodynamic properties of an aqueous electrolyte solution can be expressed in terms of the structural and electrostatic consequences of dissolving an electrolyte in H$_2$O. In addition to ideal mixing and contributions by the solvent, the conceptual model adopted in the present study takes account of (1) the intrinsic properties of the electrolyte itself, which are independent of the solvent, (2) cavity formation, electrostriction collapse, and local disruption of the solvent structure by the solute, (3) orientation of the disordered H$_2$O dipoles to form primary and (more
loosely held) secondary solvation shells about the ions, (4) long and short-range interaction of the solvated ions with one another, and (5) ion association. Although all these processes are interdependent and none is strictly separable in either a physical or thermodynamic sense, regression calculations (Helgeson and Kirkham, 1976) together with theoretical and empirical considerations discussed below indicate that each can be represented adequately by separate functions of pressure, temperature, and concentration.

Because contributions by ionic interaction to the thermodynamic properties of an electrolyte are negligible at infinite dilution, the standard partial molal properties of aqueous species depend only on their intrinsic characteristics and the calorimetric and volumetric consequences of cavity formation, electrostriction collapse, and ion solvation. Accordingly, if we regard cavity formation as part of the electrostriction process we can write

\[
\Xi^o_{i,j} = \Xi^o_{i,j} + \Delta \Xi^o_{c,j},
\]

where \(\Xi^o_{i,j}\) represents the conventional standard partial molal heat capacity, entropy, enthalpy, internal energy, Gibbs free energy, Helmholtz free energy, volume, expansibility, or compressibility of the \(j\)th aqueous ion, and the subscripts \(i\) and \(e\) denote the intrinsic and electrostriction contributions to these properties. The electrostriction term in eq (113) is given by

\[
\Delta \Xi^o_{c,j} = \Delta \Xi^o_{c,j} + \Delta \Xi^o_{s,j}
\]

and for convenience in later discussion let us define

\[
\Delta \Xi^o_{n,j} = \Xi^o_{i,j} + \Delta \Xi^o_{c,j}
\]

where the subscripts \(c\), \(s\), and \(n\) refer to the collapse, solvation, and the sum of the intrinsic and collapse contributions to the standard partial molal property, respectively. Note that cavity formation is included implicitly in \(\Delta \Xi^o_{c,j}\). Combining eqs (113) through (115) leads to

\[
\Xi^o_{i,j} = \Xi^o_{i,j} + \Delta \Xi^o_{c,j} + \Delta \Xi^o_{s,j} = \Delta \Xi^o_{n,j} + \Delta \Xi^o_{s,j}.
\]

Analogous statements of eqs (113) through (115) can be written for the standard partial molal properties of electrolytes as well as for the absolute standard partial molal properties of aqueous species, which are related to their conventional counterparts by appropriate statements of eq (17) and its analogs for \(\Xi^o_{i,j}, \Delta \Xi^o_{c,j}, \Delta \Xi^o_{n,j}\), and \(\Delta \Xi^o_{s,j}\).

Before considering the pressure and temperature dependence of the intrinsic, collapse, and solvation contributions to the standard partial molal properties of aqueous electrolytes, let us first examine the relative partial molal properties of aqueous species in terms of the theoretical model summarized above.

Any relative partial molal property of the \(j\)th ion at a given pressure and temperature \((\Delta \Xi^o_{i,j})\) can be expressed as the sum of (1) an ideal mixing term \((\Xi^o_{\text{ideal},i,j})\), (2) an electrostatic contribution representing long-range
ionic interaction described by Debye-Hückel theory \( \bar{\Xi}_{d,j} \), (3) a nonstandard state analog of the electrostatic and nonelectrostatic terms in eq (116) to provide for the concentration dependence of ion solvation \( \Delta \bar{\Xi}_{h,j} \) and the sum of the intrinsic and collapse contributions to the thermodynamic behavior of the solute, and (4) a short-range interaction term to account for the consequences of collisions among solvated ions. If we let \( \bar{\Xi}_{r,j} \) represent the intrinsic, collapse, and short-range interaction contributions to \( \Delta \bar{\Xi}_j \), we can thus write

\[
\Delta \bar{\Xi}_j = \bar{\Xi}_{\text{ideal},j} + \bar{\Xi}_{d,j} + \Delta \bar{\Xi}_{h,j} + \bar{\Xi}_{r,j}
\]  

(117)

which is consistent with

\[
\Delta \bar{\Xi}_j = \bar{\Xi}_j - \bar{\Xi}_j^{\text{obs}}
\]  

(118)

and

\[
\Delta \bar{\Xi}_{h,j} = \Delta \bar{\Xi}_{s,j} - \Delta \bar{\Xi}_j^{\text{obs}}
\]  

(119)

where \( \Delta \bar{\Xi}_{s,j} \) represents the contribution by electrostriction solvation to the partial molal property of the subscripted ion, and \( \Delta \bar{\Xi}_j^{\text{obs}} \) stands for the corresponding contribution in the standard state.* The short-range interaction term \( \bar{\Xi}_{r,j} \) in eq (117) modifies the solvation term \( \Delta \bar{\Xi}_{h,j} \), and together they are intended to provide for all departures from ideality which are not accounted for by Debye-Hückel theory. These include changes in the dielectric constant of the solution with increasing ionic strength, polarization of solvated ions, differences in the bulk dielectric constant of the solution and that in the immediate vicinity of the ions, discontinuities in the dielectric medium arising from void spaces and disorder in the solvation shells, changes in solvation with increasing ionic strength, and short-range interaction of the solvated ions. These factors contribute negligibly to departures from ideality in dilute solutions. However, at high ionic strengths they dominate the thermodynamic behavior of electrolytes.

The Debye-Hückel term in eq (117) can be taken to represent electrostatic long-range interaction among idealized solvated ions (Robinson and Stokes, 1959). Consequently, it dominates the thermodynamic behavior of electrolytes only at low concentrations. To ensure adequate provision in the model for the major contributions by ion solvation and short-range interaction to departures from ideality at high concentrations, equations representing \( \Delta \bar{\Xi}_{h,j} \) and \( \bar{\Xi}_{r,j} \) as functions of temperature, pressure, and composition are derived below from theoretical and semiempirical considerations. Other less important factors contributing to the thermodynamic behavior of electrolytes at high concentrations (see above) are then incorporated implicitly in the coefficients of the equations by regression of experimental data.

Relative Partial Molal Gibbs Free Energy

Because abundant, accurate, and precise experimental data for aqueous electrolytes are available at low pressures and temperatures where

*Although eqs (113) through (119) are written for the \( j \)th ion, analogous expressions can be written for any aqueous species.
most alkali and alkaline earth halides, as well as many other dissolved salts are "completely" dissociated, the dependence of activity coefficients, osmotic coefficients, and the relative partial molal properties of electrolytes on "true" ionic strength is well known at 25°C and 1 bar. Nevertheless, general agreement has yet to be reached on a satisfactory theoretical equation to represent these data beyond the range of concentration where Debye-Hückel theory suffices to describe departures from ideality.

Despite Bjerrum's (1926, 1929) early consideration of the effects of ion association on the thermodynamic behavior of electrolytes, few of the many extensions of the Debye-Hückel equation that have been proposed over the past 50 years contain explicit provision for complexing. The importance of including such provision is underscored by the fact that electrostatic theory requires I and not I to be used in the Debye-Hückel equation, which is almost certainly responsible for the observation (Guggenheim and Stokes, 1969) that the thermodynamic behavior of electrolytes consisting of divalent cations and anions that associate to a high degree to form species such as MgSO₄, ZnSO₄, CuSO₄, CaSO₄, and other similar complexes apparently contravenes the Debye-Hückel limiting law.

The effect of ion association on stoichiometric activity coefficients reinforces negative contributions by ionic interaction and opposes the positive consequences of ion solvation. Because complexing in electrolyte solutions increases dramatically with increasing temperature, failure to provide explicitly for its consequences may lead to calculated stoichiometric activity coefficients at high temperatures which are several order of magnitude larger than the actual values. Attempts to take account implicitly of ion association by including provision for its effects solely in short-range interaction terms leads to second, third, or even higher degree extensions of the Debye-Hückel equation (for example, see Pitzer and Mayorga, 1974) and a series of regression parameters (virial coefficients) which cannot be generalized or used with confidence to extrapolate the data from which they are generated. Regardless of the theoretical arguments responsible for the extended terms in such equations, the functional form of the expressions is sufficiently flexible, and they commonly contain enough parameters to fit almost any configuration of activity coefficient data. In such cases, "best fit" criteria are no more meaningful with respect to the validity of the theoretical origin of the equations or the physical significance of the fit parameters than they are in regression calculations with strictly empirical power functions. For example, equations derived by (1) Stokes and Robinson (1948) from hydration theory, (2) Ruff (1977, 1979) and Pytkowicz and Johnson (1979) from a lattice-like model of electrolyte solutions, (3) Pauckhurst and Macaskill (1976a and b) from provision for interaction of like-charged ions in Guggenheim's theory of specific interaction, and (4) Pytkowicz and Kester (1969) and Johnson and Pytkowicz (1978, 1979) from an ion association model all yield adequate fits of experimental data. Consequently, none of the fits neither supports nor contravenes the theoretical basis for any of the equations, all of which are capable of overfitting the data. The latter observa-
tion underscores the fact that all the models yield derivative functions which fail to account adequately for all pertinent calorimetric and volumetric data with parameters obtained by regression of activity coefficients.

Virial expansions and second and third-degree extensions of Debye-Hückel theory are commonly employed in solution chemistry to represent departures from ideality in "completely" dissociated electrolytes, notwithstanding the fact that a first-degree extension affords close approximation of experimental activity coefficients for many such electrolytes at high ionic strengths (Helgeson and James, 1968; Helgeson, 1969; Wood, ms and 1975). In most cases, higher order extensions of the Debye-Hückel equation yield improved accuracy over their first-degree counterparts only to the extent of a few percent or less at "true" ionic strengths ranging up to \( \sim 6 \, \text{m} \), depending on the electrolyte (see below). It can be argued from a pragmatic point of view that such a small increase in accuracy hardly justifies practical application of complicated theoretical equations, such as those derived by Mayer (1950), Scatchard (1961, 1968, 1969), Scatchard, Rush, and Johnson (1970), Wu, Rush, and Scatchard (1968, 1969), Friedman (1972a, b, and c), Reilly, Wood, and Robinson (1971), Robinson, Wood, and Reilly (1971), Pitzer (1973, 1975, 1979), Pitzer and Mayorga (1973, 1974), Pitzer and Kim (1974), Justice and Justice (1976, 1977), Justice (1978), and Justice and Ebeling (1979). This certainly is the case in geochemistry, where refinements of the order of 0.1 or less in the logarithm of an activity coefficient are much smaller than uncertainties in the logarithms of most equilibrium constants employed in geochemical calculations.

Regression of apparent molal heat capacities, enthalpies, and volumes of electrolytes with appropriate equations derived from second and third-degree extensions of the Debye-Hückel equation almost invariably lead to overfits of the data. Owing to the interdependence of the fit parameters in extended forms of the Debye-Hückel equation and the relatively large magnification of experimental uncertainties in apparent molal properties, most calorimetric and density data are incapable of yielding "unique" values for more than one adjustable parameter in regression calculations. The interdependence of the fit parameters in these extensions (Helgeson and James, 1968) also commonly leads to close fits of activity coefficients, regardless of whether the electrolytes are "completely" or incompletely dissociated. Ambiguities in the physical significance of the fit parameters or virial coefficients generated by such calculations preclude extrapolation of the parameters to higher temperatures or general application of the regression functions, despite the validity of any or all of the theoretical concepts responsible for the extended terms in the equations.

As a consequence of the observations summarized above, a first degree extension of Debye-Hückel theory was chosen in the present study to represent activity coefficients, osmotic coefficients, and the partial molal properties of aqueous species as a function of "true" ionic strength. The theoretical concepts, assumptions, and approximations involved in the derivation of the equations are summarized below, beginning with the
Debye-Hückel term representing the thermodynamic consequences of long-range electrostatic interaction among solvated ions.

**Long-range ionic interaction.**—Despite the fact that aqueous electrolytes are not actually composed of hard, spherical, unpolarizable, and incompressible ions in a homogeneous dielectric continuum, the Debye-Hückel equation closely represents activity coefficients in many dilute (< 0.01 m) electrolyte solutions. Although the various assumptions and approximations implicit in the Debye-Hückel equation are clearly invalid at high ionic strengths, the equation is nevertheless invoked to account for long-range interaction among solvated ions in nearly all semi-empirical and theoretical expressions describing the dependence of activity and osmotic coefficients on ionic strength at high concentrations.9

The Debye-Hückel equation for \( \Gamma_{a,j} \) can be written for the molality scale of concentration as \(^5\)

\[
\Gamma_{a,j} = \frac{\psi_j \Lambda \Gamma_{j}^{1/2}}{\Lambda} + \Gamma_a
\]

(120)

where \( \psi_j \) is defined by eq (82) and

\[
\Lambda \equiv 1 + \bar{a} B \Gamma_{j}^{1/2}
\]

(121)

where \( \bar{a} \) corresponds to the ion size parameter (see below), \( \Gamma \) designates the "true" ionic strength of the solution in molality units of concentration (eq 83), \( \Gamma_a \) denotes the mole fraction to molality conversion factor given by \(^1\)

\[
\Gamma_a \equiv -2.303RT \log(1 + 0.018015m^*)
\]

(122)

where \( m^* \) stands for the sum of the molalities of all solute species (eq 107) and

\[
\Lambda \equiv -2(2.303)RT \Lambda_{\gamma}
\]

(123)

where \( \Lambda_{\gamma} \) (and \( B_{\gamma} \) in eq 121) refer to the Debye-Hückel activity coefficient parameters in kg\(^{1/2}\) mole\(^{-1/2}\) and kg\(^{1/2}\) mole\(^{-1/2}\) cm\(^{-1}\), respectively, which are defined by eqs (C-1) and (C-2) in app. C. Note that a statement of eq (120) can be written for any charged species in solution and that eqs (C-1) and (C-2) in app. C require \( \bar{a} \) in eq (121) to be expressed in centimeters. Values of \( \Lambda_{\gamma} \), \( B_{\gamma} \), and other Debye-Hückel parameters defined below are

\(^9\)Several alternatives to this approach have been suggested recently (Pitzer, 1973, 1977; Pytkowicz, Johnson, and Curtis, 1977). For example, Pitzer's alternate form of the Debye-Hückel equation takes account of hardcore kinetic effects on departures from ideality. Activity coefficients computed from Pitzer's function differ slightly from those predicted by the Debye-Hückel equation, but both reduce to the same limiting law.

\(^10\)Eq (120) and related equations are written above in terms of \( \Gamma \) instead of \( I \) because Debye-Hückel theory strictly applies only to completely dissociated electrolytes, many of which become highly associated at high concentrations and/or temperature. Note also that although eq (120) is written for the \( j \)th ion, an analogous expression can be written for any charged aqueous species.

\(^11\)The activity coefficient analog of \( \Gamma_{a,j}/(2.303RT) \) in eq (120), which provides for conversion of the rational activity coefficient to its molal counterpart, is commonly omitted from molality expressions of the Debye-Hückel equation because \( \Gamma_{a,j}/(2.303RT) \) is insignificant in dilute solutions (Harned and Owen, 1958). However, it can be shown that the term becomes important at higher concentrations if eq (120) is used to describe long-range ionic interaction in extensions of Debye-Hückel theory.
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Debye-Hückel parameters computed from equations summarized in app. C by Helgeson and Kirkham (1974b) for 1 bar at temperatures $< 100^\circ$C and pressures corresponding to those along the liquid-vapor equilibrium curve for H$_2$O at temperatures $> 100^\circ$C.

given in table 1 for temperatures ≤ 95°C at 1 bar and 100° to 350°C at pressures corresponding to those along the vapor-liquid equilibrium curve for H₂O.

Although \( \hat{a} \) is commonly referred to as the distance of closest approach of ions in solution, it is an electrostatic parameter representing the minimum distance of charge separation among solvated ions, which does not correspond to the sum of their intrinsic radii. Considerable evidence indicates that the effective electrostatic radii of solvated cations are substantially greater than their intrinsic counterparts (Helgeson and Kirkham, 1976). The ion size parameter is defined by Robinson and Stokes (1959) as "the distance from the center of an ion within which no other ion can penetrate," which corresponds to the distance at which the electrostatic fields of the solvated ions interfere with each other to a great enough extent to cause repulsion. The extent of this interference is a function of the effective electrostatic radii of the ions (Helgeson and Kirkham, 1976).

In principle, values of \( \hat{a} \) can be computed from the Debye-Hückel equation with the aid of activity coefficients for dilute aqueous solutions. However, the experimental values of log \( \gamma_\pm \) must be accurate to at least the fourth decimal in order for the calculations to be definitive. Approximations of \( \hat{a} \) were computed from crystal radii and ionic mobilities many years ago by Kielland (1937), but most values generated since the advent of high speed computers have been obtained by regression of stoichiometric activity coefficients with extended forms of the Debye-Hückel equation (Guggenheim and Stokes, 1969; Helgeson and James, 1968; Helgeson, 1969; Hamer and Wu, 1972; Wood, ms and 1975). Although this approach commonly yields acceptable fits of the data, it is not diagnostic with respect to the relative contributions by ionic interaction, solvation, and ion association to the regression parameters, which are interdependent and commonly vary with the range of concentration considered in the fit. Values of \( \hat{a} \) generated in this manner are thus neither unique nor physically significant, despite the fact that they may be of the right order of magnitude. Depending on the degree of the extended Debye-Hückel equation employed, and the range of concentration represented by the data, any value of \( \hat{a} \) of the order of 1 to 6 Å may result from regression of activity coefficients.

Although \( \hat{a} \) should in principle be a function of solution composition at constant ionic strength, the algebraic form of eq (120) precludes any such provision for mixed electrolytes. Introduction of the ion size parameter in the derivation of the Debye-Hückel equation is responsible for the appearance of \( \Lambda \) in the denominator of the expression. As a consequence, the requirements for \( dG_{\text{solution}} \) to be an exact differential cannot be satisfied without modification of the Debye-Hückel equation (Scatchard, Rush, and Johnson, 1970), unless \( \hat{a} \) is the same for all the solute components of a mixed electrolyte solution. However, the calculations described below, together with theoretical considerations, indicate that \( \hat{a} \) is not the same for all electrolytes. It thus appears that the form of the Debye-Hückel
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equation is congenitally flawed for application to mixed electrolytes. This
observation has led to the somewhat lame but almost universal decision
by solution chemists to assign a single value of \( \hat{a} \) to all electrolytes and
make up for errors arising from this procedure by regression of experi-
mental data with extended forms of the Debye-Hückel equation (Robin-
son and Stokes, 1959; Lewis and Randall, 1961; Harned and Robinson,
1968). In contrast, the approach taken in the present study is to calculate
a characteristic but different value of \( \hat{a} \) for each electrolyte solution. As a
consequence, provision is included in eq (102) for long-range interaction
among all the abundant ions in solution. However, it also means that \( \Lambda \neq \Lambda_k \)
in the limit as \( y_k \to 1 \) in mixed electrolytes, which introduces slight
errors in computed values of \( \log \hat{\gamma}_{\pm,k} \) in multicomponent solutions (see
below). Nevertheless, from a geochemical point of view it seems prefer-
able to accept these small errors rather than introduce an empirical cor-
rection function in an attempt to resolve the dichotomy generated by the
form of the Debye-Hückel equation applied to mixed electrolytes.

The values of \( \hat{a} \) adopted in the present study were computed from
the effective electrostatic radii of aqueous ions \( (r_{e,j}) \) given by Heigeson
and Kirkham (1976) and the relation,

\[
\hat{a} = 2 \frac{\sum_k \sum_j v_{j,k} r_{e,j}}{\sum_k v_k} \frac{\sum_k v_k \hat{\alpha}_k}{\sum_k v_k} \tag{124}
\]

where

\[
\hat{\alpha}_k = 2 \sum_j v_{j,k} r_{e,j} / v_k \tag{125}
\]

where \( j \) \((j = 1, 2, \ldots, j) \) and \( k \) \((k = 1, 2, \ldots, k) \) in this case refer to the
predominant ions and components of the solute, respectively.

Eq (124) represents a weighted arithmetic mean of the \( \hat{a} \) values for
the major components of the solute. As such it takes no account of com-
plexing. The ambiguous and moot significance of the ion size parameter
in concentrated solutions precludes explicit provision for the effects of
ion association on \( \hat{a} \). Calculated values of \( \hat{\alpha}_k \) are given in table 2 for a large
number of electrolytes. The values of \( r_{e,j} \) used to evaluate eq (125) are
listed in table 3 and discussed below.

Long-range interaction of polar neutral species with each other and
with cations, anions, and charged complexes almost certainly occurs in
electrolyte solutions. However, experimental evidence (Randall and Failey,
1927a, b, and c) suggests that the effects of such interaction on the ther-
mo dynamic behavior of the species is negligible compared to contributions
by solvation and short-range interaction. Accordingly, for most practical
purposes \( \mathcal{C}_d \) for neutral species. \( (\mathcal{C}_{d,n}) \) can be approximated as

\[
\mathcal{C}_{d,n} = \Gamma G \tag{125A}
\]
Ion-solvent interaction.—The solvation contribution to the thermodynamic behavior of an electrolyte can be described in terms of the Born (1920) transfer process, which, like the Debye-Hückel theory and the Drude-Nernst equation (Drude and Nernst, 1894), is strictly applicable only to idealized incompressible, unpolarizable, spherical ions in a homogeneous dielectric continuum. Owing to structural discontinuities, asymmetric solvation shells, dielectric saturation, and the finite compressibility of solvated ions (which are not necessarily spherical and nonpolar), aqueous electrolytes fail to meet these conditions. Nevertheless, the success of

<table>
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<th>$a_k$</th>
<th>Solute</th>
<th>$a_k$</th>
<th>Solute</th>
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<td>CsOH</td>
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</tr>
</tbody>
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$^a$Angstroms. $^b$Calculated from equation (125) and values of $r_{e,i}$ given in table 3.
the equation of state proposed recently by Helgeson and Kirkham (1976) for aqueous electrolytes at infinite dilution suggests that the Born-Bjerrum theory affords close approximation of the thermodynamics of ion solvation, provided that separate provision is made for electrostriction collapse of the local solvent structure.

The applicability of the Born equation (as corrected by Bjerrum, 1929) to the solvation process in an aqueous solution can be demonstrated by taking account of

\[
\begin{align*}
D_{A^+_{\text{vacuum}}} & \quad \Delta G_1 \\
A^+_{\text{aqueous disordered}} & \quad \Delta G_2 \\
A^+_{\text{aqueous ordered}} & \quad \Delta G_3
\end{align*}
\]

\[\Delta G_s = \Delta G_1 + \Delta G_2. \tag{127}\]

where \(A^+_{\text{aqueous disordered}}\) refers to the unsolvated ion represented by \(A^+\) in an aqueous state of disorder immediately after collapse of the solvent structure, \(A^+_{\text{aqueous ordered}}\) designates the solvated ion with oriented \(H_2O\) dipoles in primary and secondary coordination, \(A^+_{\text{vacuum}}\) stands for the ion in a vacuum, \(\Delta G_1\) and \(\Delta G_2\) denote the Gibbs free energy changes attending transfer of the ion from an aqueous disordered state to a vacuum and from a vacuum to an aqueous solvated state, respectively, and \(\Delta G_s\) corresponds to the Gibbs free energy of solvation within the aqueous phase, which can be expressed as

The solvation process is commonly identified in classical solution chemistry with transfer of an ion from a vacuum (or gas state) to an aqueous solvated state without regard for local collapse of the solvent structure. Consequently, the Born theory alone, which contains no provision for electrostriction collapse, fails to describe adequately the hydration process. However, because the effects of solvent collapse cancel in the transfer of an ion both to and from a vacuum (as depicted in schematic diagram 126), the Born theory is directly applicable to the process of ion solvation within an aqueous phase. Separate provision must then be made for the statistical and structural consequences of electrostriction collapse.

The Born (1920) equation as corrected by Bjerrum (1929) can be written for solvation of one mole of the \(j\)th ion as

\[
\Delta G_{s,j} = \frac{N^o Z_j^2 e^2}{2r_{e,j}} \left( \frac{1}{\epsilon} - 1 \right) = \frac{\eta Z_j^2}{r_{e,j}} \left( \frac{1}{\epsilon} - 1 \right) = \omega_{abs} \left( \frac{1}{\epsilon} - 1 \right) \tag{128}\]

The terms "ion hydration" and "ion solvation" are used in the present communication to distinguish two different processes. The first denotes transfer of an ion from a vacuum or a gas state to an aqueous solvated state, as in step 2 of schematic diagram (126). In contrast, ion solvation refers to the process of assembly and orientation of \(H_2O\) dipoles about an aqueous ion which is already in solution.
where $\Delta G_{s,j}$ represents the partial molal Gibbs free energy of solvation of the ion, $N^0$ stands for Avogadro's number ($6.02252 \times 10^{23}$ molecules mole$^{-1}$), $e$ designates the electronic charge ($4.80298 \times 10^{-10}$ esu), $Z_j$ and $r_{e,j}$ correspond to the charge and effective electrostatic radius of the subscripted ion, $\epsilon$ denotes the dielectric constant of the solution, $\eta$ refers to a constant defined by

$$\eta \equiv \frac{N^0 e^2}{2} = 1.66027 \times 10^9 \text{ A cal mole}^{-1} \quad (129)$$

and $\omega_f^{abs}$ refers to the absolute Born coefficient for the subscripted ion, which can be expressed as

$$\omega_f^{abs} = \frac{\eta Z_j^2}{r_{e,j}}. \quad (130)$$

The fact that the dielectric constant of the bulk solution is used in eq (128) should not be interpreted as an assumption that $\epsilon$ represents a close approximation of the dielectric constant of the medium in the immediate vicinity of the ion. Instead, it means simply that the values of $r_{e,j}$ employed in eq (128) must be consistent with the electrostatic behavior of the bulk solution. For example, evaluation of the Born equation using the dielectric constant of H$_2$O to generate effective electrostatic radii (Helgeson and Kirkham, 1976) assigns implicitly the effects of dielectric saturation and reduced orientational polarizability of solvated H$_2$O dipoles to $r_{e,j}$. Nevertheless, because the dielectric constant is always multiplied by $r_{e,j}$ in calculating the thermodynamic consequences of ion solvation, no inconsistencies are introduced by this procedure.

For the $k$th completely dissociated component of an electrolyte solution, eq (128) becomes

$$\Delta G_{s,k} = \omega_k \left( \frac{1}{\epsilon} - 1 \right) \quad (131)$$

where

$$\omega_k = \eta \sum_j (v_{j,k} Z_j^2 / r_{e,j}) = \sum_j v_{j,k} \omega_f^{abs}. \quad (132)$$

If $r_{e,j}$ is taken to be independent of concentration (see below), the standard state counterpart of eq (128) can be written for the conventional analog of $\Delta G_{s,j}^{abs}$ as

$$\Delta G_{s,j}^{abs} = \omega_j \left( \frac{1}{\epsilon^o} - 1 \right), \quad (133)$$

and the standard partial molal Gibbs free energy of solvation of the $k$th electrolyte can be expressed as

$$\Delta G_{s,k} = \omega_k \left( \frac{1}{\epsilon^o} - 1 \right) \quad (134)$$

where $\epsilon^o$ stands for the dielectric constant of H$_2$O and $\omega_j$ corresponds to the conventional Born coefficient defined by

$$\omega_j \equiv \omega_f^{abs} - Z_j \omega_{1j}^{abs}. \quad (135)$$
which is consistent with
\[ \omega_k = \sum_j v_{j,k} \omega_j . \] (136)

Similarly, the absolute analog of \( \Delta G^o_{s,i} \) is given by
\[ \Delta G^o_{s,i}^{abs} = \omega_j^{abs} \left( \frac{1}{\epsilon^o} - 1 \right) . \] (137)

Although eqs (128), (130), (133), (135), and (137) apply to the \( j \)th ion, analogous expressions can be written for the \( q \)th complex.

Representing the thermodynamic consequences of ion solvation with the Born equation is not meant to imply that neutral species fail to solvate. On the contrary, ample evidence indicates that polar neutral complexes solvate to a significant degree. The formal charge of zero on a neutral species can be regarded as an additive consequence of positive and negative vector charges distributed over the polar molecule. If we designate the sum of either the positive or negative contributions to the formal charge of zero as the effective local charge \( (Z_e) \) of the complex, we can define the Born coefficient for the \( n \)th neutral species \( (n = 1, 2, \ldots, \hat{n}) \) as
\[ \omega_n^{abs} = \frac{n Z_e.n^2}{r_e.n} \] (138)
where \( r_e.n \) represents the effective electrostatic radius of the complex. The partial molal Gibbs free energy of solvation of the \( n \)th neutral species is then given by
\[ \Delta G_{s,n} = \omega_n^{abs} \left( \frac{1}{\epsilon} - 1 \right) . \] (139)
which is consistent with
\[ \Delta G_{s,n}^{abs} = \omega_n^{abs} \left( \frac{1}{\epsilon^o} - 1 \right) . \] (140)

Because \( Z_n = 0 \),
\[ \omega_n = \omega_n^{abs} \] (140A)
and
\[ \Delta G^o_{s,n} = \Delta G^o_{s,n}^{abs} . \] (141)

The effective electrostatic radius of an aqueous ion \( (r_{e,j}) \) differs from its intrinsic counterpart \( (r_{i,j}) \), and neither (except for \( r_{e,anions} \) — see below) is equivalent to its crystallographic radius \( (r_{x,j}) \). Despite arguments to the contrary (Glueckauf, 1965), \( r_{e,j} \) does not necessarily correspond to the physical radius of a solvated ion, nor does the word radius in the term imply that the ion and its solvation shell are actually spherical in symmetry. The effective electrostatic radius of a solvated ion is an idealized parameter representing the net contribution by the ion to the equilibrium distance of charge separation in solution. Although \( r_{e,j} \) is referred to a spherical idealization of the ion and its solvation shell, the actual solvated ion may
have a different shape and its contribution to charge separation may be a vector quantity. Nevertheless, the average contribution by the ion to charge separation in solution can be expressed as an equivalent contribution by a hypothetical ion with a spherical configuration. Hence the term "effective" electrostatic radius.

The dependence of $r_{_{\text{e},j}}$ on void space in the solvation shell is an electrostatic function of discontinuities in the dielectric medium, but the dependence of $r_{_{\text{i},j}}$ on void space is a configurational function of the geometry of the solvation shell. Both the intrinsic and effective electrostatic radii of an ion have physical significance only in a statistical context related to the residence time for nearest neighbor configurations of solvent dipoles about the ion (Samoilov, 1972). As a consequence, neither $r_{_{\text{i},j}}$ nor $r_{_{\text{e},j}}$ is strictly independent of short-range ionic interaction, which perturbs the solvation shells about the ions in solution (see below). Nevertheless, empirical considerations suggest that both can be regarded as concentration-independent parameters.

Regression analysis (Helgeson and Kirkham, 1976) indicates that $r_{_{\text{e},j}}$ for anions can be taken to be equivalent to their crystallographic radii.

![Graph](image)

**Fig. 1.** Correlation of $\Delta S_{_{\text{ne},j}}$ ($\Delta S_{_{\text{ne},j}} \equiv S_{_{\text{i},j}} + \Delta S_{_{\text{e},j}}$) with the reciprocal of the effective electrostatic radii ($r_{_{\text{e},j}}$) of aqueous ions at 25°C and 1 bar (Helgeson and Kirkham, 1976).
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\[ r_{e, +} = r_{x, +} + 0.94 \, Z_+ \]

(142)

where \( Z_+ \) represents the charge on the cation. Values of \( r_{e,j} \) can be computed from eq (142) or estimated from entropy correlations such as those illustrated in figures 1 and 2. The diagrams in these figures depict the difference in the conventional standard partial molal third law entropy of the \( j \)th aqueous ion and the corresponding absolute standard partial molal entropy of solvation (\( \Delta S^o_{s,j, aq} \)) as a function of \( r_{e,j}^{-1} \) and \( |Z_j|/r_{e,j}^{-1} \), respectively (Helgeson and Kirkham, 1976). Values of both \( r_{e,j} \) and \( r_{x,j} \) for aqueous species are given in table 3.

Various attempts have been made to improve the applicability of the Born theory to ion hydration (see footnote 12) by regarding \( r_{e,j} \) as a function of temperature and/or pressure (Latimer, 1955; Hamann and Strauss, 1955; Benson and Copeland, 1963). Despite the fact that the effective electrostatic radius of an ion might be expected to change as a consequence of changes in the extent to which the ion solvates with increasing pressure and temperature, calculation of the standard partial molal properties of aqueous species at high pressures and temperatures with the aid of continuum theory (Cobble and Murray, 1977; Sen, Murray, and Cobble, ms; Helgeson and Kirkham, 1976) leaves little doubt that \( r_{e,j} \) can be taken to be independent of pressure and temperature. Regression analysis of high pressure/temperature data to define values

![Diagram](image)

**Fig. 2.** Correlation of \( \Delta S^o_{s,j} \) (\( \Delta S^o_{s,j} \equiv S^o_{s,j} + \Delta S^o_{c,j} \)) with the absolute ionic potential of aqueous ions at 25°C and 1 bar (Helgeson and Kirkham, 1976).
Table 3

Effective electrostatic ($r_{\text{e},i}$) and crystal ($r_{\text{c},i}$) radii, absolute ($\omega_{\text{a}}$) and conventional ($\omega_{\text{c}}$) Born parameters, and conventional standard partial molal entropies ($S_{\text{pm}}$) of aqueous ions (designated by the subscript $j$) at 25°C and 1 bar.

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<th>$r_{\text{c},i}$</th>
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Computed from the entropy correlation shown in figure 1 with the aid of equation (74) in Helgeson and Kirkham (1976). ‡Computed from equation (135). §Calculated by equation (133). ‡Computed from equation (135). §CODATA (1975). §Hinchey and Cobble (1970). ‡Wagner and others (1968, 1969); Parker, Wagner, and Evans (1971). §Corrected typographical error in Wagner and others (1968). ‡Hemingway and Robie (1977). §Computed from data reported by Readnour and Cobble (1965). ‡Berg and Vanderzee (1972). ‡Calculated by equation (133). §Other values of $S_{\text{pm}}$ (in cal mole⁻¹ K⁻¹) adopted in the present study include: -49.4 Å for Pr⁴⁺, -49.4 Å for Nd⁴⁺, -52.5 Å for Sm⁴⁺, 45.7 Å for Ho⁴⁺, 33.5 Å for Y³⁺, 28.3 Å for Tl⁺, 39.0 Å for Br⁻, -7.0 Å for Cl⁻, -53.0 Å for PO₄³⁻, 21.9 Å for H₂PO₄⁻. §22.0 Å for HCO₃⁻.
prediction, thermodynamic behavior of aqueous electrolytes: IV. 1305

of \( \omega_h^{abs} \) (see below) indicates that \( r_{e,n} \) can also be regarded as a pressure/temperature-independent parameter. Values of \( \omega_j^{abs} \) together with its conventional analog (\( \omega_j \)) are given in table 3 for a number of aqueous ions.

If \( r_{e,j} \) is regarded as a concentration-independent parameter, \( \Delta \overline{G}_{h,j} \) as a function of ionic strength is controlled by the concentration dependence of \( \epsilon^{-1} \). The distribution of experimental data shown in figures 3 through 5 suggests that the reciprocals of the dielectric constants of electrolyte solutions are linearly related to ionic strength at 25°C and 1 bar, which is consistent with the observed behavior of activity coefficients at high ionic strengths (see below). Few comparable data are available at other temperatures and pressures, but it can be seen in figure 6 that the same behavior is exhibited by electrolytes at 1.5°C, 3°C, and 10°C. The linear dependence of \( \epsilon^{-1} \) on ionic strength in figures 3 through 5 contradicts conclusions reached by Hasted, Ritson, and Collie (1948), who represented their data as a linear function of molarity (\( M \)) to ~ 2M (fig. 5). Comparative calculations demonstrate that the latter correlation is a fortuitous consequence of the near-proportionality of \( (\epsilon^{-1} - \epsilon^0) \) to \( (\epsilon - \epsilon^0) \) and \( M \) to \( m \) for \( 70 \leq \epsilon \leq 90 \) and \( M \leq 2 \).

It can be deduced from figures 3 through 6 that increasing ionic strength affects dramatically the dielectric constant of electrolyte solutions. For example, increasing the concentration of an NaCl solution from 0.1 to ~ 6 m at 1.5°C and 1 bar causes the dielectric constant of the solution to decrease by ~ 45 (fig. 5), which is equal to the decrease in the dielectric constant of H₂O accompanying a temperature increase from 25°C to ~ 200°C at pressures corresponding to the liquid-vapor equilibrium curve.

![Fig. 3. Reciprocal of the dielectric constant of KCl solutions as a function of stoichiometric ionic strength at 25°C and 1 bar. The symbols in figures 3 through 10, 12, 14, and 131 represent values taken from the following sources: open and closed circles, Hasted, Ritson, and Collie (1948); open and closed hexagons, Pottel (1973) and Giese, Kaatze, and Pottel (1970); striped circles, Hasted and Roderick (1958); striped hexagons, Hasted and El Sabe (1953); open and closed squares, Haggis, Hasted, and Buchan (1952); triangles, Helgeson and Kirkham (1976); cross-hatched circles, Harris and O'Konski (1957). The solid curves in these figures are consistent with one another and with the electrostatic parameters and equations summarized in the tables and text. The dashed curves merely connect data points (see text).](image1)

![Fig. 4. Reciprocal of the dielectric constant of NaNO₃ and NaClO₄ solutions as a function of stoichiometric ionic strength at 25°C and 1 bar (see caption of fig. 5).](image2)
for H₂O. The disruptive effect of short-range ionic interaction on dipole alignment in the bulk solvent also promotes ion association. The effect of complexing on the dielectric constant of electrolyte solutions can be assessed in figure 7. The increasing departures of the dashed curves in this figure from their solid counterparts with increasing ionic strength is consistent with increasing ion association with increasing concentration.

It has been observed repeatedly (Hasted, Ritson, and Collie, 1948; Haggis, Hasted, and Buchanan, 1952; Hasted and El Sabe, 1953; Hasted and Roderick, 1958; Harris and O'Konski, 1957; Giese, Kaatze, and Pottel, 1970; Pottel, 1973) that the extent to which the dielectric constant of an electrolyte solution decreases with increasing concentration is highly sensitive to the size and charge of the cation, but essentially independent of the identity of the anion. In contrast to the strong forces of attraction between cations and H₂O dipoles, the solvation shells about anions are loosely held and easily perturbed. The relatively weak forces of attraction between anions and H₂O molecules are apparently of the same order of magnitude as the hydrogen bonds among solvent dipoles, which are oriented about anions with their focus of negative charge outward. Owing to the opposite orientation of H₂O dipoles about cations, few hydrogen bonds are broken by cation solvation. The smaller the cation and the greater its charge, the stronger its influence on the electrostatic behavior of H₂O dipoles in its solvation shell. Because the orientational polarizability of H₂O dipoles about anions is not reduced significantly by coordination, anions contribute only slightly to the electrostatic properties of electrolyte solutions. The solid curves shown in figures 3, 4, 6, and 7 are consistent with this

![Fig. 5. Reciprocal of the dielectric constant of NaCl solutions as a function of stoichiometric ionic strength at 1.5°C and 1 bar (see caption of fig. 3). The filled and open circles correspond to the same experimental data plotted against molal and molar ionic strength, respectively.](image-url)
observation; that is, the slopes of the curves for electrolytes consisting of a given cation and different anions (KCl and KF, NaNO₃, NaClO₄, Na₂SO₄, NaF, and NaI, and CaCl₂ and CaBr₂) are nearly (but not exactly) identical.

Considerable uncertainty and many assumptions and approximations are involved in extracting static dielectric constants from measurements of the permittivities of electrolyte solutions as a function of frequency. It

Fig. 6. Reciprocal of the dielectric constant of electrolyte solutions as a function of stoichiometric ionic strength at 3°, 10°, and 25°C and 1 bar (see text and caption of fig. 3).
is thus not surprising that many dielectric constant data reported by different investigators (or the same investigator at different times) contradict one another. For example, all the data reported by Harris and O’Konski (1957) in figure 9 are internally consistent, and the dashed curves drawn through the data points are linear with nearly the same slopes for electrolytes with a common cation. However, none of the dashed curves in figures 8 and 9 has the same slope as the corresponding solid curves for the same electrolytes in figures 3, 7, 8, and 9, which are consistent with data reported by other investigators.

Many dielectric constant data reported for temperatures above and below 25°C are also inconsistent with one another. For example, it can be seen in figure 10 that the data for NaCl at 0°, 10°, 15°, 20°, 25°, 30°, 35°, 40°, and 50°C reported by Pottel (1973) and Hasted, Ritson, and Collie (1948) are incompatible with those obtained at 3°, 25°, and 40°C by Haggis, Hasted, and Buchanan (1952) and Hasted and Roderick (1958). The solid curves in figures 6 and 10 for temperatures other than 25°C are based on consideration of the relation of \( \epsilon^{-1} \) to \( \epsilon^{0-1} \) as a function of temperature (see below).

If inconsistencies and departures from linearity in the distribution of data shown in figures 3 through 10 can be attributed to experimental

Fig. 7. Reciprocal of the dielectric constant of electrolyte solutions as a function of stoichiometric ionic strength at 25°C and 1 bar (see text and caption of fig. 3).
error and/or ion association (which seems likely) all the data shown in the figures are consistent with
\[ \epsilon_k^{-1} - \epsilon_0^{-1} = \hat{b}_k I, \]  
(143)

where \( \hat{b}_k \) corresponds to the slopes of the solid curves in figures 8 through 10. Although eq (143) applies only to single electrolytes, consideration of osmotic coefficient data for mixed electrolytes (see below) suggests that it can be generalized to
\[ \epsilon^{-1} - \epsilon_0^{-1} = \sum_k \hat{b}_k \psi_k I, \]  
(144)

which for “complete” dissociation reduces to
\[ \epsilon^{-1} - \epsilon_0^{-1} = \sum_k \hat{b}_k \psi_k m_k. \]  
(145)

At least in principle, the separation of variables represented by eq (117), together with eq (29) and constraints imposed by the fact that \( dG_{\text{solution}} \) is an exact differential requires
\[ \left( \frac{\partial (\Delta G_{s,k})}{\partial m_k} \right)_{P,T,\tilde{m}_s,n_w} = \left( \frac{\partial (\Delta G_{s,k})}{\partial \tilde{m}_k} \right)_{P,T,\tilde{m}_s,n_w} \]  
(146)

Fig. 8. Reciprocal of the dielectric constant of electrolyte solutions as a function of stoichiometric ionic strength at 25°C and 1 bar (see text and caption of fig. 3).
Fig. 9. Reciprocal of the dielectric constant of electrolyte solutions as a function of stoichiometric ionic strength at 25°C and 1 bar (see text and caption of fig. 3).
Fig. 10. Reciprocal of the dielectric constant of NaCl solutions as a function of stoichiometric ionic strength at various temperatures (labeled in °C) and 1 bar (see text and caption of fig. 3).

Fig. 11. Correlation of the Born coefficient ($\omega_b$) calculated by Helgeson and Kirkham (1976) with the electrostatic parameter $\psi_kb_k$ computed from the slopes of the solid curves for 25°C and 1 bar in figures 3, 4, and 6 through 10 (see eq 149 and fig. 12).
where \( \bar{m}_k \) and \( \bar{m}_{\bar{k}} \) \((k = i, 2, \ldots, \bar{k})\) refer to constant molalities of all components other than the \( k \)th and \( \bar{k} \)th, respectively. Taking account of eq (131), it follows that

\[
\omega_{\bar{k}} \left( \frac{\partial \epsilon}{\partial m_k} \right)_{P,T,\bar{m}_k,n_w} = \omega_k \left( \frac{\partial \epsilon}{\partial m_{\bar{k}}} \right)_{P,T,\bar{m}_{\bar{k}},n_w}
\]  

(147)

which can be combined with appropriate partial derivatives of eq (145) to give

\[
\omega_{\bar{k}} \psi_{\bar{k}} \bar{b}_k = \omega_k \psi_k \bar{b}_\bar{k}
\]  

(148)

Eq (148) is satisfied if \( \omega_k \) is proportional to \( \psi_k \bar{b}_k \) for all values of \( k \). It can be deduced from figures 11 and 12, together with the identity

\[
b_k = \frac{\bar{b}_k}{2.303RT}
\]  

(149)

that this is indeed the case. The solid symbols plotted in figures 11 and 12 correspond to values of \( b_k \) computed from the slopes of the solid curves for 25°C in figures 3, 4, and 6 through 10. Because these curves represent (within experimental uncertainty) the bulk of the dielectric constant measurements for “completely” dissociated electrolyte solutions, figures 11 and 12 strongly support the validity of eq (144). The differences between the slopes of the dashed curves (which merely connect data points) in figures 8 through 10 and the solid curves probably arise from experimental error.

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Fig. 12. Enlargement of the lower left corner of figure 11.
### Table 4
Electrostatic parameters for aqueous electrolytes (designated by the subscript $k$) at 25°C and 1 bar (see text)

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$*$Computed from equation (149). $^1b_{e}$ mole$^{-1}$. $^2S_{e}$ cal$^{-1}$. $^3$Computed from equation (150).
In any event, the dashed curves are not all consistent with one another. The equation of the curve in figures 11 and 12 can be written as

\[ b_k = \left( \omega_k / \psi_k \right) \times 10^{-11} \]

(150)

which ensures internal consistency among the slopes of the solid curves in figures 3, 4, and 6 through 10.

Values of \( b_k \) and \( \dot{b}_k \) computed from eqs (136), (149), and (150) using values of \( \omega_j \) taken from table 3 are given in table 4. It can be deduced from figure 13 that the value of \( \dot{b}_{\text{Rbl}} \) generated in this way is exactly consistent with data reported by Pottel (1973). However, corresponding values for RbF, RBr, and the cesium halides in table 4 are slightly larger than those consistent with the dashed curves for these electrolytes in figure 13, which simply connect \( \varepsilon^{\circ-1} \) and the single data point for each of the solutions. This observation is consistent with the fact that RbF, RBr, CsCl, CsI, CsBr, and CsF are all associated to a significant degree at 1 m (see below). In contrast, the discrepancy between the calculated slopes of the solid curves in figure 14 and those of the dashed curves (which are dictated by the distribution of the data points) is probably due primarily to systematic experimental error. This conclusion is supported by the fact that the values of \( b_k \) corresponding to the slopes of the dashed curves for HCl and BaCl₂ at 25°C in figure 14 fall well off the curve in figure 11, where they are plotted as open symbols. Regression of apparent molal heat capacity and volume data for electrolytes (see below) indicates similar contradictions in the data reported for HCl and BaCl₂ at 3°, 10°, and 40°C (fig. 14).

Because ion solvation contributes significantly to the standard partial molal entropies of aqueous ions, it would be reasonable to expect \( b_k \) to exhibit a close correlation with \( S^\circ \) of cations in electrolytes with a common anion. It can be seen in figure 15 that this is indeed the case for the slopes of the solid curves representing chlorides in figures 3 and 6 through 10. However, in contrast to the conclusion reached by Harris and O’Konski (1957), the correlation in figure 15 is linear, and all the curves are parallel.

![Fig. 13. Reciprocal of the dielectric constant of electrolyte solutions as a function of stoichiometric ionic strength at 25°C and 1 bar (see text and caption of fig. 3).](image-url)
Fig. 14. Reciprocal of the dielectric constant of electrolyte solutions as a function of stoichiometric ionic strength at 5°, 10°, 25°, and 40°C and 1 bar (see text and caption of fig. 3).
Fig. 15. Correlation of $\psi_k b_k$ for chlorides with the conventional standard partial molal entropies of the cations (indicated by the subscript $i$) in the chloride components at $25^\circ$C and 1 bar (see text).

with intercepts that are linearly related to $Z_i$ (the charge on the cations in the electrolytes). The equation of the curves in figure 15 is given by

$$\psi_k b_k = a - 2.1 S^0_i \times 10^{-5}$$  \hspace{1cm} (151)

where

$$a = 2.7 Z_i \times 10^{-3}$$  \hspace{1cm} (152)

which apply to $25^\circ$C and 1 bar. It can be seen in figure 15 that the value of $b_k$ for BaCl$_2$ computed from eq (150) and given in table 4 (which is represented by the open symbol) is consistent with eq (152), but similar agreement cannot be demonstrated for HCl.

Taking account of eqs (119), (128), (137), (139) through (141), and (144), the relative partial molal Gibbs free energy of solvation of aqueous species can be expressed as

$$\Delta G_{h,i} = \omega_i \sum_k b_k y_k l$$  \hspace{1cm} (153)

$$\Delta G_{h,l} = \omega_l b_k y_k l$$  \hspace{1cm} (154)

and

$$\Delta G_{h,n} = \omega_n b_k y_k l$$  \hspace{1cm} (155)
where the subscripts \( i \) and \( l \) denote positively and negatively charged species, respectively, and the index \( n \) again designates neutral complexes.\(^{13}\)

Let us now consider the extent to which contributions by long-range ionic interaction and ion solvation to the relative partial molal Gibbs free energy of an electrolyte are modified by short-range interaction.

**Short-range ionic interaction.**—Following in part the approach taken by Guggenheim (1935), Scatchard (1936), Harned and Robinson (1968), and others, \( \bar{G}_{r,j} \) in a Gibbs free energy statement of eq (117) can be represented in a first approximation by regular solution theory.\(^{14}\) Adopting Brønsted’s (1922) principle of specific interaction and taking account of differences in the forces of attraction among species in solution then leads to

\[
\bar{G}_{r,i} = \sum_{i} b_{il} m_i = \sum_{i} \frac{b_{il} \tilde{y}_l I}{\psi_i} \tag{156}
\]

and

\[
\bar{G}_{r,l} = \sum_{i} b_{li} m_i = \sum_{i} \frac{b_{li} \tilde{y}_i I}{\psi_l} \tag{157}
\]

where \( b_{il} \) represents the short-range interaction parameter for the \( i \)th positively charged species interacting with the \( l \)th negatively charged species.\(^{15}\) Because the forces of attraction among polar neutral complexes are much smaller than those among charged and neutral species, we can also write as a close approximation,

\[
\bar{G}_{r,n} = \sum_{i} b_{ni} m_i + \sum_{l} b_{nl} m_l = \sum_{i} \frac{b_{ni} \tilde{y}_i I}{\psi_i} + \sum_{l} \frac{b_{nl} \tilde{y}_l I}{\psi_l} \tag{158}
\]

where \( b_{ni} \) and \( b_{nl} \) stand for short-range interaction parameters for the \( n \)th

\(^{13}\) Note that the charged species designated by the subscripts \( i \) and \( l \) include charged complexes as well as individual cations and anions.

\(^{14}\) In a strict sense, the term regular solution employed in this context is a misnomer. A regular solution is defined as one for which \( S_{r,j} = 0 \), which is not true of the contribution by \( S_{r,j} \) to \( S_{r,j} \) for electrolyte solutions. The term is employed above to indicate that the representation of \( \bar{G}_{r,j} \) as a function of molality is consistent with a regular solution dependence of \( G_{r,j} \) on composition, which is parabolic in mole fraction space. Harned’s rule (Harned, 1935; Harned and Owen, 1958), Young’s rule (Young, 1951; Young and Smith, 1954; Young, Wu, and Krawetz, 1957; Wu, Smith, and Young, 1965; Wu, 1970) and several other similar algorithms employed in solution chemistry are also consistent with this dependence of \( G_{r,j} \) on mole fraction.

\(^{15}\) Brønsted’s principle of specific interaction states that short-range interaction among like-charged species can be considered insignificant compared to interaction among species of unlike charge. It simply recognizes that the mutual attraction of oppositely charged species makes them far more likely to come into close proximity to one another than species of like charge, which repel each other. With a few notable exceptions, Brønsted’s principle of specific interaction is consistent with the observed thermodynamic behavior of aqueous electrolytes. For this reason, terms accounting for interaction among species of like charge have been omitted from eqs (156) and (157). Similarly, because short-range interaction among charged and neutral species is negligible compared to that among cations and anions, no provision was included in eqs (156) and (157) for interaction of charged and neutral species. Although most neutral complexes are polar, the forces of attraction among neutral molecules are small compared to those among neutral molecules and charged species. Accordingly, analogous terms representing interaction of neutral species were omitted from eq (158).
neutral species interacting with the \(i\)th and \(l\)th charged species, respectively. The physical significance of the interaction parameters in eqs (156) through (158) is discussed below.

**Combined equations.**—Substituting the subscript \(i\) for \(j\) in appropriate statements of eqs (117) and (120) and combining the resulting expressions with eqs (153), (156), and

\[
\overline{\mathcal{G}}_{\text{ideal},i} = 2.303 \, RT \, \log m_i \tag{159}
\]

leads to (see footnote 13)

\[
\overline{\mathcal{G}}_i - \overline{\mathcal{G}}^o_{\text{abs}} = \frac{\psi_i A_G}{\Lambda} + \Gamma_G + \omega_i^{\text{abs}} \sum_k b_k y_k I + \sum_l b_{il} m_l + 2.303 \, RT \, \log m_i . \tag{160}
\]

Similarly, from statements of eqs (117) and (159) written for the \(l\)th and \(n\)th species, together with appropriate statements of eqs (120), (125A), (154), (155), (157), and (158) it follows that

\[
\overline{\mathcal{G}}_l - \overline{\mathcal{G}}^o_{\text{abs}} = \frac{\psi_i A_G}{\Lambda} + \Gamma_G + \omega_i^{\text{abs}} \sum_k b_k y_k I + \sum_i b_{il} m_i + 2.303 \, RT \, \log m_i \tag{161}
\]

and

\[
\overline{\mathcal{G}}_n - \overline{\mathcal{G}}^o_n = \Gamma_G + \omega_n \sum_k b_k y_k I + \sum_i b_{ni} m_i + \sum_l b_{nl} m_i + 2.303 \, RT \, \log m_n . \tag{162}
\]

We can thus write a combined statement of eq (29) and its standard state analog for the \(k\)th "completely" dissociated single electrolyte consisting of the \(i\)th and \(l\)th cation and anion, or the \(k\)th 1:1 or 2:2 electrolyte in which only a single ion pair forms to a significant degree as

\[
\overline{\mathcal{G}}_k - \overline{\mathcal{G}}^o_k = \frac{\psi_k A_G}{\Lambda} + \nu_k \Gamma_G + \frac{(\omega_k \psi_k b_k + 2 \nu_i \nu_k \nu_{l,k} b_{il}) I}{\psi_k} + \frac{2.303 \, RT \, \log(\nu_i \nu_k \nu_{l,k} \nu_{l,i})}{\psi_k} + 2.303 \nu_k \, RT \, \log \left( \frac{I}{\psi_k} \right) \tag{163}
\]

which corresponds to eq (60). Note that if we alternatively define an ideal solution as one in which all activity coefficients on the mole fraction scale of concentration are unity, eq (159) becomes

\[
\overline{\mathcal{G}}_{\text{ideal},i} = RT \, \ln m_i + \Gamma_G \tag{164}
\]

Eq (164) is consistent with

\[
\overline{\mathcal{G}}^o_{i,(x)} = \overline{\mathcal{G}}^o_i + RT \, \ln(55.51) \tag{164A}
\]

and

\[
\overline{\mathcal{G}}_{i,\text{ideal}(x)} = \overline{\mathcal{G}}_{i,\text{ideal}} - RT \, \ln(55.51) = RT \, \ln X_i \tag{164B}
\]
where $\mathcal{G}^0_i$ and $\mathcal{G}_{i,\text{ideal}}$ refer to the molality scale of concentration, $C^0_{i(X)}$ and $\mathcal{G}_{i,\text{ideal}}(X)$ represent their mole fraction analogs, and $X_i$ stands for the mole fraction of the subscripted species. Note that if eq (164) is adopted in lieu of eq (159), $\Gamma_0$ must be removed from eqs (120) and (125A), but eqs (160) through (162) are not affected. Other definitions of ideality such as the one adopted by Harned and Robinson (1968) in which the activity of the solvent in an ideal solution is taken to be equal to its mole fraction yield somewhat different expressions analogous to eq (164).

Eqs (160) through (162) describe the compositional dependence of the partial molal Gibbs free energies of all species in solution in terms of the theoretical model described above. The relative extent to which the solvation and ionic interaction parameters in these expressions affect the chemical and thermodynamic behavior of aqueous electrolytes can be assessed by comparing the consequences of differences and similarities in the sign and magnitude of $\omega_k b_k$ and $2\nu_{i,k} \nu_{1,k} b_{i1}$ in eq (163).

The effect of increasing concentration on the solvation process in electrolyte solutions is manifested by a positive change in Gibbs free energy at low temperatures, but a negative change at high temperatures. In contrast, the corresponding changes in entropy and volume are positive at all temperatures $\leq 500^\circ\text{C}$ (see below). Changes in Gibbs free energy and volume caused by the effect of increasing concentration on ion solvation may be either reinforced or opposed by short-range ionic interaction, depending on the temperature. For example, at low temperatures where $b_k$ is positive, $b_{i1}$ is commonly negative, but at high temperatures where $b_k$ is negative, $b_{i1}$ becomes positive (see below). In contrast, the entropy contribution by short-range ionic interaction opposes the corresponding contribution by ion solvation to $\mathcal{G}_i - \mathcal{G}_i^{\text{abs}}$ and $\mathcal{G}_l - \mathcal{G}_l^{\text{abs}}$ at all temperatures. Although long-range electrostatic interaction is always accompanied by a positive change in entropy and a negative change in Gibbs free energy, the sum of all three contributions by ion solvation and short-range ionic interaction may either oppose or reinforce the negative contribution by the solvent to the Gibbs free energy of the solution.

In addition to contributions by $\Delta \mathcal{G}_{n,f}$ (the sum of the intrinsic Gibbs free energy of the $j$th ion and that associated with collapse of the local solvent structure), $b_{i1}$ takes into account implicitly the effect of short-range ionic interaction on the extent to which $\text{H}_2\text{O}$ dipoles are partitioned between solvation shells and the bulk solvent. As observed by Friedman and Krishnan (1973), $\text{H}_2\text{O}$ dipoles may be "captured" or "liberated" by collisions of solvated ions. Temporary "capture" of solvent dipoles is accompanied by positive contributions to $\mathcal{G}_i - \mathcal{G}_i^{\text{abs}}$ and $\mathcal{G}_l - \mathcal{G}_l^{\text{abs}}$, which oppose the Gibbs free energy change associated with collision of the ions. To facilitate conceptual appreciation for the consequences of short-range ionic interaction with respect to the distribution of $\text{H}_2\text{O}$ dipoles in solution, four of the many possible types of collisions that might occur in an electrolyte solution are depicted schematically in figure 16, where a hypothetical cation with five $\text{H}_2\text{O}$ dipoles in its primary solvation shell is
shown colliding with a solvated anion, which has six \( \text{H}_2\text{O} \) dipoles in primary coordination. For the sake of clarity, pre-collision secondary solvation shells and \( \text{H}_2\text{O} \) dipoles in the bulk solvent have been omitted from the diagram, which portrays idealized encounters among solvated ions without provision for asymmetry, interpenetration of electron clouds, or transitory perturbation of the solvation shells.

In the case labeled \( a \) in figure 16, the attractive forces between the solvated cation and anion are too weak for the collision to expel two intervening solvent dipoles. The intervening dipoles are then no longer part of the bulk solvent but are temporarily "captured" in secondary coordination shells about the ions. In contrast, collision \( b \) results in expulsion of all intervening \( \text{H}_2\text{O} \) dipoles. However, in case \( c \), collision of the solvated ions results in a net loss of two \( \text{H}_2\text{O} \) dipoles from the solvation shells of the ions, and in collision \( d \) the attractive forces between the ions are strong enough to cause exclusion of all intervening \( \text{H}_2\text{O} \) dipoles between the cation and anion, with a net loss of four \( \text{H}_2\text{O} \) dipoles to the bulk solvent.

The distinction between ion association and short-range interaction of the type illustrated in figure 16 is largely statistical. The central configurations in collisions \( c \) and \( d \) correspond to outer sphere and inner sphere complexes, respectively. If the dissociation constants for such complexes are small, interactions of type \( c \) and \( d \) may predominate in solution. Under these circumstances, ion association dominates departures from ideality, and \( b_{11} \) is negative and varies with total concentration. In contrast, if the dissociation constants for the collisions depicted in the lower two diagrams of figure 16 are large, ion association contributes negligibly to the thermodynamic behavior of electrolytes. The consequences of in-

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**Fig. 16.** Schematic illustration of four hypothetical cases of short-range interaction among solvated cations and anions in electrolyte solutions (see text).
termediate cases between these two extremes depends on the magnitude of \( \omega_i^{abs} b_k \) and \( \omega_l^{abs} b_k \). If these parameters are large, ion association may affect the thermodynamic behavior of an electrolyte only at high concentrations. However, if the forces of attraction among the solvated cations and anions are weak and of the same order of magnitude as those between the ions and their solvated \( H_2O \) dipoles, the effects of ion association may be manifest only at low concentrations. Both kinds of behavior are exhibited by aqueous electrolytes at both low and high temperatures and pressures (Davies, 1962; Hwang, Ludemann, and Hartmann, 1970; Helgeson and Kirkham, 1976; Pitzer, in press).

The short-range interaction parameter \( \bar{b}_{il} \) for the collision depicted in figure 16. A may be either positive or negative, depending on whether the energetic consequence of “capturing” two solvent dipoles is greater or less than the loss of energy caused by the collision. In contrast, \( \bar{b}_{il} \) is always negative and opposes \( \omega_i^{abs} b_k \) and \( \omega_l^{abs} b_k \) for type b collisions. Because in both cases the forces of attraction between the solvated ions at low temperatures are relatively weak compared to those among the ions and the \( H_2O \) dipoles in their solvation shells, the net contribution by the solvation and short-range interaction terms to \( \mathcal{C}_i - \mathcal{C}_i^{0,abs} \) and \( \mathcal{C}_i - \mathcal{C}_i^{0,abs} \) for type a and b collisions in figure 16 is positive at low temperatures. However, the opposite is true at high temperatures where the forces of attraction among ions and \( H_2O \) dipoles are small compared to those among cations and anions (see below).

**Activity Coefficients**

It follows from eqs (66), (123), (149), and (160) through (162) that the activity coefficients of aqueous species can be expressed as

\[
\log \bar{y}_i = -\frac{A_i Z_i^2 \bar{I}^{1/2}}{\lambda} + \Gamma_i + \omega_i^{abs} \sum_k b_k y_k \bar{I} + \sum_l (b_{il} \bar{y}_l \bar{I}/\psi_i) \quad (165)
\]

\[
\log \bar{y}_l = -\frac{A_l Z_l^2 \bar{I}^{1/2}}{\lambda} + \Gamma_l + \omega_l^{abs} \sum_k b_k y_k \bar{I} + \sum_i (b_{li} \bar{y}_i \bar{I}/\psi_l) \quad (166)
\]

\[
\log \bar{y}_n = \Gamma_n + \omega_n \sum_k b_k y_k \bar{I} + \sum_i (b_{ni} \bar{y}_i \bar{I}/\psi_l) + \sum_l (b_{nl} \bar{y}_l \bar{I}/\psi_l) \quad (167)
\]

and for the kth component consisting of the ith and lth ions we can write

\[
\log \bar{y}_{+k} = -\frac{A_i Z_i Z_l \bar{I}^{1/2}}{\lambda} + \Gamma_i + \frac{\omega_k}{\nu_k} \sum_k b_k y_k \bar{I} + \frac{\nu_{i,k}}{\nu_k} \sum_l (b_{ili} \bar{y}_l \bar{I}/\psi_i) + \frac{\nu_{l,k}}{\nu_k} \sum_i (b_{ili} \bar{y}_l \bar{I}/\psi_i) \quad (168)
\]
where $\Lambda$ and $A_\gamma$ are defined by eqs (121) and (C-1) in app. C and

$$\Gamma_\gamma = \frac{\Gamma_0}{2.303RT},$$  \hspace{1cm} (169)

$$b_{ii} = \frac{\delta_{ii}}{2.303RT},$$  \hspace{1cm} (170)

$$b_{n,i} = \frac{\delta_{n,i}}{2.303RT},$$  \hspace{1cm} (171)

and

$$b_{nl} = \frac{\delta_{nl}}{2.303RT}.$$  \hspace{1cm} (172)

For an electrolyte solution in which the solute is composed of the $k$th "completely" dissociated component consisting of the $i$th and $l$th cation and anion, or a 1:1 or 2:2 electrolyte in which only a single ion pair forms to a significant degree, eq (168) reduces to

$$\log \tilde{\gamma}_{\pm,k} = -\frac{A_\gamma|Z_iZ_l|^{1/2}}{\Lambda} + \Gamma_\gamma + b_{\gamma,k}I$$  \hspace{1cm} (173)

where

$$b_{\gamma,k} = \frac{\omega_k\psi_kb_k + 2\nu_{l,k}\nu_{l,k}b_{il}}{\nu_k^2 \psi_k}.$$  \hspace{1cm} (174)

Similarly, for a two-component solute consisting of an undissociated component corresponding to the $n$th neutral species (for which $b_n = 0$) and the $k$th "completely" dissociated component consisting of the $i$th and $l$th cation and anion, respectively, eq (167) can be written as

$$\log \tilde{\gamma}_n = \Gamma_\gamma + b_{\gamma,n}I$$  \hspace{1cm} (175)

where

$$b_{\gamma,n} = \frac{\omega_n\psi_kb_k + \nu_{l,k}b_{nl} + \nu_{l,k}b_{nl}}{\psi_k}.$$  \hspace{1cm} (176)

Eq (173) is identical to that proposed over 50 years ago by Hückel (1925). Although Hückel derived his equation assuming that the dielectric constant of an electrolyte solution is a linear function of ionic strength (which has since been shown to be incorrect — see above), he nevertheless arrived at an expression that closely approximates the activity coefficients of "completely" dissociated electrolytes at high concentrations (see below). Similarly, eq (175) corresponds in form to the modification of the empirical Setchénow (1892) equation adopted over 50 years ago by Randall and Failey (1927a, b, and c), who demonstrated that the logarithms of the activity coefficients of neutral species in electrolyte solutions are essentially proportional to ionic strength.

The close approximation of activity coefficients afforded by eq (173) is apparent in figures 17 through 21, which depict differences in the values of $\log \gamma_{\pm}$ reported in the literature and those computed from the Debye-
Fig. 17. $\rho^*_\gamma$ (eqs 177 and 178) as a function of stoichiometric ionic strength at 25°C and 1 bar (see text). The symbols shown above and those plotted in figures 18 through 21, 23 through 34, and 45 through 47 represent experimental data taken from Hamer and Wu (1972) and/or Harned and Owen (1958), Robinson and Stokes (1959), Goldberg and Nuttal (1978), Faita, Mussini, and Oggioni (1966), Holmes, Bacs, and Mesmer (1978), Towns, Greeley, and Lietzke (1960), and Greeley and others (1960a and b).
Hückel equation as a function of stoichiometric ionic strength at 25°C and 1 bar. This difference, denoted by the symbol $\rho^*_{\gamma}$, is given by

$$
\rho^*_{\gamma} = \log \gamma_{\pm} + \frac{A_{\gamma}|Z_{\pm}Z_{\ominus}|^{1/2}}{\Lambda} - \Gamma_{\gamma}.
$$

(177)

For a "completely" dissociated single electrolyte designated by the index, $k$,

$$
\rho^*_{\gamma,k} = b_{\gamma,k} 1.
$$

(178)

The values of $A_{\gamma}$, $B_{\gamma}$, and $\Lambda$ used to calculate $\rho^*_{\gamma}$ were taken from tables 1 and 2. The experimental activity coefficients used in the calculations are those reported by Hamer and Wu (1972) and Robinson and Stokes (1959). Comparative values of $\rho^*_{\gamma,k}$ generated from activity coefficients given by Goldberg and Nuttall (1978) for CaCl$_2$ and MgCl$_2$ are shown in figure 21.

Fig. 18. $\rho^*_{\gamma}$ (eqs 177 and 178) as a function of stoichiometric ionic strength at 25°C and 1 bar (see caption of fig. 17).
It can be seen in figures 17 through 21 that the linear curves approximate closely all the values of $\rho^*_{\gamma}$ computed from the experimental data. However, it is also apparent that the distribution of data points for all the electrolytes is not strictly linear. For example, in the case of NH₄Cl (fig. 18) the configuration of the data points is slightly sigmoid. In contrast, those for KCl, KBr, and KF exhibit a slight but systematically increasing departure from linearity toward higher values of $\rho^*_{\gamma}$ with increasing ionic strength. It can also be seen that some of the data points for many of the electrolytes (notably SrBr₂, MgCl₂, HNO₃, KCl, KBr, KI, FeCl₂, and BaI₂) fall slightly above or below the linear curves in the dilute region of concentration. These observations underscore the fact that eq (173) affords no more than a close approximation of the dependence of

Fig. 19. $\rho^*_{\gamma}$ (eqs 177 and 178) as a function of stoichiometric ionic strength at 25°C and 1 bar (see caption of fig. 17).
Fig. 20. \( \rho^* \) (eqs 177 and 178) as a function of stoichiometric ionic strength at 25°C and 1 bar (see caption of fig. 17).

Fig. 21. \( \rho^* \) (eqs 177 and 178) as a function of stoichiometric ionic strength at 25°C and 1 bar (see caption of fig. 17).
Table 5
Extended term parameters (b) for computing osmotic and mean ionic activity coefficients of aqueous electrolytes at 25°C and 1 bar from eqs (173) and (193)\(^a\)

<table>
<thead>
<tr>
<th>Solute</th>
<th>( b_{\gamma} )</th>
<th>I Limit</th>
<th>Max. Resid.</th>
<th>Solute</th>
<th>( b_{\gamma} )</th>
<th>I Limit</th>
<th>Max. Resid.</th>
</tr>
</thead>
<tbody>
<tr>
<td>HCl</td>
<td>0.125</td>
<td>2.0</td>
<td>0.006</td>
<td>MgBr(_2)</td>
<td>0.132</td>
<td>5.4</td>
<td>0.011</td>
</tr>
<tr>
<td>LiCl</td>
<td>0.140</td>
<td>5.5</td>
<td>0.014</td>
<td>SrBr(_2)</td>
<td>0.079</td>
<td>3.0</td>
<td>0.005</td>
</tr>
<tr>
<td>NH(_4)Cl</td>
<td>0.020</td>
<td>7.4</td>
<td>0.009</td>
<td>CaBr(_2)</td>
<td>0.097</td>
<td>3.6</td>
<td>0.007</td>
</tr>
<tr>
<td>KCl</td>
<td>0.024</td>
<td>5.0</td>
<td>0.012</td>
<td>BaBr(_2)</td>
<td>0.056</td>
<td>3.0</td>
<td>0.006</td>
</tr>
<tr>
<td>NaCl</td>
<td>0.064</td>
<td>5.0</td>
<td>0.011</td>
<td>HI</td>
<td>0.187</td>
<td>2.5</td>
<td>0.003</td>
</tr>
<tr>
<td>MgCl(_2)</td>
<td>0.106</td>
<td>6.0</td>
<td>0.015</td>
<td>LiI</td>
<td>0.190</td>
<td>3.0</td>
<td>0.020</td>
</tr>
<tr>
<td>SrCl(_2)</td>
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<td>3.6</td>
<td>0.006</td>
<td>KI</td>
<td>0.035</td>
<td>4.5</td>
<td>0.005</td>
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<td>CaCl(_2)</td>
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<td>5.5</td>
<td>0.008</td>
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<tr>
<td>BaCl(_2)</td>
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<td>0.003</td>
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<td>3.0</td>
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</tr>
<tr>
<td>FeCl(_2)</td>
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<td>6.0</td>
<td>0.012</td>
<td>SrI(_2)</td>
<td>0.104</td>
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</tr>
<tr>
<td>MnCl(_2)</td>
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<td>CaI(_2)</td>
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<td>AlCl(_3)</td>
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<td>BaI(_2)</td>
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<tr>
<td>HBr</td>
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<td>0.005</td>
<td>KF</td>
<td>0.064</td>
<td>5.5</td>
<td>0.010</td>
</tr>
<tr>
<td>LiBr</td>
<td>0.156</td>
<td>3.0</td>
<td>0.006</td>
<td>KOH</td>
<td>0.120</td>
<td>4.5</td>
<td>0.010</td>
</tr>
<tr>
<td>KBr</td>
<td>0.023</td>
<td>5.5</td>
<td>0.010</td>
<td>NaOH</td>
<td>0.085</td>
<td>4.5</td>
<td>0.004</td>
</tr>
<tr>
<td>NaBr</td>
<td>0.077</td>
<td>4.0</td>
<td>0.008</td>
<td>HNO(_3)</td>
<td>0.062</td>
<td>6.0</td>
<td>0.011</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>LiNO(_3)</td>
<td>0.093</td>
<td>3.0</td>
<td>0.003</td>
</tr>
</tbody>
</table>

\(^a\)The values of \( b_{\gamma} \) given in this table correspond to the slopes of the linear curves in figures 17 through 21. \(^b\)kg mole\(^{-1}\). \(^c\)The I limit designates the maximum stoichiometric ionic strength for which the values of \( b_{\gamma} \) are applicable. \(^d\)The maximum residual denotes the largest discrepancy between the linear curves and data points shown in figures 17 through 21.
activity coefficients on ionic strength at high concentrations, partly because it contains no provision for short-range interaction among ions of like charge. Similarly, because eq (142) is an empirical approximation, uncertainties in the values of ā employed in the calculations are not uniform. Nevertheless, in all but two of the cases shown in figures 17 through 21, the linear curves correspond to values of \( \rho^{*\gamma} \) that are within 0.015 log units of the experimental values at ionic strengths ranging up to \( \sim 8 \), depending on the electrolyte. The two exceptions are AlCl₃ and LiI, which exhibit discrepancies of 0.032 and 0.020 log units at low ionic strengths. In contrast, the discrepancies for more than two thirds of the electrolytes shown in figures 17 through 21 are \( \leq 0.010 \), and in most cases the corresponding error in \( \gamma_\pm \) is less than 1 to 2 percent. Differences in \( \rho^{*\gamma} \) of 0.010 to 0.015 log units correspond to differences in \( \gamma_\pm \) of the order of 0.04 or less, which as a rule introduce insignificant error in geochemical calculations.

Because contributions by ion association to departures from ideality are apparently negligible for the electrolytes depicted in figures 17 through 21, \( I = I \) and the slopes of the curves correspond to \( b_{\gamma,k} \) in eqs (173) and (178). These values of \( b_{\gamma,k} \) are given in table 5, together with the maximum uncertainty and range of ionic strength represented by the experimental data from which they were derived.

The close agreement between the linear curves and symbols at both high and low ionic strengths in figures 17 through 21 strongly supports the validity of the values of ā computed independently from electrostatic considerations and the standard state properties of aqueous ions. It can be deduced from figure 22B that slight errors in ā result in a non-linear

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**Fig. 22.** Diagram A: Correlation of \( b_{\gamma} \) with ā for alkaline earth halides at 25°C and 1 bar. The values represented by the symbols correspond to those given in tables 2 and 5. Diagram B: Hypothetical values of \( \rho^{*\gamma} \) for KCl at 25°C and 1 bar generated assuming \( I = I \) from data tabulated by Hamer and Wu (1972) using alternate values of ā (labeled in angstroms) to demonstrate the sensitivity of eq (178) to errors in ā.
Fig. 23. $\rho^*_\gamma$ (eqs 177 and 178) as a function of stoichiometric ionic strength (see caption of fig. 17).
### Table 6

Alternate extended term parameters \( b_y \) for computing osmotic and mean ionic activity coefficients of highly concentrated aqueous electrolytes at 25\(^\circ\)C and 1 bar from eqs (173) and (193) — see text\(^a\)

<table>
<thead>
<tr>
<th>Solute</th>
<th>( b_y )</th>
<th>I ( \text{Limit} )</th>
<th>Max. Resid.</th>
<th>Solute</th>
<th>( b_y )</th>
<th>I ( \text{Limit} )</th>
<th>Max. Resid.</th>
</tr>
</thead>
<tbody>
<tr>
<td>HCl</td>
<td>0.139</td>
<td>12.0</td>
<td>0.030</td>
<td>CaBr(_2)</td>
<td>0.115</td>
<td>12.0</td>
<td>0.076</td>
</tr>
<tr>
<td>LiCl</td>
<td>0.146</td>
<td>12.0</td>
<td>0.036</td>
<td>BaBr(_2)</td>
<td>0.061</td>
<td>6.0</td>
<td>0.015</td>
</tr>
<tr>
<td>MgCl(_2)</td>
<td>0.117</td>
<td>12.0</td>
<td>0.058</td>
<td>HI</td>
<td>0.202</td>
<td>8.0</td>
<td>0.041</td>
</tr>
<tr>
<td>SrCl(_2)</td>
<td>0.076</td>
<td>10.5</td>
<td>0.039</td>
<td>NaI</td>
<td>0.101</td>
<td>12.0</td>
<td>0.033</td>
</tr>
<tr>
<td>CaCl(_2)</td>
<td>0.093</td>
<td>12.0</td>
<td>0.058</td>
<td>MgI(_2)</td>
<td>0.168</td>
<td>12.0</td>
<td>0.077</td>
</tr>
<tr>
<td>BaCl(_2)</td>
<td>0.041</td>
<td>5.4</td>
<td>0.008</td>
<td>SrI(_2)</td>
<td>0.112</td>
<td>6.0</td>
<td>0.020</td>
</tr>
<tr>
<td>AlCl(_3)</td>
<td>0.104</td>
<td>10.8</td>
<td>0.066</td>
<td>CaI(_2)</td>
<td>0.123</td>
<td>6.0</td>
<td>0.024</td>
</tr>
<tr>
<td>HBr</td>
<td>0.187</td>
<td>11.0</td>
<td>0.066</td>
<td>BaI(_2)</td>
<td>0.099</td>
<td>6.0</td>
<td>0.023</td>
</tr>
<tr>
<td>LiBr</td>
<td>0.178</td>
<td>10.0</td>
<td>0.074</td>
<td>KF</td>
<td>0.070</td>
<td>14.0</td>
<td>0.029</td>
</tr>
<tr>
<td>NaBr</td>
<td>0.080</td>
<td>9.0</td>
<td>0.013</td>
<td>KOH</td>
<td>0.123</td>
<td>12.0</td>
<td>0.026</td>
</tr>
<tr>
<td>MgBr(_2)</td>
<td>0.141</td>
<td>12.0</td>
<td>0.070</td>
<td>NaOH</td>
<td>0.098</td>
<td>11.0</td>
<td>0.054</td>
</tr>
<tr>
<td>SrBr(_2)</td>
<td>0.086</td>
<td>6.0</td>
<td>0.020</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\(^a\)The values of \( b_y \) shown in this table correspond to the slopes of the curves for the corresponding electrolytes in figures 23 through 25 and 27. \( b_y \) \( \text{kg mole}^{-1} \) \( \text{cm} \) is the I limit designates the maximum stoichiometric ionic strength for which the values of \( b_y \) are applicable. \(^d\)The maximum residual denotes the largest discrepancy between the linear curves and data points shown in figures 23 through 25 and 27.
dependence of $\rho^{*\gamma}$ on I. In addition, it can be seen that an error of only a few tenths of an angstrom in $\hat{a}$ may introduce a sigmoid distribution of $\rho^{*\gamma}$ as a function of I. The sensitivity of $\rho^{*\gamma}$ to such errors is largely responsible for difficulties encountered in deriving values of both $\hat{a}$ and $b_{\gamma}$ by two-parameter regression of activity coefficient data with eq (173), which commonly leads to non unique values of the fit coefficients (see above). The latter observation is consistent with the correlation of $b_{\gamma}$ with $\hat{a}$ in figure 22A, where it can be seen that $b_{\gamma}$ for electrolytes sharing a common anion or cation can be expressed as a linear function of $\hat{a}$ at 25°C and 1 bar. A similar (but not necessarily linear) correlation of $b_{\gamma}$ with $\hat{a}$ is apparent in values of these coefficients obtained by successive two-parameter regression of activity coefficient data over different ranges of concentration using eq (173) with $\Gamma_{\gamma}$ set to zero (Helgeson and James, 1968; Guggenheim and Stokes, 1969; Wood, ms and 1975).

The dependence of $\rho^{*\gamma}$ on ionic strength at $I > 6m$ is shown in figures 23 through 25, where it can be seen that the data points for AlCl$_3$ and

Fig. 24. $\rho^{*\gamma}$ (eqs 177 and 178) as a function of stoichiometric ionic strength at 25°C and 1 bar (see caption of fig. 17).
the alkaline earth halides at high ionic strengths depart to a slight but increasing extent from a linear configuration toward higher values of $\rho^*_{\gamma}$ with increasing ionic strength. The opposite behavior is exhibited by MnCl$_2$, HNO$_3$, LiCl, NaOH, KF, KOH, LiBr, LiNO$_3$, and HCl, which depart from linearity toward lower values of $\rho^*_{\gamma}$ at high ionic strengths (see below). The contrasting behavior of these two groups of electrolytes can be attributed to different relative contributions by ion solvation and association to the short range interaction of species responsible for non-Debye-Hückel departures from ideality. Where the linear curves approximate closely the distribution of data points in figures 23 through 25, the behavior of the electrolytes is consistent with "complete" dissociation up to ionic strengths as high as 14 m.

Because the deviations from linearity in the distribution of the data points in figures 23 through 25 are greater at high ionic strengths, the differences between the linear curves and the experimental data plotted in these figures are greater than the corresponding differences in figures 17 through 21. Nevertheless, all the linear curves afford close approximation of the data. It can be seen in table 6 that the values of $b_{\gamma,k}$ corresponding to the slopes of the curves in figures 23 through 25 yield corresponding values of $\rho^*_{\gamma}$ within 0.077 log units of their experimental counterparts at

![Graphs showing $\rho^*_{\gamma}$ as a function of stoichiometric ionic strength at 25°C and 1 bar (see caption of fig. 17).](image-url)
Fig. 26. $\rho^{\gamma}_e$ (eqs 177 and 178) as a function of stoichiometric ionic strength at temperatures from 0° to 100°C at 1 bar and 100° to 275°C at pressures corresponding to liquid-vapor equilibrium.
ionic strengths ranging up to 14. Even discrepancies of this order of magnitude are acceptable for many equilibrium and mass transfer calculations in geochemistry. However, few such calculations are concerned with such concentrated electrolytes. For this reason, as well as for the sake of accuracy, the values of \( b_{\gamma,k} \) given in table 5 were used in the calculations reported below in preference to those in table 6.

It can be seen in figure 26 that with the exception of HCl at temperatures \( \sim 225^\circ\text{C} \) and HBr at temperatures \( > 150^\circ\text{C} \), the linear dependence of \( \rho^*_{\gamma} \) on \( I \) for these electrolytes at \( 25^\circ\text{C} \) and 1 bar is manifested also at high temperatures. In addition to experimental uncertainties, the apparent discrepancies above \( 150^\circ\) and \( 225^\circ\text{C} \) in figure 26 almost certainly

---

**Fig. 27.** \( \rho^*_{\gamma} \) (eqs 177 and 178) as a function of stoichiometric ionic strength at \( 25^\circ\text{C} \) and 1 bar (see caption of fig. 17).
arise in part from ion association, which also affects significantly the behavior of $\rho^*_{\gamma,NaCl}$ at high temperatures and pressures (see below).

**Effects of ion association on $\rho^*_{\gamma}$**—Although contributions by ion association to departures from ideality are apparently negligible (except as noted above) for the electrolytes depicted in figures 12 through 26, those shown in figures 27 and 28 exhibit behavior consistent with increasing ion association with increasing ionic strength at $I > 8$ to 17, depending on the electrolyte. At lower concentrations, ion association is apparently negligible. Dissociation constants at low temperatures and pressures are not available for most of the electrolytes shown in figures 27 and 28, but log K values of 1.53 ($25^\circ C$), $-0.1$ ($25^\circ C$), 0.57 to 0.77 ($20^\circ C$), 1.45 ($30^\circ C$), and 1.51 ($20^\circ C$) have been reported for $\text{HNO}_3 \rightleftharpoons \text{H}^+ + \text{NO}_3^-$, $\text{MnCl}_2 \rightleftharpoons \text{Mn}^{2+} + \text{Cl}^-$, $\text{NaOH} \rightleftharpoons \text{Na}^+ + \text{OH}^-$, $\text{LiNO}_3 \rightleftharpoons \text{Li}^+ + \text{NO}_3^-$, and $\text{HCl} \rightleftharpoons \text{H}^+ + \text{Cl}^-$, respectively, at 1 bar and infinite dilution (Haase, Ducker, and Küppers, 1965; Masterton and Berka, 1966; Gimblett and Monk, 1954; von Halban and Eisenbrand, 1928; Posner, 1953). Dissocia-

**Fig. 28.** $\rho^*_{\gamma}$ (eqs 177 and 178) as a function of stoichiometric ionic strength at $25^\circ C$ and 1 bar (see caption of fig. 17).
Fig. 29. $\rho^*_\gamma$ (eqs 177 and 178) as a function of stoichiometric ionic strength at 25°C and 1 bar (see caption of fig. 17).
tion constants of this order of magnitude are consistent with formation of appreciable activities of ion pairs only at relatively high concentrations.

It can be seen in figure 29 that behavior similar to that shown in figures 27 and 28 occurs at much lower ionic strengths in the case of NaF, ZnI₂, ZnBr₂, CuCl₂, and ZnCl₂. Reliable dissociation constants are not available for NaF and ZnI₂ at 25°C and 1 bar, but log K values given in the literature for CuCl⁺ ⇌ Cu⁺⁺ + Cl⁻, CuCl₂ ⇌ Cu⁺⁺ + 2 Cl⁻, ZnCl⁺ ⇌ Zn⁺⁺ + Cl⁻, ZnCl₂ ⇌ Zn⁺⁺ + 2 Cl⁻, ZnCl₃ ⇌ Zn⁺⁺ + 3 Cl⁻, ZnCl₄⁻⁻ ⇌ Zn⁺⁺ + 4 Cl⁻, ZnBr⁺ ⇌ Zn⁺⁺ + Br⁻, ZnBr₂ ⇌ ZnBr⁺ + Br⁻, ZnBr₃⁻⁻ ⇌ ZnBr⁺ + Br⁻, and ZnBr₄⁻⁻ ⇌ ZnBr⁺ + Br⁻ at infinite dilution are -0.01, 0.69, -0.43, -0.61, -0.53, -0.20, and (at various concentrations) 0.0, 0.15, 0.30, and 0.52, respectively (Helgeson, 1969; Horne, Holm, and Meyers, 1957). Dissociation constants of this order of magnitude lead to higher degrees of formation of complexes at lower concentrations than those attained in the solutions depicted in figures 27 and 28. The several inflections in the curves for ZnBr₂, ZnI₂, and ZnCl₂ in figure 29 almost certainly result in part from stepwise formation of polyligand complexes. The linear tangents to the curves in figure 29 are explained in later discussion.

Fig. 30. ρ⁻⁻ (eqs 177 and 178) as a function of stoichiometric ionic strength at 25°C and 1 bar (see caption of fig. 17).
Fig. 31. $\rho^*$ (eqs 177 and 178) as a function of stoichiometric ionic strength at 25°C and 1 bar (see caption of fig. 17).
Even smaller dissociation constants than those adduced above result in negative values of $\rho^*\gamma$, which decrease (and in certain cases minimize) with increasing ionic strength. It can be seen in figures 30 through 32 that (among others) this behavior is exhibited by HF, ZnSO$_4$, Li$_2$SO$_4$, CuSO$_4$, Na$_2$SO$_4$, K$_2$SO$_4$, MgSO$_4$, CdBr$_2$, CdI$_2$, and CdCl$_2$, for which the first dissociation constants are $10^{-3.18}$, $10^{-2.38}$, $10^{-1.02}$, $10^{-2.38}$, $10^{-1.00}$, $10^{-1.03}$, $10^{-2.25}$, $10^{-2.23}$, $10^{-2.17}$, and $10^{-2.0}$, respectively, at 25$^\circ$C, 1 bar, and infinite dilution (Ellis, 1963; Nair and Nancollas, 1958; Fisher and Fox, 1977, 1978; Bale, Davies, and Monk, 1956; Nancollas, 1960; Kivalo and Ekari, 1957; Vasilev and Grechina, 1964; Prasad, 1968; Helgeson, 1967, 1969). Because most of these dissociation constants are $< 10^{-2}$, the bulk of the solute is complexed in all but dilute solutions of HF, ZnSO$_4$, CuSO$_4$, MgSO$_4$, CdBr$_2$, CdI$_2$, and CdCl$_2$. In the case of CdBr$_2$, CdI$_2$, and CdCl$_2$,

*Fig. 32. $\rho^*\gamma$ (eqs 177 and 178) as a function of stoichiometric ionic strength at 25$^\circ$C and 1 bar (see caption of fig. 17).*
additional dissociation constant data suggest significant formation of polyligand complexes at high concentration.

Contributions by ion association to the behavior of $\rho^*_{\gamma}$ for AgNO$_3$, KNO$_3$, NaNO$_3$, CsI, and CsBr can also be assessed in figures 30 through 32. The standard state dissociation constants for these species are reported to be $10^{-0.2}$, $10^{0.15}$, $10^{0.3}$ to $10^{0.5}$, $10^{0.03}$, and $10^{-0.03}$, respectively, at 25°C and 1 bar (Masterton and Berka, 1966; Chlebek and Lister, 1966; Robinson, 1937; Hsia and Fuoss, 1968). Corresponding values for RbCl and CsCl are $10^{0.55}$ and $10^{0.39}$ at 25°C, respectively (Smith and Martell, 1976; Dunsmore, Jalota, and Patterson, 1972; Patterson, Jalota, and Dunsmore, 1971). In all seven of these cases, relatively weak ion association appears to be coupled with weak ion solvation, which leads to negative departures from Debye-Hückel theory at low concentrations but positive departures at high. As a consequence, the degree of ion association maximizes with increasing concentration in relatively dilute solutions. In contrast, the degree of ion association as a function of ionic strength at high temperatures apparently maximizes at high concentrations (see below). Differences in the positions of the minima in the curves shown in figures 30 through 32 are a function of the relative size of the dissociation constants and the solvation and short-range interaction parameters for the various aqueous species.

The validity of attributing departures from linearity in $\rho^*_{\gamma}$ as a function of $I$ to ion association can be demonstrated by first combining

![Graphical representation](image-url)

Fig. 33. Graphic representation of eq (181) for electrolytes at 25°C and 1 bar (see text and caption of fig. 17).
eq (68) with appropriate material balance expressions and statements of
the law of mass action representing the dissociation of complexes in solution. For example, in the case of 1:1 or 2:2 electrolytes in which only a
single ion pair forms, we can write eq (68) as

\[ \gamma_{\pm I} = \gamma_{\pm 1}, \]  

(179)

which can be combined with eq (173) and values of \( A_\gamma, B_\gamma, \hat{a}, \) and \( b_\gamma \) to compute \( I \) as a function of \( I \) from experimental values of \( \gamma_{\pm 1} \) and the relation, \( m^* = 1 + I. \) The resulting values of \( I \) can then be used to calculate corresponding values of \( K_n\hat{\gamma}_n \) from the law of mass action representing the
dissociation of the \( n \)th neutral complex, which can be written as

\[ K_n\hat{\gamma}_n = \frac{\gamma_{\pm I}^2}{(1 - I)^\psi_k}, \]

(180)

where \( K_n \) and \( \hat{\gamma}_n \) refer to the dissociation constant and activity coefficient of the subscripted species. Taking account of eq (175) it follows that

\[ \log (K_n\hat{\gamma}_n) - \Gamma_\gamma = \log K_n + b_{\gamma,n}I. \]

(181)

Eq (181) describes the linear curves shown in figures 33 and 34, which
were generated from eq (180) in the manner described above using values

![Fig. 34. Graphic representation of eq (181) for electrolytes at 25°C and 1 bar (see text and caption of fig. 17).](image)
TABLE 7
Short-range interaction parameters \((b_{ij})^a\) for computing activity and osmotic coefficients at 25°C and 1 bar — see text

<table>
<thead>
<tr>
<th>Cation</th>
<th>Anion</th>
<th>(F^-)</th>
<th>(Cl^-)</th>
<th>(Br^-)</th>
<th>(I^-)</th>
</tr>
</thead>
<tbody>
<tr>
<td>H(^+)</td>
<td>(-0.083^c)</td>
<td>0.019(^b)</td>
<td>0.056(^b)</td>
<td>0.103(^b)</td>
<td></td>
</tr>
<tr>
<td>Li(^+)</td>
<td>(-0.182^c)</td>
<td>(-0.049^b)</td>
<td>(-0.024^b)</td>
<td>0.032(^b)</td>
<td></td>
</tr>
<tr>
<td>Na(^+)</td>
<td>(-0.155^c)</td>
<td>(-0.096^b)</td>
<td>(-0.075^b)</td>
<td>(-0.032^b)</td>
<td></td>
</tr>
<tr>
<td>K(^+)</td>
<td>(-0.132^b)</td>
<td>(-0.112^b)</td>
<td>(-0.101^b)</td>
<td>(-0.075^b)</td>
<td></td>
</tr>
<tr>
<td>Rb(^+)</td>
<td>(-0.127^c)</td>
<td>(-0.119^c)</td>
<td>(-0.118^c)</td>
<td>(-0.097^c)</td>
<td></td>
</tr>
<tr>
<td>Cs(^+)</td>
<td>(-0.119^c)</td>
<td>(-0.122^c)</td>
<td>(-0.121^c)</td>
<td>(-0.108^c)</td>
<td></td>
</tr>
<tr>
<td>NH(_4^+)</td>
<td>(-0.131^c)</td>
<td>(-0.114^b)</td>
<td>(-0.107^c)</td>
<td>(-0.087^c)</td>
<td></td>
</tr>
<tr>
<td>Ag(^+)</td>
<td>(-0.134^c)</td>
<td>(-0.086^c)</td>
<td>(-0.067^c)</td>
<td>(-0.037^c)</td>
<td></td>
</tr>
<tr>
<td>Au(^+)</td>
<td>(-0.131^c)</td>
<td>(-0.115^c)</td>
<td>(-0.107^c)</td>
<td>(-0.087^c)</td>
<td></td>
</tr>
<tr>
<td>Cu(^+)</td>
<td>(-0.153^c)</td>
<td>(-0.076^c)</td>
<td>(-0.047^c)</td>
<td>(-0.007^c)</td>
<td></td>
</tr>
<tr>
<td>Mg(^{++})</td>
<td>(-0.451^c)</td>
<td>(-0.256^b)</td>
<td>(-0.167^b)</td>
<td>(-0.085^b)</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Cation</th>
<th>Anion</th>
<th>(F^-)</th>
<th>(Cl^-)</th>
<th>(Br^-)</th>
<th>(I^-)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ca(^{++})</td>
<td>(-0.493^c)</td>
<td>(-0.257^b)</td>
<td>(-0.183^b)</td>
<td>(-0.100^b)</td>
<td></td>
</tr>
<tr>
<td>Sr(^{++})</td>
<td>(-0.503^c)</td>
<td>(-0.266^b)</td>
<td>(-0.204^b)</td>
<td>(-0.113^b)</td>
<td></td>
</tr>
<tr>
<td>Ba(^{++})</td>
<td>(-0.529^c)</td>
<td>(-0.292^b)</td>
<td>(-0.227^b)</td>
<td>(-0.112^b)</td>
<td></td>
</tr>
<tr>
<td>Pb(^{++})</td>
<td>(-0.554^c)</td>
<td>(-0.324^c)</td>
<td>(-0.238^c)</td>
<td>(-0.140^c)</td>
<td></td>
</tr>
<tr>
<td>Zn(^{++})</td>
<td>(-0.462^c)</td>
<td>(-0.259^c)</td>
<td>(-0.186^c)</td>
<td>(-0.098^c)</td>
<td></td>
</tr>
<tr>
<td>Cu(^{++})</td>
<td>(-0.485^c)</td>
<td>(-0.277^c)</td>
<td>(-0.202^c)</td>
<td>(-0.114^c)</td>
<td></td>
</tr>
<tr>
<td>Cd(^{++})</td>
<td>(-0.463^c)</td>
<td>(-0.256^c)</td>
<td>(-0.180^c)</td>
<td>(-0.092^c)</td>
<td></td>
</tr>
<tr>
<td>Hg(^{++})</td>
<td>(-0.491^c)</td>
<td>(-0.278^c)</td>
<td>(-0.198^c)</td>
<td>(-0.104^c)</td>
<td></td>
</tr>
<tr>
<td>Fe(^{++})</td>
<td>(-0.414^c)</td>
<td>(-0.282^b)</td>
<td>(-0.157^c)</td>
<td>(-0.071^c)</td>
<td></td>
</tr>
<tr>
<td>Mn(^{++})</td>
<td>(-0.499^c)</td>
<td>(-0.298^b)</td>
<td>(-0.210^c)</td>
<td>(-0.118^c)</td>
<td></td>
</tr>
<tr>
<td>Al(^{+++})</td>
<td>(-0.481^b)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\(^a\)kg mole\(^-1\). \(^b\)Computed from equation (136) and (174) using the values of \(\omega_1\), \(b_1\), and \(b_2\) given in tables 3 through 5, which also yield \(-0.002\), \(-0.038\), \(-0.126\), and \(-0.064\) for \(b_{\text{HNO}_3}\), \(b_{\text{LiNO}_3}\), \(b_{\text{NaOH}}\), and \(b_{\text{KOH}}\), respectively. \(^c\)Calculated from equations (136) and (174) using values of \(\omega_1\) and \(b_1\) taken from tables 3 and 4, together with estimated values of \(b_2\) taken from table 8.
of \( b_{\gamma,k} \) taken from table 5 or calculated from correlation algorithms (see below). The distribution of the symbols in these figures is a sensitive test of the effect of ion association on stoichiometric activity coefficients. The linear curves shown in figures 33 and 34 are consistent with the nonlinear stoichiometric ionic strength dependence of \( \rho^*_{\gamma} \) for LiNO\(_3\), HNO\(_3\), CsCl, RbCl, CsBr, and CsI in figures 28 and 32.

The departure from linearity of \( \log (K_n\gamma_n) \) for CsI as a function of I at high ionic strengths in figure 33 is almost certainly a manifestation of the formation of CsI\(_2^-\) with increasing I. It should perhaps be emphasized that the intercepts of the curves in figures 33 and 34 are in close agreement with the experimental values of \( \log K_n \) cited above, and the slopes of the curves are similar to those for other neutral species determined experimentally by Randall and Failey (1927a, b, and c). The same observations hold for corresponding curves representing \( \log (K_n\gamma_n) \) for CaSO\(_4\), MgSO\(_4\), CuSO\(_4\), and many other electrolytes in which ion pairs form to a significant degree at 25\(^\circ\)C and 1 bar.

![Fig. 35. Correlation of the extended term parameter \( b_\gamma \) with the standard partial molal entropies of electrolytes at 25\(^\circ\)C and 1 bar. The symbols represent values of \( b_\gamma \) taken from table 5 and those for \( S^* \) generated from the conventional standard partial molal entropies of aqueous ions given in table 3. \( S^*_i \) stands for the conventional standard partial molal entropy of the \( i \)th anion.](image)
Correlation and calculation of $b_\gamma$ and $b_{ii}$.—The values of $b_{\gamma,k}$ in table 5 permit calculation of $b_{ii}$ at 25°C and 1 bar from eq (174) and the values of $b_k$ given in table 4. The results of these calculations are shown in table 7. The other values of $b_{ii}$ shown in table 7 were generated from the estimates of $b_{\gamma,k}$ in table 8, which are consistent with the curves shown in figures 35 and 36. It should perhaps be emphasized in this regard that the estimates in table 7 for the lithium and ferrous halides are more uncertain than the others, owing to discrepancies between the curves and the symbols shown for LiCl, LiBr, LiI, and FeCl$_2$ in figures 35 and 36. Similarly, the paucity of data for fluorides in figure 35 reduces the estimates of $b_{ii}$ for these electrolytes to the status of first approximations.

The correlation of $b_{\gamma,k}$ with $S^o_k$ in figures 35 and 36 is similar to that demonstrated by Burns (1964), who correlated the extended term parameter in the Guggenheim equation with the standard molal entropies of electrolytes at 25°C and 1 bar. The equation of the linear curves in figures 35 and 36 can be expressed as

$$b_{\gamma,k} = a + bS^o_k$$  \hspace{1cm} (182)

Fig. 36. Correlation of the extended term parameter $b_\gamma$ with the standard partial molal entropies of electrolytes at 25°C and 1 bar. The symbols represent values of $b_\gamma$ taken from table 5 and those for $S^o$ generated from the conventional standard partial molal entropies of aqueous ions given in table 3. $S^o_i$ stands for the conventional standard partial molal entropy of the $i$th anion.
where \( a = 0.0755 \) for 1:1 fluorides, 0.1741 for 1:1 chlorides, 0.2518 for 1:1 bromides, and 0.3453 for 1:1 iodides. In contrast, the \( b \) parameter for all 1:1 halides is given by

\[
b = -12 \times 10^{-4} - 19.7 \times 10^{-5} S_{i}\]

where \( S_{i} \) refers to the standard molal entropy of the \( i \)th halide ligand at 25°C and 1 bar. The values of \( a \) and \( b \) for halides with the stoichiometry of MgCl₂ in figure 36 are given by

\[
a = 6.75 \times 10^{-4} + 68.1 \times 10^{-4} S_{i}\]

and

\[
b = -25.5 \times 10^{-4} + 33.5 \times 10^{-6} S_{i}\]

Despite the fact that the estimates of \( b_{\gamma,k} \) for the fluorides in table 8 were generated by extrapolation of the slopes and intercepts of the curves in figures 35 and 36, it can be seen in figure 29 that the value of \( b_{\gamma,NaF} \) (which corresponds to the slope of the linear curve for NaF in the figure) is closely consistent with the experimental data used to calculate \( \rho^{\ast \gamma,NaF} \).

**Table 8**

Estimated extended term parameters \( (b_{\gamma}) \) for computing osmotic and mean ionic activity coefficients of aqueous electrolytes at 25°C and 1 bar from eqs (173) and (193) and the standard partial molal entropies \( (S) \) used in the calculations — see text

<table>
<thead>
<tr>
<th>Solute</th>
<th>( \bar{S} \times 10^{15} )</th>
<th>( b_{\gamma} \times 10^{-3} )</th>
<th>Solute</th>
<th>( \bar{S} \times 10^{15} )</th>
<th>( b_{\gamma} \times 10^{-3} )</th>
<th>Solute</th>
<th>( \bar{S} \times 10^{15} )</th>
<th>( b_{\gamma} \times 10^{-3} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>RbCl</td>
<td>42.4</td>
<td>0.010</td>
<td>ZnBr₂</td>
<td>13.4</td>
<td>0.016</td>
<td>LiF</td>
<td>-0.5</td>
<td>0.076</td>
</tr>
<tr>
<td>CsCl</td>
<td>45.3</td>
<td>-0.001</td>
<td>CuBr₂</td>
<td>16.4</td>
<td>0.011</td>
<td>RbF</td>
<td>25.6</td>
<td>0.061</td>
</tr>
<tr>
<td>AgCl</td>
<td>31.10</td>
<td>0.054</td>
<td>CdBr₂</td>
<td>22.2</td>
<td>0.010</td>
<td>CsF</td>
<td>28.6</td>
<td>0.059</td>
</tr>
<tr>
<td>AuCl</td>
<td>40.1</td>
<td>0.019</td>
<td>HgBr₂</td>
<td>30.9</td>
<td>0.083</td>
<td>AgF</td>
<td>14.3</td>
<td>0.067</td>
</tr>
<tr>
<td>CuCl</td>
<td>23.3</td>
<td>0.004</td>
<td>Rbl</td>
<td>54.3</td>
<td>0.007</td>
<td>AuF</td>
<td>23.3</td>
<td>0.062</td>
</tr>
<tr>
<td>PbCl₂</td>
<td>31.4</td>
<td>0.033</td>
<td>CsI</td>
<td>57.25</td>
<td>-0.011</td>
<td>CuF</td>
<td>6.5</td>
<td>0.072</td>
</tr>
<tr>
<td>ZnCl₂</td>
<td>1.0</td>
<td>0.097</td>
<td>AgI</td>
<td>43.04</td>
<td>0.077</td>
<td>NH₄F</td>
<td>23.4</td>
<td>0.062</td>
</tr>
<tr>
<td>CuCl₂</td>
<td>4.0</td>
<td>0.091</td>
<td>AuI</td>
<td>52.0</td>
<td>0.022</td>
<td>Fe₂O₃</td>
<td>-39.3</td>
<td>0.094</td>
</tr>
<tr>
<td>CdCl₂</td>
<td>9.8</td>
<td>0.079</td>
<td>CuI</td>
<td>35.2</td>
<td>0.126</td>
<td>MnF₂</td>
<td>-24.0</td>
<td>0.053</td>
</tr>
<tr>
<td>HgCl₂</td>
<td>18.5</td>
<td>0.060</td>
<td>NH₄⁺</td>
<td>52.1</td>
<td>0.021</td>
<td>Mg₂O₃</td>
<td>-39.4</td>
<td>0.050</td>
</tr>
<tr>
<td>RbBr</td>
<td>48.6</td>
<td>0.004</td>
<td>Fe₂O₃</td>
<td>18.5</td>
<td>0.150</td>
<td>CaF₂</td>
<td>-19.9</td>
<td>0.038</td>
</tr>
<tr>
<td>CsBr</td>
<td>51.55</td>
<td>-0.011</td>
<td>Mn₂O₄</td>
<td>33.4</td>
<td>0.124</td>
<td>SrF₂</td>
<td>-14.2</td>
<td>0.023</td>
</tr>
<tr>
<td>AgBr</td>
<td>37.34</td>
<td>0.061</td>
<td>PbI₂</td>
<td>55.2</td>
<td>0.087</td>
<td>BaF₂</td>
<td>-4.1</td>
<td>0.004</td>
</tr>
<tr>
<td>AuBr</td>
<td>46.3</td>
<td>0.016</td>
<td>ZnI₂</td>
<td>24.8</td>
<td>0.130</td>
<td>PbF₂</td>
<td>-2.2</td>
<td>0.005</td>
</tr>
<tr>
<td>CuBr</td>
<td>29.5</td>
<td>0.101</td>
<td>CuI₂</td>
<td>27.8</td>
<td>0.133</td>
<td>ZnF₂</td>
<td>-32.6</td>
<td>0.076</td>
</tr>
<tr>
<td>NH₄Br</td>
<td>46.4</td>
<td>0.015</td>
<td>CdI₂</td>
<td>33.6</td>
<td>0.123</td>
<td>CuF₂</td>
<td>-29.6</td>
<td>0.068</td>
</tr>
<tr>
<td>FeBr₂</td>
<td>6.7</td>
<td>0.129</td>
<td>HgI₂</td>
<td>42.3</td>
<td>0.103</td>
<td>CdF₂</td>
<td>-23.8</td>
<td>0.053</td>
</tr>
<tr>
<td>MnBr₂</td>
<td>22.0</td>
<td>0.100</td>
<td>HgF₂</td>
<td>-3.2</td>
<td>0.077</td>
<td>HgF₂</td>
<td>-15.1</td>
<td>0.030</td>
</tr>
<tr>
<td>PbBr₂</td>
<td>43.8</td>
<td>0.059</td>
<td>NaF</td>
<td>10.8</td>
<td>0.069</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\( a \) Calculated from the conventional standard partial molal entropies of ionic species given in table 3. \( \bar{S} \) cal mol⁻¹ (*K*)⁻¹. \( \bar{S} \) Calculated from equations (182) through (184) and the standard partial molal entropies given above. \( d_{kg} \) mol⁻¹.
Even more convincing agreement is apparent in figure 29 between experimental values of $\rho^*_\gamma$ for CuCl$_2$, ZnCl$_2$, ZnBr$_2$, and ZnI$_2$ and the linear curves in the dilute range of concentration, which were generated from the estimates of $b_{\gamma,k}$ for these electrolytes in table 8. It thus appears that short-range interaction parameters computed from estimated values of $b_{\gamma,k}$ can be used with confidence to predict the thermodynamic behavior of partially associated electrolyte solutions.

It can be seen in table 7 that all the short-range interaction parameters are negative, except those for HCl, HBr, HI, and LiI. Note also that the values of $b_{ij}$ for certain cations (such as Rb$^+$ and Cs$^+$) interacting with F$^-$, Cl$^-$, Br$^-$, and I$^-$ differ only slightly from one another. In others, $b_{ij}$ increases substantially for successive interaction of a given cation with F$^-$, Cl$^-$, Br$^-$, and I$^-$. In general, the values of $b_{ij}$ for divalent and trivalent cations interacting with Cl$^-$ are two to three times more negative than those for monovalent interactions.

**Mixed electrolytes.**—The equations and parameters summarized above permit calculation of activity coefficients in mixed as well as single electrolytes at 25°C and 1 bar. For example, the mean ionic activity coefficient of the $k$th component of a mixture of “completely” dissociated electrolytes can be expressed in terms of the ionic strength fractions of the components by combining the identities, $\bar{y}_j = y_j$, $\bar{I} = I$, and $\bar{y}_{\pm,k} = y_{\pm,k}$ with eqs (69) and (144) to give

$$\log \gamma_{\pm,k} = - \frac{\Lambda y_i |Z_i| Z_i|^{1/2}}{\Lambda} + \Gamma_\gamma + \frac{\omega_k}{\nu_k} \sum_k b_{k,y_k} I +$$

$$\frac{\nu_{i,k}}{\nu_k} \sum_l b_{i,l} \left( \sum_k \frac{\nu_{i,k} y_k I}{\psi_k} \right) + \frac{\nu_{l,k}}{\nu_k} \sum_i b_{l,i} \left( \sum_k \frac{\nu_{i,k} y_k I}{\psi_k} \right)$$

(185)

![Figure 57](image)  

Fig. 57. $\delta^*_k$ at 25°C and 1 bar computed from eq (189) as a function of $\hat{\alpha} - \hat{\alpha}_k$ at constant $I$ (indicated by the numbers on the curves) (see text).
where the subscripts $i$ and $l$ again designate the cation and anion in the $k$th component. Similarly, we can write eq (107) for a "completely" dissociated electrolyte solution as

$$m^* = \sum \sum \frac{\nu_{j,k}y_{k}I}{\psi_{k}}$$  \hspace{1cm} (186)

which permits eqs (122) and (169) to be combined to give

$$\Gamma_{\gamma} = -\log \left(1 + 0.0180153 \sum \sum \frac{\nu_{j,k}y_{k}I}{\psi_{k}}\right)$$  \hspace{1cm} (187)

where the subscript $j$ designates both cations and anions.

The difference between the logarithm of the mean ionic activity coefficient of the $k$th component of a mixed electrolyte ($\log \tilde{\gamma}_{\pm,k}$) and that of the component in a single electrolyte solution with the same ionic strength ($\log \tilde{\gamma}^{\circ}_{\pm,k}$) can be expressed as

$$\delta_k \equiv \log \tilde{\gamma}_{\pm,k} - \log \tilde{\gamma}^{\circ}_{\pm,k}$$  \hspace{1cm} (188)

which at $y_{k} = 1$ reduces to

$$\delta^{\circ}_{k} = -\left|Z_i Z_l\right|A_{\gamma}I^{1/2}\left(\frac{1}{\Lambda} - \frac{1}{\Lambda_k}\right)$$  \hspace{1cm} (189)

where $\Lambda$ and $\Lambda_k$ stand for $(1 + \tilde{\alpha}B_{\gamma}I^{1/2})$ and $(1 + \tilde{\alpha}_kB_{\gamma}I^{1/2})$, respectively. It follows that $\delta^{\circ}_{k}$ represents the error in $\log \tilde{\gamma}_{\pm,k}$ resulting from the approximation represented by eq (124) for mixed electrolytes (see above). The magnitude of this error can be assessed in figure 37, where $\delta^{\circ}_{k}$ is plotted as a function of $\tilde{\alpha} - \tilde{\alpha}_k$. Consideration of the range of $\tilde{\alpha}$ values shown in table 2 suggests that in most cases $|\delta^{\circ}_{k}|$ for 1:1 components of mixed electrolyte solutions is $< 0.02$ at $I \leq 3$. The corresponding error for electrolytes with the stoichiometry of MgCl$_2$ is $< 0.04$. As a rule, errors of this order of magnitude are insignificant compared to other uncertainties attending calculation of equilibrium and mass transfer in geochemical processes.

Values of $\log \tilde{\gamma}_{\pm,k}$ computed from eqs (124), (132), and (185) through (187) using values of $A_{\gamma}$, $B_{\gamma}$, $\tilde{\alpha}_k$, $\omega_p$, $b_i$, and $b_{ii}$ given in tables 1 through 4 and 7 are shown in figures 38 through 43 as a function of ionic strength fraction at 25°C, 1 bar, and various ionic strengths. It should perhaps be emphasized that the symbols in these figures represent experimental data which were not used to produce the curves. It can be seen in figures 38 through 43 that in a number of instances the curves generated from the calculations are in close agreement with the experimental data represented by the symbols, but in others the slopes and/or intercepts of the curves differ significantly from those consistent with the trend of the symbols. Nevertheless, all the predicted values of $\log \tilde{\gamma}_{\pm}$ in figures 38 and 39 fall within $\sim 0.02$ log units of the independent experimental values, and those in figure 40 differ from their experimental counterparts by $< 0.04$ log units. As noted above, errors of this order of magnitude are insignificant.
Fig. 38. Predicted (curves) and experimental (symbols) stoichiometric mean activity coefficients of the components of mixed electrolytes at 25°C, 1 bar, and constant ionic strength as a function of the stoichiometric ionic strength fraction of the components (see text). The experimental data represented by the symbols shown above and those in figures 59 through 43 were taken from: cross-hatched circles, Harned (1959, 1960); half-filled circles, Lietzke and Stoughton (1964b); filled squares, Stakhanova, Vasilev, and Epikhin (1963); filled circles, Robinson and Bower (1965, 1966a and b); filled hexagons, Robinson (1961); striped circles, Wu, Rush, and Scatchard (1968); open circles, Lanier (1965); open diamonds, Platford (1968); striped hexagons, Robinson and Coving- ton (1968); circles enclosing dot, Harned and Mason (1931); striped triangles, Harned and Gary (1955a and b). The curves were generated assuming complete dissociation.
in most geochemical calculations. In fact, even the larger discrepancies apparent in figures 41 through 43 have a negligible effect on many equilibrium calculations for natural systems.

The configuration of the curves in figures 38 through 43 is consistent with Harned's rule (Harned, 1935; Harned and Owen, 1958). The fact that in many cases the predicted curves are in remarkable agreement with independent experimental data taken from the literature strongly supports the theoretical basis for eq (185). However, the discrepancies in figures 41 through 43 similarly underscore the fact that (owing in part to

Fig. 39. Predicted (curves) and experimental (symbols) stoichiometric mean activity coefficients of the components of mixed electrolytes at 25°C, 1 bar, and constant ionic strength as a function of the stoichiometric ionic strength fractions of the components (see text and caption of fig. 38).
the oversimplification represented by eq 142) the predictive equations afford no more than a close approximation of non-ideality in electrolyte solutions. At the same time it can be shown that where large differences occur between the curves and symbols, the theoretical calculations are not necessarily responsible for all the error. Relatively large experimental uncertainties attend extraction of activity coefficients from vapor pressure data for mixed electrolytes, and contradictions in values of $\gamma_\pm$ reported by different investigators for the components of the same system are not uncommon. One such example is apparent in figure 43, where the values of $\log \gamma_\pm$, NaCl in NaCl-CaCl$_2$ solutions at $y_{\text{CaCl}_2} = 1$ and $I = 3$ reported by Lanier (1965) and Robinson and Bower (1966b) differ by 0.04 log units.

Fig. 40. Predicted (curves) and experimental (symbols) stoichiometric mean activity coefficients of the components of mixed electrolytes at 25°C, 1 bar, and constant ionic strength as a function of the stoichiometric ionic strength fractions of the components (see text and caption of fig. 38).
Fig. 41. Predicted (curves) and experimental (symbols) stoichiometric mean activity coefficients of the components of mixed electrolytes at 25°C, 1 bar, and constant ionic strength as a function of the stoichiometric ionic strength fractions of the components (see text and caption of fig. 38).
It can be deduced from figures 38 through 43 that in many cases the errors in the predicted values of $\log \gamma_{\pm,k}$ shown in the figures arise solely from $\delta^0_k$, which is independent of the composition of the solution. It thus appears (at least in those instances) that eqs (153), (154), (156), and (157) afford accurate representation of the compositional dependence of $\log \tilde{\gamma}_{\pm,k}$ at constant $I = I$. In other cases, the discrepancies between the trend of the predicted curves and that of the experimental data suggest significant ion association and/or interaction among ions of like charge.

Fig. 42. Predicted (curves) and experimental (symbols) stoichiometric mean activity coefficients of the components of mixed electrolytes at 25°C, 1 bar, and constant ionic strength as a function of the stoichiometric ionic strength fractions of the components (see text and caption of fig. 38).
However, as noted above, the extent to which compositional variation affects $\tilde{\gamma}_{\pm,k}$ is for many electrolytes insignificant compared to uncertainties in dissociation constants used in geochemical calculations.

The relative contributions by $\Gamma_y$ and the solvation and short-range interaction contributions to $\log \tilde{\gamma}_{\pm,k}$ at constant $I = 1$ for a number of mixed electrolytes are depicted in figure 44, where it can be seen that the latter two terms are comparable in magnitude but of opposite sign. Note also that in certain cases at high ionic strength, $\Gamma_y$ contributes significantly to $\log \tilde{\gamma}_{\pm,k}$ by reinforcing the negative contribution by short-range ionic interaction.

---

Fig. 43. Predicted (curves) and experimental (symbols) stoichiometric mean activity coefficients of the components of mixed electrolytes at 25°C, 1 bar, and constant ionic strength as a function of the stoichiometric ionic strength fractions of the components (see text and caption of fig. 38).
Fig. 44. Calculated contributions by $\Gamma_y$, $\tilde{\gamma}_y$, and $\tilde{\gamma}_s$ to $\tilde{\gamma}_s$ of the components of mixed electrolyte solutions at 25°C, 1 bar, and constant “true” ionic strength as a function of the “true” ionic strength fraction of the components. The curves were generated from equations given in the text using parameters taken from tables 3, 4, and 7.
OSMOTIC COEFFICIENTS

Recalling that the subscript \( j \) designates only "free" cations and anions, it follows from eqs (111), (160), and (161) that we can write

\[
\phi = -\frac{2.303}{m^*} \sum_j m_{t,j} \left( \frac{A_r Z_j^2 \sigma(\tilde{A} B_r \tilde{I}^{1/2})}{3} + \frac{\Gamma_\gamma}{0.0180153 m^*} \right.
\]

\[
-\frac{1}{2} \left( \omega_j \sum_k b_{ky} \psi + \sum_i b_{i} \psi_i + \sum_l b_{l} \psi_l \right)
\]

\[
= -\frac{2.303}{u^*} \sum_j \left( \sum_k \nu_{j,k} \tilde{\psi}_k \right) \left( \frac{A_r Z_j^2 \sigma(\tilde{A} B_r \tilde{I}^{1/2})}{3} \right.
\]

\[
+ \frac{\Gamma_\gamma}{0.0180153 m^*} - \frac{1}{2} \left( \omega_j \sum_k b_{ky} \psi_k + \sum_i b_{i} \tilde{\psi}_i + \sum_l b_{l} \tilde{\psi}_l \right)
\]

(190)

where \( b_{ij} \) and \( b_{jl} \) correspond to \( b_{il} \) above with the stipulation that \( b_{ij} = 0 \) if \( j \) designates a cation but \( b_{jl} = 0 \) if \( j \) designates an anion, and

\[
\sigma(\tilde{A} B_r \tilde{I}^{1/2}) \equiv \frac{3}{\tilde{A}^2 B_r \tilde{I}^{1/2}} \left( \Lambda - \frac{1}{\Lambda} - 2 \ln \Lambda \right)
\]

(191)

where \( m^*, \Lambda, \tilde{A}, \) and \( \Gamma_\gamma \) are defined by eqs (107), (121), (124), and (169), and

\[
u^* \equiv \sum_j \frac{\tilde{\psi}_j}{\psi_j} + \sum_q \frac{\tilde{\psi}_q}{\psi_q} = \frac{m^*}{\tilde{I}}
\]

(192)

where the subscript \( q \) again designates complexes in solution. Eq (190) can be written for a "completely" dissociated electrolyte or a 1:1 or 2:2 electrolyte in which only a single ion pair forms to a significant degree as

\[
\phi = -2.303 \left( \frac{|Z_+ Z_-| A_r \tilde{I}^{1/2} \sigma(\tilde{A} B_r \tilde{I}^{1/2})}{3} + \frac{\psi_k \Gamma_\gamma}{0.0180153 \nu_k \tilde{I}} - \frac{b_{y} \tilde{I}}{2} \right)
\]

(193)

The osmotic coefficient analog of eq (177) for \( \rho^*_\phi \) can now be expressed in terms of eq (193) as

\[
\rho^*_\phi = \frac{\phi}{2.303} + \frac{|Z_+ Z_-| A_r \tilde{I}^{1/2} \sigma(\tilde{A} B_r \tilde{I}^{1/2})}{3} + \frac{\psi_k \Gamma_\gamma}{0.0180153 \nu_k \tilde{I}}
\]

(194)

For the \( P \)th completely dissociated single electrolyte solution,

\[
\rho^*_\phi = \frac{b_{y,P} \tilde{I}}{2}
\]

(195)

Values of \( \rho^*_\phi \) computed from experimental osmotic coefficients at 25°C and 1 bar are plotted against \( \tilde{I} \) in figures 45 through 47. As expected,
Fig. 45. $\rho^*_o$ (eqs 194 and 195) as a function of stoichiometric ionic strength at 25°C and 1 bar (see caption of fig. 17.)

Fig. 46. $\rho^*_o$ (eqs 194 and 195) as a function of stoichiometric ionic strength at pressures corresponding to liquid-vapor equilibrium at the temperatures in °C shown in the figure (see caption of fig. 17.)
the slopes of the curves in these figures are consistent with those of the curves representing \( \rho^*_v \) for the same electrolytes in figures 17 through 32. However, the apparent discrepancies between the curves and symbols representing \( \rho^*_v \) are magnified by a factor of 2 in \( \rho^*_v \). The nonlinear distribution of the symbols in figure 47 can be attributed to increasing ion association with increasing concentration (see above). Note in figure 46 that eq (195) is closely consistent with the recent isopiestic measurements reported by Holmes, Baes, and Mesmer (1978) for CaCl\(_2\) at temperatures to 200°C. Similarly, experimental osmotic coefficients for NaCl-BaCl\(_2\) solutions in figure 48 compare favorably with those predicted independently from eq (190).

RELATIVE PARTIAL MOLAL AND APPARENT MOLAL ENTHALPY, HEAT CAPACITY, VOLUME, COMPRESSIBILITY, AND EXPANSIBILITY

Combining appropriate partial derivatives of eqs (165) and (166) with eqs (B-9) through (B-14) in app. B yields expressions for the relative partial molal volumes, enthalpies, entropies, heat capacities, expansi-

**Fig. 47.** \( \rho^*_v \) (eqs 194 and 195) as a function of stoichiometric ionic strength at 25°C and 1 bar (see caption of fig. 17).

**Fig. 48.** Calculated (assuming I = 1) and experimental osmotic coefficients of NaCl-BaCl\(_2\) solutions as a function of the stoichiometric ionic strength fraction of NaCl at 25°C, 1 bar, and I = 3.
bilities, and compressibilities of aqueous ions. Before writing these equations, let us again adopt the index $j$ to designate "free" cations and anions and represent eqs (165) and (166) with the general expression,

$$\log \tilde{\gamma}_j = -\frac{A_y Z_j \Gamma^{1/2}}{\Lambda} + \Gamma_\gamma + \omega_j \sum_k b_k y_k \Gamma + \sum_i \omega_{ij} m_i + \sum_l b_{ji} m_j,$$

(196)

where the subscripts $i$ and $l$ designate cations and anions, respectively, and $b_{ij}$ and $b_{ji}$ correspond to $b_{ii}$ in eqs (165) and (166) with the stipulation that $b_{ij} = 0$, if $j$ refers to a cations, but $b_{ji} = 0$ if $j$ designates an anion. If $r_{e,i}$ is independent of temperature and pressure (see above), it follows from eq (196) and those given in app. A, B, and C that we can write,

$$\nabla_j - \nabla_j^o = \psi_j \left( 2A_y \Lambda \Gamma + A_g \Lambda' \Gamma' - 2A_g \Lambda' \Gamma' \right) / 2 \Gamma^{1/2} \Lambda^2 + \Gamma_\gamma$$

$$+ \sum_k \left( b_{v,k} y_k \Gamma + 2b_{v,k} y_k \Gamma' \right) / 2 + \sum_i \left( b_{v,ij} m_i + 2b_{v,ij} m'_{i,j} \right) / 2$$

$$+ \sum_l \left( b_{v,ji} m_l + 2b_{v,ji} m'_{i,j} \right) / 2 + RT m_{j,L,T} / m_j,$$

(197)

$$L_j = \nabla_j - \nabla_j^o = \psi_j \left( (2A_y \Lambda \Gamma - A_g \Lambda' \Gamma' + 2A_g \Lambda' \Gamma') / 2 \Gamma^{1/2} \Lambda^2 \right) - \Gamma_\gamma$$

$$- \omega_j^{obs} \sum_k \left( b_{v,k} y_k \Gamma + 2b_{v,k} y_k \Gamma' \right) / 2 - \sum_i \left( b_{v,ij} m_i + 2b_{v,ij} m'_{i,j} \right) / 2$$

$$- \sum_l \left( b_{v,ji} m_l + 2b_{v,ji} m'_{i,j} \right) / 2 - RT^2 m_{j,p} / m_j,$$

(198)

and

$$J_j = J_j - \nabla_j^o = \psi_j \left( (\Gamma^{1/2} \Lambda (2A_y \Lambda \Gamma + 3A_g \Lambda' \Gamma' - 2A_g \Lambda' \Gamma' - A_g \Lambda' \Gamma' (\Lambda' + 2A_g \Gamma' + A_g \Gamma' \Lambda' \Gamma' - 2 \Lambda' \Gamma' + (\Lambda' / 2 \Gamma^{1/2} \Lambda)) \right) (2A_y \Lambda 

- A_g \Lambda' \Gamma' + 2A_g \Gamma' \Lambda' \Gamma') / 2 \Gamma^{1/2} \Lambda^2 \right) - \Gamma_j - \omega_j^{obs} \sum_i \left( b_{v,k} y_k \Gamma + b_{v,k} y_k \Gamma' \right) + ((b_{v,k} + 2b_{v,k} y_k \Gamma' / \Gamma) / 2 - \sum_i \left( b_{v,ij} m_i + b_{v,ij} m'_{i,j} \right) + (b_{v,ij} + 2b_{v,ij} m'_{i,j} / \Gamma) / 2 - \sum_l \left( b_{v,ji} m_l + b_{v,ji} m'_{i,j} \right) + (b_{v,ji} + 2b_{v,ji} m'_{i,j} / \Gamma) / 2 - \sum_i \left( b_{v,ij} m_i + b_{v,ij} m'_{i,j} \right) + (b_{v,ij} + 2b_{v,ij} m'_{i,j} / \Gamma) / 2 - \sum_l \left( b_{v,ji} m_l + b_{v,ji} m'_{i,j} \right) + (b_{v,ji} + 2b_{v,ji} m'_{i,j} / \Gamma) / 2$$

(199)
where \( b_k, b_{ij}, b_{jl}, A_G, A_V, b_{v,k}, b_{v,ij} b_{v,jl}, A_H, b_{H,k}, b_{H,ij}, b_{H,jl}, A_J, b_{J,k}, b_{J,ij}, \) and \( b_{J,jl} \) are defined in the text and app. C and

\[
\Lambda'_T \equiv \left( \frac{\partial \Lambda}{\partial P} \right)_{T, \tilde{\mu}(k)} = \frac{\bar{\Delta} B_T I_T}{2 I^{1/2}} + \bar{\Delta} I^{1/2} \left( \frac{\partial B_T}{\partial P} \right)_T ,
\]

\[
\Lambda'_P \equiv \left( \frac{\partial \Lambda}{\partial T} \right)_{P, \tilde{\mu}(k)} = \frac{\bar{\Delta} B_P I_P}{2 I^{1/2}} + \bar{\Delta} I^{1/2} \left( \frac{\partial B_P}{\partial T} \right)_P ,
\]

and

\[
\Lambda''_P \equiv \left( \frac{\partial \Lambda'_P}{\partial T} \right)_P = \frac{\bar{\Delta} B_P (I_P')^2}{4 I^{3/2}} + \bar{\Delta} I^{1/2} \left( \frac{\partial^2 B_P}{\partial T^2} \right)_P + \frac{\bar{\Delta} I'_P}{2 I^{3/2}} \left( \frac{\partial B_P}{\partial T} \right)_P ,
\]

where \((\partial B_P/\partial P)_T, (\partial B_P/\partial T)_P, \) and \((\partial^2 B_P/\partial T^2)_P, \) correspond to partial derivatives of eq (C-2) in app. C, which can be computed from the values of \( B_v, B_H, \) and \( B_J \) given by Helgeson and Kirkham (1974b) with the aid of eqs (C-9) through (C-11) in app. C, and

\[
I'_T \equiv \left( \frac{\partial I}{\partial P} \right)_{T, \tilde{\mu}(k)} ,
\]

\[
I'_P \equiv \left( \frac{\partial I}{\partial T} \right)_{P, \tilde{\mu}(k)} ,
\]

\[
I''_P \equiv \left( \frac{\partial I'_P}{\partial T} \right)_{P, \tilde{\mu}(k)} ,
\]

\[
m'_j,T \equiv \left( \frac{\partial m_j}{\partial P} \right)_{T, \tilde{\mu}(k)} ,
\]

\[
m'_j,P \equiv \left( \frac{\partial m_j}{\partial T} \right)_{P, \tilde{\mu}(k)} ,
\]

and

\[
m''_j,P \equiv \left( \frac{\partial m'_j,P}{\partial T} \right)_{P, \tilde{\mu}(k)} ,
\]

where the subscript \( j \) represents \( j, i, \) and \( l \) in eqs (197) through (199) and

\[
\Gamma_v \equiv - \frac{0.0180153RT}{1 + 0.0180153 m^*} \left( \frac{\partial m^*}{\partial P} \right)_{T, \tilde{\mu}(k)} ,
\]

\[
\Gamma_H \equiv \frac{0.0180153RT^2}{1 + 0.0180153 m^*} \left( \frac{\partial m^*}{\partial T} \right)_{P, \tilde{\mu}(k)} ,
\]

and

\[
\Gamma_J = \frac{0.0180153RT^2}{1 + 0.0180153 m^*} \left( \frac{\partial^2 m^*}{\partial T^2} \right)_{P, \tilde{\mu}(k)} + \frac{0.0180153RT}{1 + 0.0180153 m^*} \left( \frac{\partial m^*}{\partial T} \right)_{P, \tilde{\mu}(k)} .
\]
where

\[
\left( \frac{\partial m^*}{\partial \bar{P}} \right)_{T, \bar{m}(k)} = \sum_j m'_{j,T} + \sum_q m'_{q,T} \quad (212)
\]

\[
\left( \frac{\partial m^*}{\partial \bar{T}} \right)_{P, \bar{m}(k)} = \sum_j m'_{j,P} + \sum_q m'_{q,P} \quad (213)
\]

and

\[
\left( \frac{\partial^2 m^*}{\partial \bar{T}^2} \right)_{P, \bar{m}(k)} = \sum_j m''_{j,P} + \sum_q m''_{q,P} \quad (214)
\]

where \(m'_{q,T}, m'_{q,P}, \) and \(m''_{q,P}\) correspond to the analogs for the \(q\)th complex of \(m'_{j,T}, m'_{j,P}, \) and \(m''_{j,P}\) for the \(j\)th ion in eqs (206) through (208). Similar equations can be written for the relative partial molal expansibility and compressibility of the \(j\)th ion.

Eqs (197) through (199) permit calculation of the relative partial molal properties of all the components of an electrolyte solution from conservation of mass constraints and the partial derivatives of the law of mass action for the dissociation of complexes in solution. The law of mass action for dissociation of the \(q\)th aqueous complex can be written as

\[
\prod_j \frac{m_{j}^{\nu_{j,a}} \tilde{C}_{j}^{\nu_{j,a}}}{m_{q} \bar{y}_{q}} = \beta_{q}, \quad (214A)
\]

which is consistent with

\[
\sum_j \nu_{j,a} \left( \bar{V}_{j} - \bar{V}^{o}_{j,abs} \right) \frac{1}{RT} - \bar{V}_{q} - \bar{V}^{o}_{q,abs} = \frac{\Delta \bar{V}^{o}_{r,q}}{RT}, \quad (214B)
\]

\[
\frac{\bar{L}_{q}}{RT^2} - \sum_j \nu_{j,a} \frac{\bar{L}_{j}}{RT^2} = \frac{\Delta \bar{H}^{o}_{r,q}}{RT^2}, \quad (214C)
\]

and

\[
\sum_j \left( \frac{2\nu_{j,a} \bar{L}_{j}}{RT^3} - \nu_{j,a} \frac{\bar{L}_{j}}{RT^2} \right) + \frac{J_{q}}{RT^2} - \frac{2\bar{L}_{q}}{RT^2} = \frac{\Delta C^{o}_{p,r,q}}{RT^2} - \frac{2\Delta \bar{H}^{o}_{r,q}}{RT^3} \quad (214D)
\]

where \(\beta_{q}, \Delta \bar{V}^{o}_{r,q}, \Delta \bar{H}^{o}_{r,q}, \) and \(\Delta C^{o}_{p,r,q}\) stand for the overall standard molal dissociation constant and standard partial molal volume, enthalpy, and isobaric heat capacity of dissociation for the \(q\)th aqueous complex, and

\[
\bar{V}_{q} - \bar{V}^{o}_{q,abs} = RT \left( \left( \frac{\partial \ln \bar{y}_{q}}{\partial \bar{P}} \right)_{T, \bar{m}(k)} + \frac{m'_{q,T}}{m_{q}} \right), \quad (214E)
\]
\[ L_q = \Phi_q^* - \Phi_{q, abs} = -RT^2 \left( \left( \frac{\partial \ln \tilde{\gamma}_q}{\partial T} \right)_{P, \tilde{\mu}(k)} + \frac{m_{q,p}}{m_q} \right), \] (214F)

and

\[ J_q = \Phi_{P, q} - \Phi_{P, q, abs} = -R \left( 2T \left( \left( \frac{\partial \ln \tilde{\gamma}_q}{\partial T} \right)_{P, \tilde{\mu}(k)} + \frac{m_{q,p}}{m_q} \right) \right. \]
\[ + \left. T^2 \left( \left( \frac{\partial^2 \ln \tilde{\gamma}_q}{\partial T^2} \right)_{P, \tilde{\mu}(k)} + \frac{m_{q, r}}{m_q} - \frac{m_{q, r, s}}{m_q^2} \right) \right) \] (214G)

where \( m_{q,T}, m_{q,P}, \) and \( m_{q, r, s} \) represent the analogs for the qth aqueous complex of \( m_{j,T}, m_{j,P}, \) and \( m_{j, r, s} \) defined by eqs (206) through (208), and \( \left( \frac{\partial \ln \tilde{\gamma}_q}{\partial P} \right)_{T, \tilde{\mu}(k)}, \left( \frac{\partial \ln \tilde{\gamma}_q}{\partial T} \right)_{P, \tilde{\mu}(k)}, \) and \( \left( \frac{\partial^2 \ln \tilde{\gamma}_q}{\partial T^2} \right)_{P, \tilde{\mu}(k)} \) correspond to partial derivatives (multiplied by 2.303 of eqs (165), (166), or (167), depending on the charge (or lack thereof) on the qth aqueous species. For example, in the case of a single 1:1 or 2:2 electrolyte denoted by the subscript \( k \) in which only one ion pair forms to a significant degree, it follows that we can substitute \( q = n \) in eq (175) and write

\[ \left( \frac{\partial \ln \tilde{\gamma}_q}{\partial P} \right)_{T, \tilde{\mu}(k)} = \frac{\Gamma_v}{RT} + \frac{b_{v,q}^I}{2RT} + b_{v,q}^I T, \] (214H)

\[ \left( \frac{\partial \ln \tilde{\gamma}_q}{\partial T} \right)_{P, \tilde{\mu}(k)} = -\frac{\Gamma_H}{RT^2} + \frac{b_{H,q}^I}{2RT^2} + b_{H,q}^I T, \] (214I)

and

\[ \left( \frac{\partial^2 \ln \tilde{\gamma}_q}{\partial T^2} \right)_{P, \tilde{\mu}(k)} = \frac{\Gamma_J - 2\Gamma_H}{RT^3} + \frac{b_{J,q}^I}{2RT^2} - \frac{b_{H,q}^I}{RT^2} + \frac{b_{H,q}^I P}{2RT^2} + b_{v,q}^I T' \] (214J)

where \( I, I', \) and \( I'' \) are defined by eqs (203) through (205) and

\[ b_{v,q} = 2(2.303)RT \left( \frac{\partial b_{v,q}}{\partial P} \right)_T = \frac{\omega_q^{abs} \psi_k b_{v,k} + \nu_{v,k} b_{v,q} + \nu_{v,k} b_{v,q}}{\psi_k}, \] (214K)

\[ b_{H,q} = 2(2.303)RT^2 \left( \frac{\partial b_{H,q}}{\partial T} \right)_P = \frac{\omega_q^{abs} \psi_k b_{H,k} + \nu_{i,k} b_{H,q} + \nu_{i,k} b_{v,q}}{\psi_k}, \] (214L)

and

\[ b_{I,q} = \left( \frac{\partial b_{H,q}}{\partial T} \right)_P = \frac{\omega_q^{abs} \psi_k b_{I,k} + \nu_{i,k} b_{I,q} + \nu_{i,k} b_{I,q}}{\psi_k}, \] (214M)

where \( b_{v,k}, b_{H,k}, \) and \( b_{I,k} \) are defined by eqs (C-15) through (C-17) in app. C. \( \omega_q^{abs} \) stands for the analog of \( \omega_q^{abs} \) for the qth aqueous complex, \( v_i \) and \( v_i \) refer to the number of moles of the cation and anion, respectively (mole of the kth electrolyte)\(^{-1}\), and \( b_{v,q}, b_{H,q}, b_{H,q}, b_{I,q}, b_{I,q}, b_{I,q}, \) and \( b_{I,q} \) correspond to short-range interaction parameters for the qth, ith, and \( l \)th aqueous species analogous to those defined for the jth and \( j \)th species by eqs (C-20) through (C-22) in app. C.
Note that in addition to eqs (214H) through (214M), we can also write for the \( k \)th 1:1 or 2:2 electrolyte in which a single ion pair forms to a significant degree,
\[
I = \psi_k m_i = \psi_k m_i
\]
\[
I = \psi_k m_k = \psi_k m_i + \psi_k m_q
\]
\[
m^* = (1 + I)/\psi_k
\]
and
\[
m_q = (1 - I)/\psi_k
\]
where \( i, l, \) and \( q \) again refer to the cation, anion, and ion pair, respectively, in the \( k \)th electrolyte. Hence,
\[
I'_T/\psi_k = m'_{i,T} = - m'_{q,T} = \left( \frac{\partial m^*}{\partial P} \right)_{T,\tilde{n}(k)}
\]
\[
I'_P/\psi_k = m'_{i,P} = m'_{q,P} = \left( \frac{\partial m^*}{\partial T} \right)_{P,\tilde{n}(k)}
\]
and
\[
I''_P/\psi_k = m''_{i,P} = m''_{q,P} = \left( \frac{\partial^2 m^*}{\partial T^2} \right)_{P,\tilde{n}(k)}
\]

Fig. 49. Relative apparent molal enthalpy of NaCl as a function of stoichiometric ionic strength at 1 bar and 25°, 50°, 75°, and 100°C. The solid and dashed curves, respectively, were generated with and without provision for ion association from equations given in the text.
The equations summarized above permit calculation of the relative partial molal properties of aqueous electrolytes from standard partial molal properties of dissociation and the regression parameters summarized below. Integration of eqs (197) through (199) with respect to I at constant pressure, temperature, and $\bar{y}_k$ permits calculation of the apparent molal properties of the solute and the relative partial molal properties of the solvent from eqs (103) and (104). Values of $\phi_L$ computed in this way for NaCl can be compared with their experimental counterparts in figure 49, where it can be seen that the calculated relative apparent molal enthalpies are in close agreement with those reported by Messikomer and Wood (1975) at temperatures to 100°C. It is also apparent in figure 49 that $\phi_L$ for NaCl is affected dramatically by ion association as a function of concentration and temperature. The parameters used to generate the curves shown in figure 49 are derived below, together with equations for extending the calculations to 5 kb and 600°C. The latter equations together with eq (96) permit comprehensive prediction of the densities and heat capacities of NaCl solutions at high pressures and temperatures (Helgeson, 1981).

The relative partial molal properties of the kth "completely" dissociated single electrolyte for which $\bar{\alpha}$ is independent of pressure and temperature can be computed from

$$V_k - V_0^k = \frac{\psi_k A_y \bar{\alpha} B_0 I^{1/2}}{\Lambda} + \frac{\psi_k A_y \bar{\alpha} B_y I}{\Lambda^2} + \frac{\nu_k b_{y,k} I}{2},$$  \hfill (222)

$$L_k \equiv H_k - H_0^k = \frac{\psi_k A_y \bar{\alpha} B_0 I^{1/2}}{\Lambda} - \frac{\psi_k A_y \bar{\alpha} B_y I}{\Lambda^2} - \frac{\nu_k b_{y,k} I}{2},$$  \hfill (223)

$$J_k \equiv C_{P,k} - C_{P,0,k} = \frac{\psi_k A_y \bar{\alpha} B_0 I^{1/2}}{\Lambda} - \frac{\psi_k \bar{\alpha} (2A_0 B_H - A_G B_T) I}{2(2.303) R T^2 \Lambda^2} + \frac{\psi_k A_y \bar{\alpha}^2 B_H H_I^{3/2}}{2.303 R T^2 \Lambda^3} - \frac{\nu_k b_{y,k} I}{2}$$  \hfill (224)

$$E_{x,k} - E_{x,0,k} = \frac{\psi_k A_y \bar{\alpha} B_0 I^{1/2}}{\Lambda} - \frac{\psi_k \bar{\alpha} (A_y B_H - A_H B_y + A_G B_T) I}{2(2.303) R T^2 \Lambda^2}$$

$$- \frac{\psi_k A_y \bar{\alpha} B_H B_y I^{3/2}}{2.303 R T^2 \Lambda^3} + \frac{\nu_k b_{x,k} I}{2}$$  \hfill (225)

and

$$- (\tilde{k}_k - \tilde{k}_0^k) = \frac{\psi_k A_y \bar{\alpha} B_0 I^{1/2}}{\Lambda} - \frac{\psi_k \bar{\alpha} (2A_y B_T + A_G B_y) I}{2(2.303) R T \Lambda^2}$$

$$- \frac{\psi_k A_y \bar{\alpha} B_T I^{3/2}}{2.303 R T \Lambda^3} - \frac{\nu_k b_{x,k} I}{2}$$  \hfill (226)

where $A_y, B_y, b_{y,k}, A_H, B_H, b_{H,k}, A_J, B_J, b_{J,k}, A_{E_x}, B_{E_x}, b_{E_x,k}, A_K, B_K,$ and $b_{x,k}$ are defined by eqs (C-4) through (C-13) and (C-26) through (C-30) in app. C. Eqs (222) through (226) follow from eqs (B-2) through (B-7) in app. B and the partial derivatives of eq (173) at constant $I = 1$. 

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Taking account of eqs (103) and (222) through (226), the apparent molal volume, enthalpy, heat capacity, expansibility, and compressibility of the solute in a single "completely" dissociated electrolyte solution can be expressed as

\[
\phi_{v,k} = \tilde{V}^o_k + \psi_k A_{v1}^{1/2} \tilde{\alpha}^* + 2\psi_k A_{v2} B_{v} \beta(\tilde{a} B_{v} I^{1/2}) + \frac{\nu_k b_{v,k} I}{4},
\]

(227)

\[
\phi_{h,k} = \tilde{H}^o_k + \psi_k A_{h1}^{1/2} \tilde{\alpha}^* - 2\psi_k A_{h2} B_{h} \beta(\tilde{a} B_{h} I^{1/2}) - \frac{\nu_k b_{h,k} I}{4},
\]

(228)

\[
\phi_{c_{p},k} = C_{p,k}^o + \psi_k A_{c1}^{1/2} \tilde{\alpha}^* - \frac{\psi_k (2A_{h1} B_{h} - A_{c} B_{c} T) \beta(\tilde{a} B_{c} I^{1/2})}{2.303 R T^2} + \frac{2\psi_k A_{h2} B_{h}^2 \xi(\tilde{a} B_{h} I^{1/2})}{2.303 R T^2} - \frac{\nu_k b_{c,k} I}{4},
\]

(229)

\[
\phi_{e_{x},k} = E_{x,k}^o + \psi_k A_{e1}^{1/2} \tilde{\alpha}^* - \frac{\psi_k (A_{v1} B_{v} - A_{h1} B_{h} + A_{c} B_{c} E_{x} T) \beta(\tilde{a} B_{c} I^{1/2})}{2.303 R T^2} - \frac{2\psi_k A_{v2} B_{v} \xi(\tilde{a} B_{v} I^{1/2})}{2.303 R T^2} + \frac{\nu_k b_{e,k} I}{4},
\]

(230)

and

\[
-\phi_{s,k} = -\tilde{R}^o_k + \psi_k A_{s1}^{1/2} \tilde{\alpha}^* - \frac{\psi_k (2A_{v2} B_{v} + A_{c} B_{c} \beta(\tilde{a} B_{c} I^{1/2})}{2.303 R T} - \frac{2\psi_k A_{v2} B_{v} \xi(\tilde{a} B_{v} I^{1/2})}{2.303 R T} - \frac{\nu_k b_{s,k} I}{4}
\]

(231)

where

\[
\tilde{\alpha}^* \equiv \frac{1}{\Lambda} - \frac{\sigma(\tilde{a} B_{c} I^{1/2})}{3}
\]

(232)

\[
\beta(\tilde{a} B_{c} I^{1/2}) \equiv \frac{1}{3 \tilde{a} B_{c}^2 I} \left( \frac{\Lambda^2}{2} - 3\Lambda + \frac{1}{\Lambda} + 3 \ln \Lambda + 1.5 \right)
\]

(233)

and

\[
\xi(\tilde{a} B_{v} I^{1/2}) \equiv \frac{1}{3 \tilde{a} B_{v}^2 I} \left( \frac{\Lambda^2}{2} - 4\Lambda + 6 \ln \Lambda + \frac{4}{\Lambda} - \frac{1}{2\Lambda^2} \right)
\]

(234)

where \( \sigma(\tilde{a} B_{c} I^{1/2}) \) is defined by eq (191).

**Evaluation of Extended Term Parameters**

Calculation of activity coefficients at high pressures and temperatures from the theoretical equations derived above requires equations of state to represent \( A_{v}, B_{v}, b_{v,k}, b_{v}, b_{v} \), and their partial derivatives as a function of pressure and temperature. These equations have already been derived for the Debye-Hückel parameters (table 1) from the thermodynamic/electrostatic properties of \( \text{H}_2\text{O} \) (Helgeson and Kirkham, 1974b). However, because values of \( b_{v,k}, b_{v}, b_{v} \) and their partial derivatives cannot yet be calculated \textit{a priori} from theoretical considerations, derivation of corresponding equations for the extended term parameters (see below) requires
analysis of experimental enthalpy, heat capacity, volume, and compressibility data for a large number of electrolytes to assess the behavior of these parameters as a function of pressure and temperature.

Enthalpy.—The enthalpy analog of $\rho^*_{\gamma}$ and $\rho^*_{\phi}$ ($\rho^*_{H}$) for the $k$th "completely" dissociated single electrolyte is given by

$$\rho^*_{H,k} \equiv \phi_{H,k} - \psi_k \alpha_{H}^{1/2} \alpha^* + 2 \psi_k \alpha_{H} \beta \beta_{H} \beta_{H}^{1/2} = \Omega_{H,k} - \frac{\nu_k b_{H,k} I}{4}$$  \hspace{1cm} (235)

which can also be expressed in terms of the heat of dilution of the electrolyte ($\Delta H_{d,k}$) by first taking account of the identity,

$$\Delta H_{d,k} = \phi_{H,k,f} - \phi_{H,k,i}$$  \hspace{1cm} (236)

Fig. 50. $\rho^*_{\Delta H_d}$ (eq 237) computed (assuming $I = 1$) from heats of dilution reported by Leung and Millero (1975a and b) as a function of the ionic strength of the solution after dilution from the same initial concentration at 30°C and 1 bar.
Fig. 51. $\rho^*_{\Delta H_d}$ (eq 237) computed (assuming I = 1) from heats of dilution reported by Leung and Millero (1975a and b) as a function of the ionic strength of the solution after dilution from the same initial concentration at 30°C and 1 bar.
where \( A_H, A_r, B_H, \) and \( b_H \) are defined in app. C, \( \bar{\alpha}^* \) and \( \bar{\beta}(\bar{d}_B, I^{1/2}) \) are given by eqs (232) and (233), and the subscripts \( i \) and \( f \) stand for the initial and final states of the system. It follows from eqs (235) and (236) that

\[
\rho^{\Delta H_{d,k}} = \rho^{\star H,k,f} - \rho^{\star H,k,i} = \Delta H_{d,k} - \psi_k A_H (I_f^{1/2} \bar{\alpha}^*_f - I_i^{1/2} \bar{\alpha}^*_i) + 2 \psi_k A_r B_H (\beta(\bar{d}_B, I^{1/2})_f - \beta(\bar{d}_B, I^{1/2})_i) = \nu_k b_H (I_i - I_f) \overline{4} .
\]

(237)

Hence, for constant \( I_i \), a plot of \( \rho^{\Delta H_{d,k}} \) against \( I_f \) should yield a straight line with a slope and intercept equal to \( -\nu_k b_H I_i/4 \) and \( \nu_k b_H I_i/4 \), respectively. Curves of this kind are depicted in figures 50 and 51, which were generated assuming \( I = I \) from heats of dilution at 30°C and 1 bar given by Leung and Millero (1975a and b) using Debye-Hückel parameters taken from table 1. It can be deduced from these figures that eq (237) is closely consistent with the experimental data for all 17 electrolytes. Similar agreement is apparent in figures 52 and 53 between eq (237) and values of \( \rho^{\Delta H_{d,k}} \) for \( I_f = 0 \) at 25°C and 1 bar reported by Parker (1965) and Lewis and Randall (1961). However, it can be seen in figures 54 and 55 that the heats of dilution given by Fortier, Leduc, and Desnoyers (1974) for NaCl, RbBr, RbCl, RbI, CsBr, LiCl, LiBr, and CsI at 25°C and 1 bar are consistent with eq (237) only at values of \( (I_i - I_f) > 0.1 \). In contrast, their

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Fig. 52. \( \rho^{\Delta H_d} \) (eq 237) computed (assuming \( I = I \)) from heats of dilution reported by Parker (1965) as a function of the ionic strength of the solution before diluting to "infinite" dilution at 25°C and 1 bar.
data for NaF, RbF, CsF, and NaI closely represent the values of $\rho^{\ast}_{\Delta H_d,k}$ at low values of $I_i - I_f$.

Comparison of figures 50, 51, 54, and 55 indicates that the experimental data reported by Fortier, Leduc, and Desnoyers (1974) for dilute solutions of NaCl, LiCl, and RbCl are inconsistent with the experimental heats of dilution reported by Leung and Millero (1975a and b). Nevertheless, the slopes of all the linear curves in figures 54 and 55 are compatible with those shown for the same electrolytes in figure 50 (see below). However, because $\rho^{\ast}_{\Delta H_d}$ is plotted against $I_i - I_f$ in figures 54 and 55, the curves in these figures should pass through the origin. It can be seen that the distribution of the symbols representing the experimental data for NaCl, LiCl, and RbCl in figures 54 and 55 precludes a zero intercept for curves with slopes consistent with those for these electrolytes in figure 50.

Fig. 53. $\rho^{\ast}_{\Delta H_d}$ (eq 287) computed (assuming $I = I$) from heats of dilution taken from Lewis and Randall (1961) as a function of the ionic strength of the solutions before diluting to “infinite” dilution at 25°C and 1 bar.

Fig. 54. $\rho^{\ast}_{\Delta H_d}$ (eq 237) computed (assuming $I = I$) from heats of dilution reported by Fortier, Leduc, and Desnoyers (1974) as a function of the difference between the initial and final ionic strengths of the solution at 25°C and 1 bar.
Fig. 55. \( \rho^*_{\text{H}_{\text{I}l}} \) (eq 237) computed (assuming \( I = I_l \)) from heats of dilution reported by Fortier, Leduc, and Desnoyers (1974) as a function of the difference between the initial and final ionic strengths of the solution at 25°C and 1 bar.
It thus appears that in certain cases the curvature in the distribution of data points in figures 54 and 55 is a consequence of relatively large uncertainties attending measurement of $\Delta H_d$ at small values of $I_i - I_f$. In others (such as RbF and CsF) it almost certainly results from changes in the degree of formation of complexes during the dilution process (see below). If $\Delta \Pi^o_{r,q}$ in eq (214C) is positive, dissociation of complexes accompanying dilution commonly causes $\rho^{\ast}_{\Delta H_d}$ as a function of $I_i$ to depart from linearity toward less negative values with increasing concentration. This behavior is exhibited by all the electrolytes shown in figures 56 through 60. All the linear curves shown in these figures for a given electrolyte are consistent with one another. Note that in the case of both LiCl and NaCl, the change in the departure from linearity with increasing temperature is consistent with an extremum in the curves representing $\log K_{LiCl^+}$ and $\log K_{NaCl^+}$ as a function of temperature, which leads to negative values of $\Delta \Pi^o_{r,LiCl^+}$ and $\Delta \Pi^o_{r,NaCl^+}$ at high temperatures. In the case of NaCl, the existence of such an extremum can be verified independently (see below). Note also in figures 59 and 60 that the values of $\rho^{\ast}_{\Delta H_d}$ for

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Fig. 56. $\rho^{\ast}_{\Delta H_d}$ (eq 237) computed (assuming $I = 1$) from heats of dilution reported by Parker (1965) as a function of the ionic strength of the solution before diluting to "infinite" dilution at 25°C and 1 bar.
NaCl at temperatures $\leq 100^\circ$C computed from relative apparent molal enthalpies generated by Gibbard and others (1974) from vapor pressure data are consistent with the calorimetric measurements reported by Messikomer and Wood (1975).

The heat of solution ($\Delta H_s$) analog to $\rho^*_{\text{H}} (\rho^*_{\Delta H_d})$ is defined by an expression analogous to eq (237) in which $\Delta H_d$ is replaced by $\Delta H_s$. The values of $\rho^*_{\Delta H_s}$ represented by the symbols in figures 61 through 64 were computed from experimental heats of solution reported in the literature. Because all the data shown in figures 61 through 64 were obtained in dilute solutions ($I < 0.01$), the ionic strength distribution of the heat of solution data is not definitive with respect to $b_{H,k}$. In fact, analysis of heat capacity data (see below) indicates that the absolute values of $b_{H,k}$ for most electrolytes are $< 500$ cal mole$^{-1}$ at temperatures $< 200^\circ$C, which is consistent with essentially horizontal curves representing $\rho^*_{\Delta H_s}$ as a function of $I$ on the scale of figures 60 through 64. It thus appears that in contrast to the assumption made by Cobble and his coworkers (Criss and Cobble, 1961; Mitchell and Cobble, 1964; Ahluwalia and Cobble, 1964a and b; Gardner, Jeckel, and Cobble, 1969a; Gardner, Mitchell, and Cobble, 1969b;

![Diagrams](image_url)

Fig. 57. $\rho^*_{\Delta H_d}$ (eq 237) computed (assuming $I = I$) from heats of dilution reported by Parker (1965) as a function of the ionic strength of the solution before diluting to "infinite" dilution at 25°C and 1 bar.
Jeckel, Criss, and Cobble, 1964), the trends of the data points in the figures are not significant. In addition, the accuracy of the data is apparently far less than that implied by the reported experimental uncertainties. These observations lead to the large symbols in the figures, which become larger with increasing dilution to represent increasing uncertainty in $\rho^*_{\Delta H_d}$ with decreasing concentration. The intercepts of the curves were used to calculate standard partial molal heat capacities ($C^0_P$) from finite difference derivatives of the extrapolated heat of solution data (see below). The constraint that $C^0_P$ must vary smoothly with temperature, together with the fact that $b_{H,k}$ is too small to affect significantly the horizontal approximation represented by the linear curves for BaCl$_2$, GdCl$_3$, NaReO$_4$, HReO$_4$, and NaClO$_4$, provided the basis for selecting the relative positions of the intercepts for these electrolytes in figures 61 through 64. In certain instances the scatter in the heat of solution data precluded selection of definitive intercepts. The slopes of the curves shown for NaCl, CsI, and Na$_2$SO$_4$ were calculated independently from predicted values of $b_{H,k}$ as a function of temperature (see below).

Fig. 58. $\rho^*_{\Delta H_d}$ (eq 237) computed (assuming $I = I$) from heats of dilution reported by Parker (1965) as a function of the ionic strength of the solution before diluting to "infinite" dilution at 25°C and 1 bar.
Fig. 59. \( \rho^*_{\Delta H_d} \) (eq 237) computed (assuming \( I = 1 \)) from values of \( \phi_{li,k} \) (\( \phi_{li,k} = \phi_{li,k} - \phi_{li} \)) reported by Ensor and Anderson (1973) at 1 bar and 40°, 50°, 60°, 70°, and 80°C.
Values of \( b_H \) computed from the slopes of the linear curves shown in figures 50 through 58 are given in table 9. Comparative calculations (see below) indicate that the two sets of values at 25\(^\circ\) and 50\(^\circ\)C are consistent with one another as well as with the apparent molal heat capacity data for the electrolytes.

Heat capacity.—The apparent molal heat capacity function analogous to \( \rho^{\ast}_{H,k} (\rho^{\ast}_{J,k}) \) is defined by

\[
\rho^{\ast}_{J,k} \equiv \phi_{cP,k} - \psi_k A_J I^{1/2} \tilde{\alpha} + \psi_k (2A_H B_H - A_G B_J) \beta (\tilde{\alpha} B_J I^{1/2}) / 2.303 R T^2 + \frac{2 \psi_k A_J B_H^2 \zeta (\tilde{\alpha} B_J I^{1/2})}{2.303 R T^2} = C^{\circ}_{P,k} - \nu_k b_{J,k} I \quad (238)
\]

where \( \zeta (\tilde{\alpha} B_J I^{1/2}) \) is given by eq (234). It can be deduced from figures 65 through 69 that eq (238) is closely consistent with experimental values of \( \phi_{cP} \) for a large number of electrolytes at 25\(^\circ\)C and 1 bar. Figures 70 through 78 leave little doubt that this is also true at higher temperatures. The values of \( \rho^{\ast}_{J} \) shown in these (and subsequent) figures were calculated assuming \( I = I \) from eq (238) using values of \( A_J, A_H, B_H, A_G, B_J, A_y, \) and \( B_J \) taken from table 1 and/or computed from equations given in app. C. Although the agreement at high temperatures is to some extent a consequence of the magnification in \( \phi_{cP,k} \) of experimental uncertainties in \( C_{P,solution} \) at \( m_k < 1 \), it can be seen that the bulk of the data is consistent with eq (238). The uncertainty in \( \phi_{cP,k} (\delta_{\phi_{cP,k}}) \) arising from experimental uncertainty in \( C_{P,solution} (\delta_{C_{P,solution}}) \) is given by

\[
\delta_{\phi_{cP,k}} = \delta_{C_{P,solution}} / m_k, \quad (239)
\]

Fig. 60. \( \rho^{\ast}_{\Delta H_d} \) (eq 237) computed (assuming \( I = I \)) from values of \( \phi_{c,m} (\phi_{c,m} = \phi_{H,m} - \tilde{\Omega}^{\ast}_{\eta}) \) reported by Gibbard and Scatchard (1973) and Messikomer and Wood (1975) at 1 bar and 0\(^\circ\), 25\(^\circ\), 50\(^\circ\), 75\(^\circ\), and 100\(^\circ\)C.
Fig. 61. $p^*_\Delta H_s$ (see text) computed (assuming $I = 1$) from heats of solution taken from the literature. The symbols shown above and those in figures 62 through 64 represent experimental data reported by Criss and Cobble (1961), Mitchell and Cobble (1964), Ahiuwarala and Cobble (1964a and b), Gardner, Jeckel, and Cobble (1969), Gardner, Mitchell, and Cobble (1969b), and Jeckel, Criss, and Cobble (1964) for 1 bar and the temperatures in °C shown in the figures.
which is responsible for the increasing symbol size with decreasing concentration in figures 74 through 78. The much greater accuracy of the experimental data plotted in figures 65 through 69 led to the decision to represent all the data shown in these figures with symbols of the same size.

In certain cases (for example, HCl and LiBr in fig. 77) no curves could be drawn through the data points with slopes and intercepts corresponding to a smooth distribution of $-\nu B_1/4$ and $C_P^o$ with temperature. The latter constraints were used to control interpretation of the apparent molal heat capacities represented by the linear curves in figures 70 through 78, which led to the conclusion that many of the data points shown for the more dilute concentrations in these figures are unreliable. The slopes of all the curves in figures 70 through 78 that are drawn through data points reported by different investigators for the same electrolyte are equivalent. However, the intercepts are not (see below).

The effect of ion association on $\rho_{\text{mol}}^*$ as a function of $I$ is manifested by increasing departures from linearity toward more negative values with increasing $I$ at high ionic strengths. It can be seen in figures 78 through

![Fig. 62. $\rho_{\text{mol}}^*$ (see text) computed (assuming $I = 1$) from heats of solution taken from the literature (see caption of fig. 61).](image-url)
Fig. 68. $\rho_{\Delta H_{\text{aq}}}^*$ (see text) computed (assuming $I = 1$) from heats of solution taken from the literature (see caption of fig. 61).
that NaCl as well as many other electrolytes exhibit this behavior at 25°C and 1 bar, which is undoubtedly enhanced (as in the case of Na₂SO₄ in fig. 78) at higher temperatures. Note in figure 82 that ρ*ₗ for MgSO₄ exhibits considerable scatter and nonlinearity in the distribution of the data points from I = 0.5 to 4. In contrast, φ_{Cp,MgSO₄} is a linear function of I, which is consistent with theoretical considerations as well as the stoichiometric analog of eq (175) and the high degree of ion association in magnesium sulfate solutions. For an electrolyte that is "completely" associated in a solvated polar neutral complex, apparent molal properties approximate linear functions of stoichiometric ionic strength.

Fig. 64. ρ*ₗ{ΔHs} (see text) computed (assuming I = I) from heats of solution taken from the literature (see caption of fig. 61).
Values of $b_2$ computed from the slopes of the linear curves for $25^\circ$C in figures 65 through 70, 72, 73, and 79 through 81 are given in table 10. The slopes of the linear curves for temperatures other than $25^\circ$C in figures 70 through 78, together with the intercepts of all the curves in figures 65 through 82 (which correspond to extrapolated values of $C^\circ$) are considered in later discussion.

**Volume and compressibility.**—The volume and compressibility analogs of eq (238) can be written as

$$\rho^*, k = \phi_{\nu,k} - \psi_k A \lambda I^{1/2} \bar{\alpha}^* - 2\psi_k A \lambda B \bar{\beta}(\bar{\alpha} B I^{1/2}) = \bar{\nu}^* \bar{k} + \frac{\nu_k b_{\nu,k} I}{4}$$  \hspace{1cm} (240)

and

$$-\rho^* \kappa, k = -\phi_k, k - \psi_k A \lambda I^{1/2} \bar{\alpha}^* + \frac{\psi_k (2A \lambda B \nu + A \kappa B \nu) \bar{\beta}(\bar{\alpha} B \lambda I^{1/2})}{2.303RT}$$

$$+ \frac{2\psi_k A \lambda B \nu^2 \bar{\varepsilon}(\bar{\alpha} B \lambda I^{1/2})}{2.303RT} = -\bar{\nu}^* \bar{k} - \frac{\nu_k b_{\kappa,k} I}{4}.$$  \hspace{1cm} (241)

**Table 9**

Extended term parameters ($b_\nu$) for computing the relative apparent and partial molal enthalpies of aqueous electrolytes at $25^\circ$C and $30^\circ$C and 1 bar from eqs (223) and (228)

<table>
<thead>
<tr>
<th>Solute</th>
<th>$b^a_\nu$ $25^\circ$C</th>
<th>$b^a_\nu$ $30^\circ$C</th>
</tr>
</thead>
<tbody>
<tr>
<td>HCl</td>
<td>-420$^b$</td>
<td></td>
</tr>
<tr>
<td>LiCl</td>
<td>-146$^b$, -180$^b$</td>
<td></td>
</tr>
<tr>
<td>NaCl</td>
<td>470$^b$, 408$^b$</td>
<td></td>
</tr>
<tr>
<td>KCl</td>
<td>445$^c$, 400$^b$</td>
<td></td>
</tr>
<tr>
<td>RbCl</td>
<td>490$^b$, 440$^b$</td>
<td></td>
</tr>
<tr>
<td>CsCl</td>
<td>620$^c$, 560$^b$</td>
<td></td>
</tr>
<tr>
<td>NH$_4$Cl</td>
<td>120$^b$</td>
<td></td>
</tr>
<tr>
<td>MgCl$_2$</td>
<td>-216$^b$, -232$^b$</td>
<td></td>
</tr>
<tr>
<td>SrCl$_2$</td>
<td>-32$^b$, -31$^b$</td>
<td></td>
</tr>
<tr>
<td>CaCl$_2$</td>
<td>-30$^b$, -103$^b$</td>
<td></td>
</tr>
<tr>
<td>BaCl$_2$</td>
<td>-59$^b$, 9$^b$</td>
<td></td>
</tr>
<tr>
<td>HBr</td>
<td>-274$^b$</td>
<td></td>
</tr>
<tr>
<td>LiBr</td>
<td>-80$^b$</td>
<td></td>
</tr>
<tr>
<td>NaBr</td>
<td>520$^b$</td>
<td>454$^b$</td>
</tr>
<tr>
<td>KBr</td>
<td>540$^b$</td>
<td></td>
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<tr>
<td>RbBr</td>
<td>600$^b$</td>
<td></td>
</tr>
<tr>
<td>CsBr</td>
<td>740$^b$</td>
<td></td>
</tr>
<tr>
<td>NaI</td>
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</tr>
<tr>
<td>KI</td>
<td>660$^b$</td>
<td></td>
</tr>
<tr>
<td>RbI</td>
<td>740$^b$</td>
<td></td>
</tr>
<tr>
<td>CsI</td>
<td>1000$^b$</td>
<td></td>
</tr>
<tr>
<td>NaF</td>
<td>374$^b$, 192$^b$</td>
<td></td>
</tr>
<tr>
<td>KF</td>
<td>80$^b$</td>
<td></td>
</tr>
<tr>
<td>RbF</td>
<td>180$^b$</td>
<td></td>
</tr>
<tr>
<td>CsF</td>
<td>240$^b$</td>
<td></td>
</tr>
<tr>
<td>LiOH</td>
<td>-240$^b$</td>
<td></td>
</tr>
<tr>
<td>NaNO$_3$</td>
<td>615$^c$</td>
<td>520$^b$</td>
</tr>
<tr>
<td>K$_2$SO$_4$</td>
<td>513$^b$</td>
<td></td>
</tr>
<tr>
<td>LiSO$_4$</td>
<td>87$^b$</td>
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<tr>
<td>NH$_4$NO$_3$</td>
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<td>NaNO$_3$</td>
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<tr>
<td>HNO$_3$</td>
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<tr>
<td>AgNO$_3$</td>
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<tr>
<td>LiNO$_3$</td>
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<tr>
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<tr>
<td>KOH</td>
<td>40$^b$</td>
<td></td>
</tr>
</tbody>
</table>

$^a$Cal kg mole$^{-2}$. $^b$Computed from the slopes of the linear curves in figures 50 through 57 (see text). $^c$Calculated from equation (292) for $P = P_f$ using the values of $b_\nu$ shown above for $30^\circ$C and parameters taken from table 19. $^d$Estimated from the value of $b_\nu$ for $30^\circ$C given above and that for $b_j$ at $25^\circ$C in table 10, assuming $b_j$ constant from $25^\circ$ to $30^\circ$C.
It can be seen in figures 83 through 85 (which were generated assuming $I = 1$) that with only a few exceptions, the values of $\phi_v$ reported by Fortier, Leduc, and Desnoyers (1974) for 1:1 electrolytes at 25°C and 1 bar are closely consistent with eq (240). However, in a number of instances (for example, NaCl, KI, KF, CsF, RbBr, KBr, and KCl), the experimental values of $\rho^{*}_{v,k}$ depart from the linear curves toward lower values at high ionic strengths, which can be attributed to the effects of increasing ion association with increasing concentration. In contrast, the curvature in the distribution and apparent displacement of the data points for NaBr, CsI, and CsBr in the dilute region of concentration is probably a consequence of experimental error (see below).

Fig. 65. $\rho^{*}_{v}$ (eq 238) computed (assuming $I = 1$) from apparent molal heat capacities at 25°C and 1 bar taken from the literature. The circles shown above and those in figures 66 through 82 represent experimental data reported by Fortier, Leduc, and Desnoyers (1974), Perron, Desnoyers, and Millero (1974), Perron, Fortier, and Desnoyers (1975), Parker (1965), Lewis and Randall (1961), Tanner and Lamb (1978), Likke and Bromley (1973), Ruterjans and others (1969), Eigen and Wicke (1951, 1954), Wicke, Eigen, and Ackermann (1954), and Ackermann (1958). The apparent molal heat capacities given in the first three of these references were corrected to be consistent with Desnoyers and others (1976). The calculated values of $C^*_v$ represented by the open triangles on the ordinates of the diagrams correspond to those given in tables 13 and 16. The latter values do not in all instances necessarily correspond to the extrapolated intercepts of the linear curves in the diagrams, which are denoted in figures 74 through 76, 78, and 82 by closed triangles (see text).
Fig. 66. $\rho^*(\text{eq 238})$ computed (assuming $I = 1$) from apparent molal heat capacities at 25°C and 1 bar (see caption of fig. 65).
Fig. 67. \( \rho^* \) (eq 238) computed (assuming \( I = 1 \)) from apparent molal heat capacities at 25°C and 1 bar (see caption of fig. 65).

Fig. 68. \( \rho^* \) (eq 238) computed (assuming \( I = 1 \)) from apparent molal heat capacities at 25°C and 1 bar (see caption of fig. 65).
Fig. 69. $\rho^* (eq \ 238)$ computed (assuming $I = 1$) from apparent molal heat capacities at 25°C and 1 bar (see caption of fig. 65).

Fig. 70. $\rho^* (eq \ 238)$ computed (assuming $I = 1$) from apparent molal heat capacities at 1 bar and 15°, 20°, 25°, and 30°C (see caption of fig. 65).
Density data reported by Allam (1963), most of which are for highly concentrated electrolyte solutions, were used to calculate (assuming $I = I$) the values of $\rho_{v,k}^*$ shown in figures 86 through 88 from the relation,

$$
\phi_{v,k} = \frac{1}{m_k} \left( \frac{1000 + m_k \bar{M}_k}{\rho} - \frac{1000}{\rho_{w}^0} \right)
$$

(242)

where $\rho$ and $\rho_{w}^0$ stand for the density of the solution and that of pure H$_2$O, respectively, in g cm$^{-3}$, and $\bar{M}_k$ represents the molecular weight of the $k$th electrolyte in g mole$^{-1}$. Although in certain cases (for example, NaCl, KNO$_3$, AgNO$_3$, KI, NH$_4$I, and KOH) the computed values of $\rho_{v,k}^*$ in figures 86 through 88 exhibit considerable scatter, it can be deduced that most of the densities reported by Allam for ionic strengths < 2 to 8 (depending on the electrolyte) are in close agreement with eq (240). At higher ionic strengths, the data points for all the electrolytes except NH$_4$Cl, LiI, and HNO$_3$ depart to an increasing degree from the linear curves toward lower values of $\rho_{v,k}^*$ with increasing ionic strength, which is consistent with increasing ion association with increasing concentration.

Fig. 71. $\rho_{v,x}^*$ (eq 238) computed (assuming $I = I$) from apparent molal heat capacities at 1 bar (see caption of fig. 65).
With the exception of NaCl, NaBr, KBr, and KI, the slopes of the linear curves for the same electrolyte in figures 83 through 88 are identical. Of the two sets of data for these four electrolytes, those reported by Fortier, Leduc, and Desnoyers (1974) were accepted in the present study. In general, the data reported by Allam are more uncertain than those obtained by Fortier, Leduc, and Desnoyers. This observation is consistent with the fact that the intercepts of the curves (which correspond to $\bar{V}^o_k$) in figures 86 through 88 differ from the values of $\bar{V}^o_k$ represented by the symbols on the ordinate (which correspond to values of $\bar{V}^o_k$ calculated by Helgeson and Kirkham, 1976) by as much as 1.5 cm$^3$ mole$^{-1}$. Although most of these differences are $< 0.5$ cm$^3$ mole$^{-1}$, which is not excessive, the discrepancies at $I = 0$ in figures 86 through 88 are considerably larger than those in figures 83 through 85. Note that most of the intercepts of the linear curves in the latter figures are in remarkably close agreement.

Fig. 72. $\rho^{s_2}$ (eq 238) computed (assuming $I = 1$) from apparent molal heat capacities at 1 bar (see caption of fig. 65).
with the symbols on the ordinate representing values of \( \bar{V}^o \_k \) computed by Helgeson and Kirkham (1976); the maximum difference (which is exhibited by KCl) is < 0.26 cm³ mole⁻¹. This observation, together with the fact that Helgeson and Kirkham used the Redlich-Meyer equation (Redlich and Meyer, 1964) to represent \( \phi_V \) as a function of \( I \) underscores the insensitivity of extrapolated values of \( \bar{V}^o \) to the equation used to represent \( \phi_V \) as a function of ionic strength. The Redlich-Meyer equation is an improvement over the Masson (1929) equation, but it is nevertheless incompatible with eq (178).

It can be deduced from figures 89 through 93 that eq (240) also represents closely density data for a large number of electrolytes as a function of ionic strength at high temperatures. In most cases, comparison of the extrapolated and computed values of \( \bar{V}^o \) reported by Helgeson and Kirkham (1976), which are represented by the solid and open symbols, respec-

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Fig. 73. \( \rho^*_s \) (eq 238) computed (assuming \( I = I \)) from apparent molal heat capacities at 1 bar (see caption of fig. 69).
Fig. 74. $\rho^*_s$ (eq 238) computed (assuming $I = I$) from apparent molal heat capacities at 1 bar and 80°C, together with those at pressures corresponding to liquid-vapor equilibrium and the temperatures $\geq 100^\circ$C shown in the diagrams (see caption of fig. 65).

Fig. 75. $\rho^*_s$ (eq 238) computed (assuming $I = I$) from apparent molal heat capacities at temperatures from 100° to 100°C at 1 bar, together with those at 100° to 130°C and pressures corresponding to liquid-vapor equilibrium (see caption of fig. 65).
Fig. 76. $\rho^*_T$ (eq 238) computed (assuming $I = 1$) from apparent molal heat capacities at temperatures from $10^\circ$ to $100^\circ$C at 1 bar, together with those at $100^\circ$ to $130^\circ$C and pressures corresponding to liquid-vapor equilibrium (see caption of fig. 65).
Fig. 77. $\rho^*$ (eq 238) computed (assuming $I = I$) from apparent molal heat capacities at temperatures from $10^\circ$ to $100^\circ$C at 1 bar, together with those at $100^\circ$ to $130^\circ$C and pressures corresponding to liquid-vapor equilibrium (see caption of fig. 65).

Fig. 78. $\rho^*$ (eq 238) computed (assuming $I = I$) from apparent molal heat capacities at 1 bar and $80^\circ$C, together with those at pressures corresponding to liquid-vapor equilibrium and the temperatures $\geq 100^\circ$C shown in the figure (see caption of fig. 65).
Fig. 80. $p^*$ (kg/cm$^2$) computed assuming $I = 1$ from apparent molal heat capacity at 25°C and 1 bar (see caption of Fig. 69).

Fig. 79. $p^*$ (kg/cm$^2$) computed assuming $I = 1$ from apparent molal heat capacity at 25°C and 1 bar (see caption of Fig. 69).

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Fig. 81. $\rho^*_{ij}$ (eq 238) computed (assuming $I = 1$) from apparent molal heat capacities at $25^\circ C$ and 1 bar (see caption of fig. 65).

Fig. 82. $\rho^*_{ij}$ and $\Phi_{cp}$ (eqs 229 and 238) for MgSO$_4$ computed (assuming $I = 1$) from experimental data reported in the literature (see caption of fig. 65).
### Table 10

Extended term parameters \((b_j)\) for computing the relative apparent and partial molal heat capacities of aqueous electrolytes at 25°C and 1 bar from eqs (224) and (229)

<table>
<thead>
<tr>
<th>Solute</th>
<th>(b_j)</th>
<th>Solute</th>
<th>(b_j)</th>
<th>Solute</th>
<th>(b_j)</th>
</tr>
</thead>
<tbody>
<tr>
<td>HCl</td>
<td>-4.2</td>
<td>NaBr</td>
<td>-13.4</td>
<td>LiNO(_3)</td>
<td>-9.0</td>
</tr>
<tr>
<td>LiCl</td>
<td>-7.2</td>
<td>KBr</td>
<td>-10.2</td>
<td>AgNO(_3)</td>
<td>-22.0</td>
</tr>
<tr>
<td>NaCl</td>
<td>-12.8</td>
<td>RbBr</td>
<td>-12.2</td>
<td>HNO(_3)</td>
<td>-8.6</td>
</tr>
<tr>
<td>KCl</td>
<td>-9.3</td>
<td>CsBr</td>
<td>-14.0</td>
<td>KNO(_3)</td>
<td>-30.0</td>
</tr>
<tr>
<td>RbCl</td>
<td>-10.4</td>
<td>NaI</td>
<td>-16.0</td>
<td>NaNO(_3)</td>
<td>-31.0</td>
</tr>
<tr>
<td>CsCl</td>
<td>-12.6</td>
<td>KI</td>
<td>-12.2</td>
<td>NaN(_2)</td>
<td>-25.2</td>
</tr>
<tr>
<td>NH(_4)Cl</td>
<td>-5.4</td>
<td>RbI</td>
<td>-12.8</td>
<td>LiOH</td>
<td>-12.0</td>
</tr>
<tr>
<td>CaCl(_2)</td>
<td>-4.7</td>
<td>CsI</td>
<td>-18.0</td>
<td>KOH</td>
<td>-13.0</td>
</tr>
<tr>
<td>MgCl(_2)</td>
<td>-3.5</td>
<td>NaF</td>
<td>-19.0</td>
<td>NaOH</td>
<td>-21.0</td>
</tr>
<tr>
<td>BaCl(_2)</td>
<td>-8.7</td>
<td>KF</td>
<td>-11.4</td>
<td>NaBrO(_3)</td>
<td>-27.0</td>
</tr>
<tr>
<td>SrCl(_2)</td>
<td>-5.6</td>
<td>RbF</td>
<td>-11.0</td>
<td>HIO(_3)</td>
<td>-96.0</td>
</tr>
<tr>
<td>HBr</td>
<td>-5.2</td>
<td>CsF</td>
<td>-9.4</td>
<td>KIO(_3)</td>
<td>-130.0</td>
</tr>
<tr>
<td>LiBr</td>
<td>-3.6</td>
<td>NH(_4)NO(_3)</td>
<td>-18.0</td>
<td>KNO(_3)</td>
<td>-64.4</td>
</tr>
</tbody>
</table>

\(a\text{cal kg mole}^{-2} \text{(°K)}^{-1}\). \(b\) Computed from the slopes of the linear curves in figures 65 through 69 and 79 through 81 (see text).

### Table 11

Extended term parameters \((b_Y)\) for computing the relative apparent and partial molal volumes of aqueous electrolytes at 25°C and 1 bar from eqs (222) and (227)

<table>
<thead>
<tr>
<th>Solute</th>
<th>(b_Y^{a,b})</th>
<th>Solute</th>
<th>(b_Y^{a,b})</th>
<th>Solute</th>
<th>(b_Y^{a,b})</th>
</tr>
</thead>
<tbody>
<tr>
<td>HCl</td>
<td>0.60</td>
<td>RbBr</td>
<td>1.96</td>
<td>RbF</td>
<td>2.80</td>
</tr>
<tr>
<td>LiCl</td>
<td>1.00</td>
<td>CsBr</td>
<td>1.92</td>
<td>CsF</td>
<td>2.52</td>
</tr>
<tr>
<td>NaCl</td>
<td>1.90</td>
<td>NH(_4)Br</td>
<td>1.48</td>
<td>HNO(_3)</td>
<td>0.44</td>
</tr>
<tr>
<td>KCl</td>
<td>2.30</td>
<td>LiI</td>
<td>-0.10</td>
<td>LiNO(_3)</td>
<td>0.34</td>
</tr>
<tr>
<td>RbCl</td>
<td>2.20</td>
<td>NaI</td>
<td>0.92</td>
<td>NaNO(_3)</td>
<td>1.32</td>
</tr>
<tr>
<td>CsCl</td>
<td>2.08</td>
<td>KI</td>
<td>1.64</td>
<td>KNO(_3)</td>
<td>1.90</td>
</tr>
<tr>
<td>NH(_4)Cl</td>
<td>0.60</td>
<td>RbI</td>
<td>1.62</td>
<td>NH(_4)NO(_3)</td>
<td>0.46</td>
</tr>
<tr>
<td>MgCl(_2)</td>
<td>0.88</td>
<td>CsI</td>
<td>1.64</td>
<td>AgNO(_3)</td>
<td>1.80</td>
</tr>
<tr>
<td>LiBr</td>
<td>0.54</td>
<td>NH(_4)I</td>
<td>0.36</td>
<td>LiOH</td>
<td>2.24</td>
</tr>
<tr>
<td>NaBr</td>
<td>1.52</td>
<td>NaF</td>
<td>2.90</td>
<td>NaOH</td>
<td>3.08</td>
</tr>
<tr>
<td>KBr</td>
<td>2.04</td>
<td>KF</td>
<td>2.94</td>
<td>KOH</td>
<td>3.08</td>
</tr>
</tbody>
</table>

\(a\text{cm}^3 \text{kg mole}^{-2} \text{bar}^{-1}\). \(b\) Computed from the slopes of the linear curves in figures 83 through 88. In cases where two curves with different slopes are shown for the same electrolyte, those in figures 83 through 85 were accepted in preference to those in figures 86 through 88, which are more uncertain.

### Table 12

Extended term parameters \((b_K)\) for computing the relative apparent and partial molal compressibilities of aqueous electrolytes at 25°C and 20 bars from eqs (226) and (231)

<table>
<thead>
<tr>
<th>Solute</th>
<th>(b_K^{a,b} \times 10^b)</th>
<th>Solute</th>
<th>(b_K^{a,b} \times 10^b)</th>
<th>Solute</th>
<th>(b_K^{a,b} \times 10^b)</th>
</tr>
</thead>
<tbody>
<tr>
<td>HCl</td>
<td>1.6</td>
<td>KBr</td>
<td>6.8</td>
<td>NaNO(_3)</td>
<td>3.0</td>
</tr>
<tr>
<td>LiCl</td>
<td>3.2</td>
<td>NH(_4)Br</td>
<td>2.2</td>
<td>KNO(_3)</td>
<td>5.6</td>
</tr>
<tr>
<td>NaCl</td>
<td>8.0</td>
<td>LiI</td>
<td>1.6</td>
<td>NH(_4)NO(_3)</td>
<td>1.2</td>
</tr>
<tr>
<td>KCl</td>
<td>8.4</td>
<td>NaI</td>
<td>2.0</td>
<td>AgNO(_3)</td>
<td>4.0</td>
</tr>
<tr>
<td>NH(_4)Cl</td>
<td>3.4</td>
<td>KI</td>
<td>5.0</td>
<td>LiOH</td>
<td>8.4</td>
</tr>
<tr>
<td>MgCl(_2)</td>
<td>4.3</td>
<td>NH(_4)I</td>
<td>0.8</td>
<td>NaOH</td>
<td>15.6</td>
</tr>
<tr>
<td>LiBr</td>
<td>2.0</td>
<td>HNO(_3)</td>
<td>0.8</td>
<td>KOH</td>
<td>13.0</td>
</tr>
<tr>
<td>NaBr</td>
<td>5.2</td>
<td>LiNO(_3)</td>
<td>2.0</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\(a\text{cm}^3 \text{kg mole}^{-2} \text{bar}^{-1}\). \(b\) Computed from the slopes of the linear curves in figures 95 through 97.
Fig. 83. $\rho V^*$ (eq 240) at 25°C and 1 bar computed (assuming I = 1) from density data taken from the literature. The circles shown above and those in figure 84 through 94 represent experimental data reported by Fortier, Leduc, and Desnoyers (1974), Dunn (1968), Allam (ms), Ellis (1966, 1967, 1968; written commun., 1968) and Ellis and McFadden (1968, 1972). The values of $V^*$ corresponding to the open triangles on the ordinates of the diagrams represent those computed from regression calculations by Helgeson and Kirkham (1976). The latter values do not in all instances necessarily correspond to the extrapolated intercepts of the linear curves in the diagrams (see text). The values represented by the closed triangles on the ordinates of figures 89 through 92 correspond to extrapolated values of $V^*$ generated from the Redlich-Meyer equation (Redlich and Meyer, 1964) by Helgeson and Kirkham (1976).
Fig. 84. \( \rho^* \) (eq 240) at 25°C and 1 bar computed (assuming \( I = I \)) from density data taken from the literature (see caption of fig. 83).
Fig. 85. $\rho_v^*$ (eq 240) at 25°C and 1 bar computed (assuming $I = 1$) from density data taken from the literature (see caption of fig. 83).
tively, on the ordinate, with the intercepts of the curves in figures 89 through 93 reveals discrepancies of the order of 1 cm³ mole⁻¹ or less. Uncertainties of this order of magnitude have a negligible effect on most geochemical calculations. Accordingly, values of $b_{v,k}$ computed from the slopes of the linear curves in figures 83 through 93 were accepted without reassessing $\tilde{V}_o$. As a consequence, the conventional standard partial molal volumes of ionic species adopted in the present study correspond to those generated by Helgeson and Kirkham (1976). The values of $b_{v,k}$ at 25°C and 1 bar are given in table 11; those for higher temperatures are discussed in detail below.

The departures from linearity exhibited by $\rho^{*}_v$ for Na₂SO₄, K₂SO₄, and BaCl₂ in figures 89 and 91 are consistent with increasing ion associa-

Fig. 86. $\rho^{*}_v$ (eq 240) at 25°C and 1 bar computed (assuming $I = 1$) from density data taken from the literature (see caption of fig. 83).
tion with increasing temperature. Note in figure 94, which corresponds to the volume analog of figure 82B, that \( \phi_{\text{v}, \text{MgSO}_4} \) as a function of I exhibits evidence of polyligand association at high concentrations. However, no such evidence is apparent in the ionic strength dependence of \( \phi_{\text{c}, \text{MgSO}_4} \) (fig. 82B). It should perhaps be emphasized in this regard that the effects of ion association on different apparent molal properties may be manifested at different ionic strengths, depending on differences in the standard molal Gibbs free energies, enthalpies, heat capacities, and volumes of dissociation (see below). The fact that \( \phi_{\text{v}} \) and \( \phi_{\text{c}} \) for MgSO\(_4\) are both linear functions of I below I \( \approx 3 \) is consistent with predominance of the ion pair at I < 3.

Fig. 87. \( \rho^*_{\text{v}} \) (eq 240) at 25°C and 1 bar computed (assuming I = I) from density data taken from the literature (see caption of fig. 88).
Compressibility data reported by Allam (1963) were used to compute the values of $\rho_{*,k}^*$ shown in figures 95 through 97 by taking account of the relation,

$$\phi_{k,k} = \frac{1000\beta - n_{*,w}^* V_{*,w}^* \beta_{*,w}^*}{M_k},$$

(243)

where $\beta$ stands for the coefficient of isothermal compressibility of the solution $(\partial \ln \rho / \partial P)_T$, $n_{*,w}^*$ denotes the number of moles of H$_2$O liter$^{-1}$ of solution, $V_{*,w}^*$ and $\beta_{*,w}^*$ represent the standard partial molar volume and coefficient of isothermal compressibility of pure H$_2$O, and $M_k$ refers to the molarity of the $k$th electrolyte, which is related to the molality of the electrolyte by

$$m_k = \frac{1000M_k}{18.0153n_{*,w}^*}.$$  (244)

The values of $V_{*,w}^*$ and $\beta_{*,w}^*$ employed in the calculations were taken from Helgeson and Kirkham (1974a).

With a few exceptions (for example NaOH and KOH) the computed values of $\rho_{*,k}^*$ shown in figures 95 through 97 for ionic strengths $\leq 4$ (depending on the electrolyte) are in close agreement with eq (241). However, at lower ionic strengths the data points for all but a few of the electrolyte depart to an increasing degree from the linear curves toward more negative values of $\rho_{*,k}^*$ with decreasing concentration. It can be deduced from comparison of the intercepts of the linear curves with the symbols on the ordinates representing values of $-\tilde{\kappa}_{*,k}$ taken from Mathieson and Conway (1974) and Helgeson and Kirkham (1976) that this trend is inconsistent with values of $\tilde{\kappa}_{*,k}$ derived from other experimental studies. The latter observation suggests systematic errors in Allam’s compressibility measurements at low ionic strengths. For this reason the conventional standard partial molar compressibilities of aqueous species computed by Helgeson and Kirkham (1976) were accepted in the present study in pref-

Fig. 88. $\rho_{*,r}^*$ (eq. 240) at 25°C and 1 bar computed (assuming $I = 1$) from density data taken from the literature (see caption of fig. 83).
Fig. 89. $p^*_V$ (eq 240) at 20 bars and various temperatures (indicated in °C) computed (assuming $I = I$) from density data taken from the literature (see caption of fig. 83).
Fig. 90. \( \rho^*_v \) (eq 249) at 20 bars and various temperatures (indicated in °C) computed (assuming I = I) from density data taken from the literature (see caption of fig. 83).
Fig. 91. $\rho^* V$ (eq 240) at 20 bars and various temperatures (indicated in °C) computed (assuming $I = I$) from density data taken from the literature (see caption of fig. 83).
Fig. 92. $\rho^* v$ (eq 240) at various temperatures (indicated in °C) at 1 (CaCl₂, KBr, BaCl₂) and 20 (HCl) bars computed (assuming I = 1) from density data taken from the literature (see caption of fig. 83).

Fig. 93. $\rho^* v$ (eq 240) at various temperatures (indicated in °C) and 1 bar computed (assuming I = 1) from density data taken from the literature (see caption of fig. 83).
Fig. 94. Apparent molal volume of MgSO₄ as a function of stoichiometric ionic strength at 25°C and 1 bar (see text and caption of fig. 83).

Fig. 95. \( \rho^* \) (eq 241) at 25°C and 1 bar computed (assuming I = 1) from compressibility data reported by Alfam (ns). The values of \( \bar{\rho}^* \) represented by the half-pentagons and triangles on the ordinates of the diagrams shown above and those in figures 96 through 99 correspond to standard partial molal compressibilities given by Mathieson and Conway (1974) and Helgeson and Kirkham (1976), respectively.
Fig. 96. $\rho^*_{\infty}$ (eq 241) at 25°C and 1 bar computed (assuming $I = I$) from compressibility data reported by Allam (ms) (see caption of fig. 95).

Fig. 97. $\rho^*_{\infty}$ (eq 241) at 25°C and 1 bar computed (assuming $I = I$) from compressibility data reported by Allam (ms) (see caption of fig. 95).
Fig. 98. $\rho^*_\infty$ (eq 241) at various temperatures (indicated in °C) and 1 bar computed (assuming $I = 1$) from compressibility data reported by Millero and others (1974) (see caption of fig. 95).

Fig. 99. $\rho^*_\infty$ (eq 241) for MgSO$_4$ as a function of stoichiometric ionic strength at various temperatures (indicated in °C) and 1 bar computed (assuming $I = 1$) from data reported by Millero and others (1974) (see caption of fig. 95).

Fig. 100. Correlation of apparent molal volume extended term parameters with the standard partial molal volumes of aqueous electrolytes at 25°C and 1 bar (see text). The various symbols represent the slopes of the linear curves for the electrolytes at 25°C in figures 83 through 97 and corresponding values of $V^\circ$ given by Helgeson and Kirkham (1976).
ference to the intercepts of the linear curves shown in figures 95 through 99.

Values of \( b_{v,k} \) computed from the slopes of the linear curves shown in figures 95 through 97 are given in table 12. Note in figure 98 that the experimental data for NaCl, Na\(_2\)SO\(_4\), and MgCl\(_2\) at temperatures above and below 25°C are also consistent with eq (241). However, it can be deduced from figure 98 that \( \rho^*_{v,k} \) as a function of 1 for these three electrolytes, as well as those for NaOH, AgNO\(_3\), LiCl, KI, and KOH in figure 97 exhibit departures from linearity consistent with increasing ion association with increasing concentration at high ionic strength. Note in figure 99 that \( \rho^*_{v,MgSO_4} \) is a non-linear function of ionic strength at all temperatures \( \leq 45°C \).

**Corresponding states relations.**—It can be deduced from figure 100 that \( b_v \) is linearly related to \( \nu^o \) for 1:1 electrolytes with a common cation. The curves in figure 100 are consistent with

\[
\nu_k b_{v,k}/4 = a_i + b_i \nu^o_k
\]

which can be used to estimate values of \( b_{v,k} \) in cm\(^3\) kg mole\(^{-2}\) from standard partial molal volumes expressed in cm\(^3\) mole\(^{-1}\). The requisite coefficients for eq (245) are:

<table>
<thead>
<tr>
<th>Electrolyte</th>
<th>( a_i )</th>
<th>( b_i \times 10^3 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Li — electrolys</td>
<td>0.96</td>
<td>-28.2</td>
</tr>
<tr>
<td>Na — electrolys</td>
<td>1.38</td>
<td>-25.9</td>
</tr>
<tr>
<td>K — electrolys</td>
<td>1.61</td>
<td>-17.5</td>
</tr>
<tr>
<td>Rb — electrolys</td>
<td>1.59</td>
<td>-15.4</td>
</tr>
<tr>
<td>Cs — electrolys</td>
<td>1.50</td>
<td>-11.9</td>
</tr>
<tr>
<td>NH(_4) — electrolys</td>
<td>0.54</td>
<td>-6.6</td>
</tr>
</tbody>
</table>

Fig. 101. Correlation of apparent molal compressibility extended term parameters with the standard partial molal compressibilities of aqueous electrolytes at 25°C and 1 bar (see text). The symbols represent the slopes of the linear curves for the electrolytes at 25°C in figures 95 through 97 and corresponding values of \( \kappa^o \) given by Helgeson and Kirkham (1976).
Note that the progression in these coefficients is roughly related to the effective electrostatic radii of the cations in table 8, which is also true of the coefficients in the other algorithms discussed below.

A corresponding states relation for compressibility similar to that for volume in figure 100 is shown in figure 101, where it can be seen that \( \nu_k b_{c,k}/4 \) is linearly related to \( \bar{\kappa}^o_k \) for 1:1 electrolytes with a common cation. Note also in figure 102 that the same "families" of electrolytes exhibit a linear correlation of \( \bar{\kappa}^o_k \) to \( \bar{\nu}^o_k \). Estimation algorithms corresponding to the curves in figures 101 and 102 can be written for \( b_{c,k} \) in cm\(^3\) kg mole\(^{-2}\) bar\(^{-1}\), \( \bar{\kappa}^o \) in cm\(^3\) mole\(^{-1}\) bar\(^{-1}\), and \( \bar{\nu}^o \) in cm\(^3\) mole\(^{-1}\) as

\[
\nu_k b_{c,k}/4 = a_2 + b_2 \bar{\kappa}^o_k
\]

and

\[
\bar{\kappa}^o_k = a_1 + b_1 \bar{\nu}^o_k
\]

where \( b_2 = 5 \times 10^{-4} \) and

<table>
<thead>
<tr>
<th></th>
<th>( a_2 \times 10^5 )</th>
<th>( b_2 \times 10^2 )</th>
<th>( a_3 \times 10^4 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Li</td>
<td>-16</td>
<td>-9.2</td>
<td>-51</td>
</tr>
<tr>
<td>Na</td>
<td>-37</td>
<td>-16.0</td>
<td>-63</td>
</tr>
<tr>
<td>K</td>
<td>2</td>
<td>-9.2</td>
<td>-68</td>
</tr>
<tr>
<td>Rb</td>
<td></td>
<td></td>
<td>-60</td>
</tr>
<tr>
<td>Cs</td>
<td></td>
<td></td>
<td>-74</td>
</tr>
<tr>
<td>NH(_4)</td>
<td>3</td>
<td>-7.2</td>
<td>-54</td>
</tr>
</tbody>
</table>

Discussion.—The remarkable agreement and internal consistency of the many sets of experimental activity coefficient, osmotic coefficient, enthalpy, heat capacity, volume, and compressibility data represented by the symbols in figures 17 through 21, 23 through 29, 45 through 47, and 50 through 99 strongly support the theoretical model and equations responsible for the linear curves shown in the figures, as well as the \( \bar{\kappa} \) values used in the calculations. The fact that the latter values were derived independently from standard partial molal volumes and entropies of aque-

![Fig. 102. Correlation of the standard partial molal compressibilities and volumes of aqueous electrolytes at 25°C and 1 bar. The symbols represent values given by Helgeson and Kirkham (1976) and Mathieson and Conway (1974).](image)
ous species underscores the generality of the approach. The agreement is particularly impressive in view of the relatively large uncertainties in apparent molal heat capacities, volumes, and compressibilities that result from experimental measurements of the properties of dilute solutions. Measurements in highly concentrated solutions yield more reliable values of apparent molal properties, but they are subject to significant contributions by ion association. The extent to which complexing affects the different apparent molal properties of an electrolyte at the same ionic strength may be quite different, depending on the electrolyte and the relative magnitude of the property compared to the change in the property attending ion association. For example, \((\partial^2 m_j / \partial T^2)_{P,T,\mu(k)}\) may be small compared to \(\phi_{\text{CP}}\) and \((\partial m_j / \partial P)_{T,\mu(k)}\) so that ion association might have a significant effect on \(\rho^*_y\) but not on \(\rho^*_j\). Similarly, the standard molal enthalpy of dissociation for a given complex may be small (of the order of a few hundred cal mole\(^{-1}\)), but the effect on \(\rho^*_\Delta H_d\) could be dramatic if \(\phi_{\Pi}\) is also small, despite the fact that the degree of formation of the complex might be no more than a few percent. In other cases, log \(K\) for the dissociation of a complex may be substantially negative, causing \(\gamma_{\pm}\) to be affected in dilute as well as concentrated solutions. Nevertheless, the standard partial molal enthalpy, heat capacity, and/or volume of dissociation may have an insignificant effect on \(\rho^*_\Delta H_d\), \(\rho^*_j\), or \(\rho^*_y\) if the degree of formation of the complex is insensitive to changes in temperature and pressure. In still others, the effects of ion association may be significant only in the dilute range of concentration. In general, it appears from the data considered above that \(\rho^*_\Delta H_d\) is more sensitive to ion association than \(\rho^*_j\) or \(\rho^*_y\), but that \(\rho^*_\Lambda\) is relatively insensitive to complexing. Similarly, \(\rho^*_y\) is affected by ion association to a much greater extent than is \(\rho^*_j\).

**Prediction of Thermodynamic Properties at High Pressures and Temperatures**

Calculation of the thermodynamic behavior of hydrothermal/geothermal systems at high pressures and temperatures requires predictive equations describing the pressure and temperature dependence of the standard partial molal properties of aqueous species as well as corresponding equations for the extended term parameters in the equations summarized above. Equations of this kind can be derived from the theoretical model proposed by Helgeson and Kirkham (1976).

**Standard Partial Molal Properties**

As noted above any standard partial molal property of an aqueous species can be viewed as the sum of its intrinsic counterpart and contributions by electrostriction collapse of the local solvent structure and solvation of the species in solution (eq 118). Although these contributions are not independent of one another, analysis of the thermodynamic behavior of electrolyte solutions as a function of pressure and temperature indicates that eq (116) affords close approximation of the standard partial molal properties of aqueous species at high as well as low temperatures and pressures.
The standard partial molal properties of aqueous species at high pressures and temperatures can be calculated by combining eq (116) with an equation of state describing the volumetric consequences of electrostriction collapse and ion solvation as a function of pressure and temperature, together with a corresponding equation representing the dependence of the standard partial molal heat capacity of the species on temperature at a reference pressure of 1 bar. The equation of state adopted in the present study corresponds to that derived by Helgeson and Kirkham (1976), which is reviewed briefly below.

Equation of state.—Consideration of standard partial molal volume and compressibility data for aqueous electrolytes as a function of temperature at low pressures indicates that \( \nabla_{i,j}^{\text{o}} \) (the conventional intrinsic standard partial molal volume of the \( j \)th ion) can be regarded as a temperature/pressure-independent constant and that \( \Delta \nabla_{i,j}^{\text{o}} \) (the collapse contribution to \( \nabla_{i,j}^{\text{o}} \)) is an asymptotic function of temperature, which is linearly dependent on pressure (Helgeson and Kirkham, 1976). These observations are consistent with

\[
\begin{align*}
\nabla_{i,j}^{\text{o}} &= a_{i,j} \\
\nabla_{i,j}^{\text{o}} &= \left( \frac{\partial \nabla_{i,j}^{\text{o}}}{\partial P} \right)_T = E_{x,i,j}^{\text{o}} & \left( \frac{\partial \nabla_{i,j}^{\text{o}}}{\partial T} \right)_P = 0,
\end{align*}
\]

\[
\Delta \nabla_{i,j}^{\text{o}} = a_{z,j} P + \frac{a_{z,j} T + a_{z,j} P T}{T - \theta_j},
\]

\[
\Delta \nabla_{i,j}^{\text{o}} = \left( \frac{\partial \Delta \nabla_{i,j}^{\text{o}}}{\partial P} \right)_T + \left( \frac{\partial \Delta \nabla^{\text{o}}_{i,j}}{\partial T} \right)_P = \Delta \nabla_{i,j}^{\text{o}} = a_{z,j} + \frac{a_{z,j} T}{T - \theta_j},
\]

\[
\Delta \nabla_{i,j}^{\text{o}} = a_{1,j} + a_{2,j} P + \frac{a_{3,j} T + a_{4,j} P T}{T - \theta_j},
\]

where \( a_{1,j} \), \( a_{2,j} \), \( a_{3,j} \), \( a_{4,j} \), and \( \theta_j \) (which can be regarded as a “structural” temperature analogous to that introduced by Bernal and Fowler, 1933) correspond to temperature/pressure-independent coefficients characteristic of the \( j \)th aqueous species. Note that it follows from eqs (116), (253), and the partial derivative of eq (133) at constant temperature that we can write

\[
\nabla_j^{\text{o}} = a_{1,j} + a_{2,j} P + \frac{a_{3,j} T + a_{4,j} P T}{T - \theta_j} - \omega_j Q,
\]
which is consistent with
\[ \Delta V_{s,j}^o = - \omega_j Q \] (255)
where \( \omega_j \) is defined by eq (130) and
\[ Q = - \left( \frac{\partial (1/\epsilon^o)}{\partial P} \right)_T = \frac{1}{\epsilon^o} \left( \frac{\partial \ln \epsilon^o}{\partial P} \right)_T \] (256)
where \( \epsilon^o \) denotes the dielectric constant of the solvent. Note that it follows from eq (254) that we can also write
\[ \tilde{\epsilon}^o_{x,j} = \left( \frac{\partial V_{o,j}^o}{\partial T} \right)_P = - \left( \frac{\partial S_{o,j}^o}{\partial P} \right)_T = - \frac{\theta_j (a_{x,j} + a_{x,j} P)}{(T - \theta_j)^2} - \omega_j U \] (257)
and
\[ - \tilde{r}^o_j = \left( \frac{\partial \tilde{V}_{o,j}^o}{\partial P} \right)_T = a_{x,j} + \frac{a_{x,j} T}{T - \theta_j} - \omega_j N \] (258)
where
\[ U \equiv \left( \frac{\partial Q}{\partial T} \right)_P = \frac{1}{\epsilon^o} \left( \left( \frac{\partial (\ln \epsilon^o/\partial P)}{\partial T} \right)_P - \left( \frac{\partial \ln \epsilon^o}{\partial P} \right)_T \left( \frac{\partial \ln \epsilon^o}{\partial T} \right)_P \right) \] (259)
and
\[ N \equiv \left( \frac{\partial Q}{\partial P} \right)_T = \frac{1}{\epsilon^o} \left( \left( \frac{\partial^2 \ln \epsilon^o}{\partial P^2} \right)_T - \left( \frac{\partial \ln \epsilon^o}{\partial P} \right)_T \right) \] (260)
which represent the electrostatic contributions by the solvent to the conventional standard partial molal expansibility and compressibility of solvation. These latter properties are defined by
\[ \Delta \tilde{\epsilon}^o_{x,s,j} = - \omega_j U \] (261)
and
\[ \Delta \tilde{r}^o_{x,s,j} = \omega_j N \] (262)

The equations summarized above afford close approximation of the pressure dependence of the thermodynamic properties of aqueous species in the standard state at both high and low pressures and temperatures (Helgeson and Kirkham, 1976). In addition, consideration of standard partial molal heat capacity data for aqueous electrolytes as a function of temperature indicates that the calorimetric consequences of electrostriction collapse and ion solvation as a function of temperature can be described in terms of the same theoretical model used to represent the temperature dependence of \( \Delta V_{o,j}^o \) in eq (254), which constitutes a general equation of state for aqueous species at infinite dilution.

**Calorimetric consequences of electrostriction collapse and ion solvation.**—Calculation of \( \Delta C_{p,n,k}^o \) (the sum of the intrinsic and collapse contributions to the standard partial molal heat capacity of the \( k \)th electrolyte) from calorimetric data at temperatures from 0° to 200°C along the
liquid-vapor equilibrium curve for \( \text{H}_2\text{O} \) suggests that \( \mathcal{C}^o_{P,i,j} \) corresponds to a pressure/temperature-independent constant and that \( \Delta \mathcal{C}^o_{P,e,j} \) is essentially proportional to \( \Delta \mathcal{V}^o_{o,j} \) as a function of temperature. These calculations were carried out by first taking the partial derivative of eq (133) with respect to temperature at constant pressure, which can be written as

\[
\Delta \mathcal{S}^o_{o,j} = - \left( \frac{\partial \Delta \mathcal{C}^o_{o,j}}{\partial T} \right)_P = \omega_j Y
\]

which is consistent with

\[
\Delta \mathcal{C}^o_{P,s,j} = T \left( \frac{\partial \Delta \mathcal{S}^o_{o,j}}{\partial T} \right)_P = \left( \frac{\partial \Delta \mathcal{H}^o_{o,j}}{\partial T} \right)_P = \omega_j T X
\]

where \( \Delta \mathcal{S}^o_{o,j} \) and \( \Delta \mathcal{C}^o_{P,s,j} \) stand for the conventional standard partial molal entropy and heat capacity of solvation and

\[
Y = - \left( \frac{\partial (1/e^o)}{\partial T} \right)_P = \frac{1}{e^o} \left( \frac{\partial \ln e^o}{\partial T} \right)_P
\]

and

\[
X = \left( \frac{\partial Y}{\partial T} \right)_P = \frac{1}{e^o} \left( \left( \frac{\partial^2 \ln e^o}{\partial T^2} \right)_P - \left( \frac{\partial \ln e^o}{\partial T} \right)_P^2 \right)
\]

where \( e^o \) again refers to the dielectric constant of the solvent. The values of \( \Delta \mathcal{C}^o_{P,e,k} \) were then computed from eqs (29), (116), and (264) using the experimental standard partial molal heat capacities of electrolytes represented by the intercepts of the linear curves in figures 70 through 78, together with values of \( \omega_j \) taken from table 3 and those of \( X \) given by Helgeson and Kirkham (1974a). Because the effect on \( \mathcal{C}^o_{P,k} \) of the 15 bar pressure variation from 0\(^\circ\)C to 200\(^\circ\)C along the liquid-vapor equilibrium curve for \( \text{H}_2\text{O} \) is negligible compared to the consequences of experimental uncertainties on the intercepts of the linear curves in these figures, no correction was made for pressure differences in the experiments at temperatures \( > 100\^\circ\)C. For the same reason, no correction for changes in vapor pressure as a function of concentration, and no conversion of \( \mathcal{C}^o_{sat,k} \) (the standard partial molal heat capacity of the \( k \)th electrolyte along the liquid-vapor equilibrium curve for the solvent) to \( \mathcal{C}^o_{P,k} \) was necessary. The latter two variables are related by

\[
\mathcal{C}^o_{P,k} = \mathcal{C}^o_{sat,k} + T \left( \frac{\partial \mathcal{V}^o_{k}}{\partial T} \right)_P \left. \frac{dP}{dT} \right|_{sat}
\]

where \( dP/dT \)

\( \bigg|_{sat} \) stands for the derivative of pressure with respect to temperature along the liquid-vapor equilibrium curve for \( \text{H}_2\text{O} \). The last term on the right side of eq (267) is insignificant for most electrolytes at temperatures and pressures \( \leq 200\^\circ\)C and 15 bars, as is the difference between \( \mathcal{C}_{P,k} \) and \( \mathcal{C}_{sat,k} \).
It follows from the observations made above that we can write for the reference pressure ($P_r$),

$$
\Delta C^o_{P_r,c,j} = \frac{c_{z,j} T}{T - \theta_j},
$$

(268)

and

$$
\Delta C^o_{P,r,u,j} = c_{i,j} + \frac{c_{z,j} T}{T - \theta_j},
$$

(269)

where $c_{i,j}$ and $c_{z,j}$ stand for temperature/pressure-independent coefficients analogous to $a_i$ and $a_z$ in eq (254), and $\theta_j$ is again the “structural” temperature for the $j$th species. Summing eqs (268), (269), and a statement of eq (264) for $P = P_r$ then leads to

$$
C^o_{P_r,j} = c_{i,j} + \frac{c_{z,j} T}{T - \theta_j} + \omega_j T X,
$$

(270)

which is consistent with

$$
C^o_{P_r,i,j} = C^o_{P,i,j} = c_{i,j}.
$$

(271)

Taking account of the partial derivative of eq (257) with respect to temperature at constant pressure and the relation

$$
\left(\frac{\partial C^o_P}{\partial P}\right)_T = -T \left(\frac{\partial E^o_x}{\partial T}\right)_P
$$

(272)

leads to

$$
C^o_{P,j} = \left(\frac{\partial E^o_x}{\partial T}\right)_P = -T \left(\frac{\partial E^o_x}{\partial T}\right)_P = c_{i,j} + \frac{c_{z,j} T}{T - \theta_j}
$$

$$
- \frac{\theta_j T (2a_{z,j} (P - P_r) + a_{z,j} (P^2 - P_r^2))}{(T - \theta_j)^3} + \omega_j T X
$$

(273)

which describes the pressure and temperature dependence of the standard partial molar heat capacities of aqueous species.

**Standard partial molar entropy, enthalpy, and Gibbs free energy.**—Integration of eq (273) with respect to temperature at constant pressure leads to

$$
S^o_{j,P,T} = S^o_{j,P_r,T_r} + c_{i,j} \ln \left(\frac{T}{T_r}\right)
$$

$$
+ c_{z,j} \ln \left(\frac{T - \theta_j}{T_r - \theta_j}\right)
$$

$$
+ \theta_j \left(2a_{z,j} (P - P_r) + a_{z,j} (P^2 - P_r^2)\right)
$$

$$
2(T - \theta_j)^2
$$

$$
+ \omega_j \left(Y_{P,T} - Y_{P_r,T_r}\right),
$$

(274)

and
\[ \Delta \tilde{\Pi}^{0}_{j,P,T} = \Delta \Pi^{0}_{j,P,T} + (c_{i,j} + c_{z,j}) (T - T_r) \\
+ c_{z,j} \theta_j \ln \left( \frac{T - \theta_j}{T_r - \theta_j} \right) \\
+ a_{i,j} (P - P_r) + a_{z,j} (P^2 - P_r^2)/2 \\
+ \frac{2a_{i,j}T (P - P_r) + a_{z,j}T (P^2 - P_r^2)}{2(T - \theta_j)} \\
+ \frac{2\theta_j a_{z,j}T (P - P_r) + \theta_j a_{z,j}T (P^2 - P_r^2)}{2(T - \theta_j)^2} \\
+ \omega_j (TY_{P,T} - T_r Y_{P_r,T_r} - Z_{P,T} + Z_{P_r,T_r}) \] (275)

which lead to

\[ \Delta \tilde{G}^{0}_{j,P,T} = \Delta \tilde{G}^{0}_{j,P,T} - S^{0}_{i,P_r,T_r} (T - T_r) \\
- c_{i,j} (T \ln(T/T_r) - T + T_r) \\
+ c_{z,j} \left( T - T_r - (T - \theta_j) \ln \left( \frac{T - \theta_j}{T_r - \theta_j} \right) \right) \\
+ \frac{2(a_{i,j} (T - \theta_j) + a_{z,j}T (P - P_r) + (a_{z,j} (T - \theta_j) + a_{z,j}T (P^2 - P_r^2))}{2(T - \theta_j)} \\
- \omega_j (Z_{P,T} - Z_{P_r,T_r} - Y_{P_r,T_r} (T - T_r)) \] (276)

where \( \Delta \tilde{\Pi}^{0}_{j,P,T} \) and \( \Delta \tilde{G}^{0}_{j,P,T} \) refer to the apparent standard partial molal enthalpy and Gibbs free energy of formation of the \( j \)th ion at the subscripted pressure and temperature (eqs 1 and 2) and

\[ Z_{P,T} \equiv -\frac{1}{\varepsilon_{P,T}^{0}} . \] (277)

The reliability of eqs (274) through (276) rests solely on the validity of the equation of state (eq 254) and the corresponding equation for the standard partial molal heat capacity of an electrolyte as a function of temperature at \( P_r \) (eq 270). The validity of the equation of state has already been demonstrated in a previous communication (Helgeson and Kirkham, 1976), and the reliability of eq (270) is substantiated by regression calculations summarized below.

Regression calculations.—The intercepts of the linear curves for 25°C in figures 65 through 82, which correspond to \( C_{P,r,k}^{0} \), are listed in table 13, together with the conventional standard partial molal heat capacities of aqueous ions. The intercepts for higher and lower temperatures are plotted as symbols in figures 103 through 106, where it can be seen that in every case except MgSO\(_{4}\), \( C_{P,r,k}^{0} \) maximizes as a function of temperature at pressures corresponding to those along the liquid-vapor equilibrium curve for \( H_2O \). Other symbols shown in figures 103 through 105 represent values of \( C_{P,r,k}^{0} \) computed from finite difference derivatives of the heats of solution at infinite dilution shown in figures 61 through 64.
Table 13
Standard partial molal heat capacities of aqueous electrolytes designated by the subscript \(k\) and the conventional standard partial molal heat capacities of aqueous ions denoted by the subscript \(j\) at 25°C and 1 bar

<table>
<thead>
<tr>
<th>Electrolyte ( \overline{C}_p^{\circ} ) ( _{P,K} )</th>
<th>Electrolyte ( \overline{C}_p^{\circ} ) ( _{P,K} )</th>
<th>Electrolyte ( \overline{C}_p^{\circ} ) ( _{P,K} )</th>
<th>Ion ( \overline{C}_p^{\circ} ) ( _{P,\perp} )</th>
<th>Ion ( \overline{C}_p^{\circ} ) ( _{P,\perp} )</th>
<th>Ion ( \overline{C}_p^{\circ} ) ( _{P,\perp} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>HCl -30.2</td>
<td>NaBr -21.2</td>
<td>NaN_3O_3 -9.2</td>
<td>H(^+) 0.0</td>
<td>Zn(^{2+}) -5.9</td>
<td>Br(^-) -31.2</td>
</tr>
<tr>
<td>LiCl -15.8</td>
<td>KBr -28.2</td>
<td>KNO_3 -16.2</td>
<td>Li(^+) 14.4</td>
<td>Cu(^{2+}) -5.7</td>
<td>I(^-) -29.0</td>
</tr>
<tr>
<td>NaCl -20.2</td>
<td>RbBr -33.4</td>
<td>NH_4NO_3 -2.5</td>
<td>Na(^+) 10.0</td>
<td>Ni(^{2+}) -10.3</td>
<td>ReO_4(^-) -1.8</td>
</tr>
<tr>
<td>KCl -27.2</td>
<td>CsBr -36.7</td>
<td>AgNO_3 -13.0</td>
<td>K(^+) 3.6</td>
<td>Mn(^{2+}) -3.1</td>
<td>BrO_3(^-) -23.5</td>
</tr>
<tr>
<td>RbCl -32.4</td>
<td>NaI -19.0</td>
<td>NaN_2O_3 -12.0</td>
<td>Rb(^+) -2.2</td>
<td>Gd(^{3+}) -11.9</td>
<td>I_3(^-) -28.0</td>
</tr>
<tr>
<td>CsCl -35.7</td>
<td>KI -26.0</td>
<td>HI_3O_3 -28.0</td>
<td>Cs(^+) -5.5</td>
<td>Al(^{3+}) 3.9</td>
<td>MnO_4(^-) -17.5</td>
</tr>
<tr>
<td>NH_4Cl -13.5</td>
<td>RbI -31.2</td>
<td>KIO_3 -25.0</td>
<td>Ag(^+) 6.2</td>
<td>Fe(^{3+}) 4.9</td>
<td>NO_3(^-) -19.2</td>
</tr>
<tr>
<td>MgCl_2 -64.7</td>
<td>CsI -34.5</td>
<td>NaNBrO_3 -13.5</td>
<td>NH_4(^+) 16.7</td>
<td>La(^{3+}) -10.4</td>
<td>NO_2(^-) -22.0</td>
</tr>
<tr>
<td>CaCl_2 -67.0</td>
<td>NaF -18.2</td>
<td>KMnO_4 -14.5</td>
<td>Mg(^{2+}) -4.3</td>
<td>Sm(^{3+}) -12.4</td>
<td>HCO_3(^-) -11.7</td>
</tr>
<tr>
<td>SrCl_2 -68.9</td>
<td>KF -25.2</td>
<td>LiOH -18.6</td>
<td>Ca(^{2+}) -6.6</td>
<td>Pr(^{3+}) -14.4</td>
<td>HCOO(^-) -21.0</td>
</tr>
<tr>
<td>BaCl_2 -71.6</td>
<td>RbF -30.4</td>
<td>NaOH -23.0</td>
<td>Sr(^{2+}) -8.5</td>
<td>Na(^{+}) -12.4</td>
<td>CIO_4(^-) -5.8</td>
</tr>
<tr>
<td>HBr -31.2</td>
<td>HNO_3 -19.2</td>
<td>KOH -30.0</td>
<td>Ba(^{2+}) -11.2</td>
<td>OH(^-) -33.0</td>
<td>HSO_4(^-) 14.6</td>
</tr>
<tr>
<td>LiBr -16.8</td>
<td>LINO_3 -4.8</td>
<td>CD(^{+}) -0.1</td>
<td>F(^-) -28.2</td>
<td>SO_4(^{2-}) -68.5</td>
<td></td>
</tr>
</tbody>
</table>

\( ^a \) Cal mol\(^{-1}\) (K\(^{-}\)\(^{-1}\) \( ^b \) The values of \( \overline{C}_p^{\circ} \) \( _{P,K} \) shown in these columns correspond to the interceps of the linear curves in figures 65 through 69 and 79 through 81. \( ^c \) Computed from values of \( \overline{C}_p^{\circ} \) \( _{P,K} \) shown above. \( ^d \) Calculated from the mean of \( \overline{C}_p^{\circ} \) \( _{P,K} \) for Ca(CIO_4)_2 and Mg(CIO_4)_2, CoCl_2 and Co(CIO_4)_2, NiCl_2 and Ni(CIO_4)_2, and Cd(CIO_4)_2 and Cd(NO_3)_2, respectively, given by Spitzer and others (1978 a and b) using the values of \( \overline{C}_p^{\circ} \) \( _{P,\perp} \) shown above for ClO_4\(^{-}\), Cl\(^{-}\), and NO_3\(^{-}\). \( ^e \) Calculated from data reported by Schumm and others (1973) using the value of \( \overline{C}_p^{\circ} \) \( _{P,\perp} \) shown above. \( ^f \) Readman and Cobble (1969). \( ^g \) Calculated from the sum of \( \overline{C}_p^{\circ} \) \( _{P,\perp} \) for Na\(^{+}\) and HCO_3\(^{-}\) given by Demuynck and others (1976) using the value of \( \overline{C}_p^{\circ} \) \( _{P,\perp} \) shown above. \( ^h \) Calculated from the values of \( \overline{C}_p^{\circ} \) \( _{P,K} \) for GdCl_3, HREO_4, and Na_2SO_4 at 25°C in table 16 using the values of \( \overline{C}_p^{\circ} \) \( _{P,\perp} \) for Gd\(^{3+}\) and Na\(^{+}\) shown above. \( ^i \) Estimated from equation (7) of Criss and Cobble (1964b) using values of \( \overline{C}_p^{\circ} \) taken from table 3 and \( \overline{C}_p^{\circ} \) \( _{P,H} \) \( _{abs} \) = 28 cal mol\(^{-1}\) (K\(^{-}\)\(^{-1}\). \( ^j \) Estimated from values of \( \overline{C}_p^{\circ} \) \( _{P,K} \) reported by Spitzer and others (1978 a and b) for Cu(CIO_4)_2, Zn(CIO_4)_2, and Mn(CIO_4)_2 using the value shown above for \( \overline{C}_p^{\circ} \) \( _{P,\perp} \) \( _{ClO_4} \). \(^{k}\) Wagman and others (1968). \(^{l}\)owing to vagaries in regression results reported in the literature, these values were accepted in preference to those reported by Roux and others (1978), Singh and others (1978), and Spitzer and others (1979 b).
Fig. 103. Calculated (curves) and experimental (symbols) standard partial molal heat capacities of aqueous electrolytes as a function of temperature (T) at 1 bar for T < 100°C and pressures corresponding to those along the liquid-vapor equilibrium curve for H₂O at T ≥ 100°C. The symbols correspond to the intercepts of the linear curves for the electrolytes in figures 65 through 82 and/or values of C°ₚ, computed from finite difference derivatives of the intercepts of the linear curves in figures 61 through 64 using thermodynamic data for the crystalline solids taken from Stull and Prophet (1971). The c₁ and c₂ parameters used to generate the solid curves from eq (273) were obtained by regression of the values represented by the symbols, but those responsible for the dashed curves were estimated (see text). The different symbols represent extrapolation of experimental data taken from the various sources cited in the caption of figure 65.
The values of $C^0_{P,HCl}$ represented by the solid diamonds in figure 103 were computed from

$$C^0_{P,HCl} = C^0_{P,HReO_4} + C^0_{P,NaCl} - C^0_{P,NaReO_4}$$  \hspace{1cm} (278)

Eq (278) was also used by Ahluwalia and Cobble (1964b), who extrapolated their heat of solution data to infinite dilution with the aid of the Guggenheim (1935) equation. It can be deduced from figure 103 that the results of their extrapolation differ considerably from those in the present study.

Two-parameter regression of the data represented by the symbols for HCl, NaCl, KCl, LiCl, MgCl$_2$, BaCl$_2$, LiBr, CsBr, CsI, KF, and Na$_2$SO$_4$ in figures 103 through 105 with an appropriate statement of eq (273) generated the solid curves shown for these electrolytes and the corresponding values of $c_{1,k}$, $c_{2,k}$, and $C^0_{P,k}$ given in tables 14 and 15, respectively. The values of $\omega_k$, $\theta_k$, $a_{3,k}$, $a_{4,k}$, and $X$ used in the calculations were taken from Helgeson and Kirkham (1974a, 1976), which insured consistency of the fit results with the temperature dependence of the standard partial molal volume data represented by the open symbols on the ordinates of figures 89 through 93. The electrostatic properties of H$_2$O computed by Helgeson

Fig. 104. Calculated (curves) and experimental (symbols) standard partial molal heat capacities of aqueous electrolytes as a function of temperature (T) at 1 bar for T < 100°C and pressures corresponding to those along the liquid-vapor equilibrium curve for H$_2$O at T $\geq$ 100°C (see caption of fig. 103).
and Kirkham (1974a) were adopted for this purpose in preference to those calculated by Silvester and Pitzer (1976) and Bradley and Pitzer (1979), which apply only to pressures and temperatures \( \leq 1 \text{ kb and } 350^\circ C \), because they are consistent with those used below to predict the thermodynamic properties of aqueous species to 5 kb and 600°C. Because no equation of state parameters are available for HReO₄, NaReO₄, GdCl₃, and NaClO₄, \( a_{\mathrm{s},k} \) and \( a_{\mathrm{g},k} \) for these electrolytes were set to zero before regressing the data to obtain values of \( c_{\mathrm{s},k} \) and \( c_{\mathrm{g},k} \), as well as \( \theta_k \). It can be shown that this procedure introduces negligible regression errors at temperatures \( \leq 200^\circ C \) along the liquid-vapor equilibrium curve for H₂O. The dashed

Fig. 105. Calculated (curves) and experimental (symbols) standard partial molal heat capacities of aqueous electrolytes as a function of temperature (T) at 1 bar for T < 100°C and pressures corresponding to those along the liquid-vapor equilibrium curve for H₂O at T \( \geq 100^\circ C \) (see caption of fig. 103).
### Table 14
Regression coefficients for calculating the standard partial molal properties of the $k$th aqueous electrolyte from appropriate statements of eqs (273) through (276)

<table>
<thead>
<tr>
<th>Solute</th>
<th>$a_{-1,k}$</th>
<th>$a_{2,k}$</th>
<th>$b_{k}$</th>
<th>$c_{x10^{-5}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>HCl</td>
<td>-4.8384</td>
<td>-2.0410</td>
<td>246.02</td>
<td>1.4560</td>
</tr>
<tr>
<td>LiCl</td>
<td>23.0954</td>
<td>-3.5676</td>
<td>246.49</td>
<td>1.9421</td>
</tr>
<tr>
<td>NaCl</td>
<td>30.3726</td>
<td>-7.8764</td>
<td>228.58</td>
<td>1.7865</td>
</tr>
<tr>
<td>KCl</td>
<td>13.4374</td>
<td>-4.8696</td>
<td>240.20</td>
<td>1.6487</td>
</tr>
<tr>
<td>CsCl</td>
<td>22.5549</td>
<td>-9.8123</td>
<td>231.08</td>
<td>1.5534</td>
</tr>
<tr>
<td>MgCl$_2$</td>
<td>36.90252</td>
<td>-10.17292</td>
<td>247.32</td>
<td>4.4492</td>
</tr>
<tr>
<td>BaCl$_2$</td>
<td>24.1261</td>
<td>-10.4036</td>
<td>245.52</td>
<td>3.8970</td>
</tr>
<tr>
<td>GdCl$_3$</td>
<td>14.5745</td>
<td>-8.8690</td>
<td>249.16</td>
<td>6.6945</td>
</tr>
<tr>
<td>LiBr</td>
<td>41.7539</td>
<td>-12.2178</td>
<td>209.09</td>
<td>1.8719</td>
</tr>
<tr>
<td>CsI</td>
<td>36.0092</td>
<td>-19.1509</td>
<td>198.67</td>
<td>1.3908</td>
</tr>
<tr>
<td>KF</td>
<td>18.0446</td>
<td>-3.1746</td>
<td>259.66</td>
<td>1.3797</td>
</tr>
<tr>
<td>Na$_2$SO$_4$</td>
<td>73.8864</td>
<td>-16.9270</td>
<td>239.39</td>
<td>3.8468</td>
</tr>
<tr>
<td>NaClO$_4$</td>
<td>61.0754</td>
<td>-7.2356</td>
<td>249.45</td>
<td>1.3317</td>
</tr>
<tr>
<td>NaReO$_4$</td>
<td>47.8022</td>
<td>-3.3271</td>
<td>262.48</td>
<td>1.2617</td>
</tr>
<tr>
<td>HReO$_4$</td>
<td>19.6883</td>
<td>-1.2232</td>
<td>269.45</td>
<td>0.9312</td>
</tr>
</tbody>
</table>

$^a$Cal mole$^{-1}$ (*K)$^{-1}$, $^b*K$: Cal mole$^{-1}$.

### Table 15
Standard partial molal heat capacities$^o$ of aqueous electrolytes computed from an appropriate statement of eq (273) for the $k$th electrolyte as a function of temperature at 1 bar for temperatures < 100°C and pressures corresponding to those along the liquid-vapor equilibrium curve for H$_2$O at temperatures $\geq$ 100°C — see text

<table>
<thead>
<tr>
<th>Temperature, °C</th>
<th>25</th>
<th>50</th>
<th>75</th>
<th>100</th>
<th>125</th>
<th>150</th>
<th>175</th>
<th>200</th>
</tr>
</thead>
<tbody>
<tr>
<td>HCl</td>
<td>-30.2</td>
<td>-28.5</td>
<td>-29.4</td>
<td>-32.6</td>
<td>-37.1</td>
<td>-43.8</td>
<td>-54.0</td>
<td>-70.8</td>
</tr>
<tr>
<td>LiCl</td>
<td>-15.8</td>
<td>-12.5</td>
<td>-13.1</td>
<td>-16.5</td>
<td>-22.2</td>
<td>-30.9</td>
<td>-44.4</td>
<td>-66.6</td>
</tr>
<tr>
<td>NaCl</td>
<td>-20.2</td>
<td>-15.5</td>
<td>-14.7</td>
<td>-16.7</td>
<td>-21.2</td>
<td>-28.6</td>
<td>-40.5</td>
<td>-60.5</td>
</tr>
<tr>
<td>KCl</td>
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<td>-23.0</td>
<td>-22.7</td>
<td>-24.9</td>
<td>-29.3</td>
<td>-36.4</td>
<td>-47.6</td>
<td>-66.3</td>
</tr>
<tr>
<td>CsCl</td>
<td>-35.7</td>
<td>-28.3</td>
<td>-25.8</td>
<td>-26.5</td>
<td>-29.6</td>
<td>-35.4</td>
<td>-45.3</td>
<td>-67.4</td>
</tr>
<tr>
<td>MgCl$_2$</td>
<td>-64.7</td>
<td>-53.5</td>
<td>-53.3</td>
<td>-59.8</td>
<td>-72.3</td>
<td>-91.8</td>
<td>-122.2</td>
<td>-172.9</td>
</tr>
<tr>
<td>BaCl$_2$</td>
<td>-71.6</td>
<td>-60.4</td>
<td>-59.4</td>
<td>-64.6</td>
<td>-75.1</td>
<td>-91.9</td>
<td>-118.4</td>
<td>-162.6</td>
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<tr>
<td>GdCl$_3$</td>
<td>-102.5</td>
<td>-95.0</td>
<td>-99.4</td>
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<td>-133.0</td>
<td>-163.7</td>
<td>-210.7</td>
<td>-267.7</td>
</tr>
<tr>
<td>LiBr</td>
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<td>-12.7</td>
<td>-12.0</td>
<td>-14.0</td>
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<td>-26.2</td>
<td>-38.6</td>
<td>-59.4</td>
</tr>
<tr>
<td>CsI</td>
<td>-34.5</td>
<td>-28.4</td>
<td>-25.8</td>
<td>-25.8</td>
<td>-27.9</td>
<td>-32.7</td>
<td>-41.0</td>
<td>-55.9</td>
</tr>
<tr>
<td>KF</td>
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<td>-19.1</td>
<td>-18.9</td>
<td>-22.0</td>
<td>-27.7</td>
<td>-36.5</td>
<td>-50.2</td>
<td>-72.9</td>
</tr>
<tr>
<td>Na$_2$SO$_4$</td>
<td>-48.3</td>
<td>-32.1</td>
<td>-27.9</td>
<td>-30.9</td>
<td>-39.7</td>
<td>-55.2</td>
<td>-80.4</td>
<td>-123.3</td>
</tr>
<tr>
<td>NaClO$_4$</td>
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<td>15.3</td>
<td>19.1</td>
<td>19.3</td>
<td>17.1</td>
<td>12.3</td>
<td>3.9</td>
<td>-10.6</td>
</tr>
<tr>
<td>NaReO$_4$</td>
<td>8.1</td>
<td>16.7</td>
<td>18.7</td>
<td>17.7</td>
<td>14.7</td>
<td>9.5</td>
<td>1.1</td>
<td>-13.1</td>
</tr>
<tr>
<td>HReO$_4$</td>
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<td>2.5</td>
<td>2.8</td>
<td>1.4</td>
<td>-1.3</td>
<td>-5.5</td>
<td>-11.9</td>
<td>-22.6</td>
</tr>
</tbody>
</table>

$^a$Cal mole$^{-1}$ (*K)$^{-1}$

### Table 16
Regression coefficients for calculating the conventional standard partial molal properties of the $j$th aqueous ion from eqs (273) through (276)

<table>
<thead>
<tr>
<th>Ion</th>
<th>$a_{-1,j}$</th>
<th>$a_{2,j}$</th>
<th>$b_{j}$</th>
<th>$c_{x10^{-5}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Na$^+$</td>
<td>35.7153</td>
<td>-6.2856</td>
<td>215.17</td>
<td>0.3306</td>
</tr>
<tr>
<td>K$^+$</td>
<td>18.4682</td>
<td>-2.9265</td>
<td>234.19</td>
<td>0.1927</td>
</tr>
<tr>
<td>Cs$^+$</td>
<td>29.9613</td>
<td>-9.3863</td>
<td>217.04</td>
<td>0.0974</td>
</tr>
<tr>
<td>Li$^+$</td>
<td>27.2981</td>
<td>-1.2342</td>
<td>253.94</td>
<td>0.4862</td>
</tr>
<tr>
<td>Ba$^{++}$</td>
<td>34.2215</td>
<td>-6.4428</td>
<td>245.02</td>
<td>0.9851</td>
</tr>
<tr>
<td>Mg$^{++}$</td>
<td>39.0257</td>
<td>-2.6665</td>
<td>270.56</td>
<td>1.5372</td>
</tr>
<tr>
<td>Cl$^-$</td>
<td>-4.8384</td>
<td>-2.0410</td>
<td>246.02</td>
<td>1.4560</td>
</tr>
<tr>
<td>Br$^-$</td>
<td>6.4161</td>
<td>-7.5141</td>
<td>206.75</td>
<td>1.3858</td>
</tr>
<tr>
<td>F$^-$</td>
<td>0.8814</td>
<td>-1.0551</td>
<td>272.42</td>
<td>1.7870</td>
</tr>
<tr>
<td>I$^-$</td>
<td>1.3667</td>
<td>-6.8157</td>
<td>186.51</td>
<td>1.2934</td>
</tr>
<tr>
<td>OH$^-$</td>
<td>24.7208</td>
<td>-6.2074</td>
<td>253.52</td>
<td>1.7246</td>
</tr>
<tr>
<td>SO$_4^{2-}$</td>
<td>1.3418</td>
<td>-4.6940</td>
<td>263.03</td>
<td>3.1857</td>
</tr>
</tbody>
</table>

$^a$Cal mole$^{-1}$ (*K)$^{-1}$, $^b*K$: Cal mole$^{-1}$. 
curves in figures 103 through 105 were generated from a correlation algorithm (see below), but that shown in figure 106 merely connects the data points for MgSO₄. Insufficient data are available for the latter electrolyte to permit definitive regression of $\Delta C^\circ_{p,MgSO_4}$ as a function of temperature.

It can be deduced from figures 103 through 105 that the analog of eq (273) for the $k$th electrolyte represents closely the experimental heat capacity data corresponding to the symbols at temperatures $\geq 15^\circ C$. The discrepancies below $\sim 15^\circ C$ can be attributed to inconsistencies in the calorimetric and volumetric data for the electrolytes at low temperatures. The agreement between the regression curves and symbols in figures 103 through 105 (which correspond to the calculated and observed intercepts of the linear curves in figures 70 through 76 and 78) is particularly impressive in view of the fact that independent values of $\theta$ and/or $\omega$ derived from standard partial molal volume and compressibility data were specified in the calculations. The validity of eqs (254), (270), and (273) are further substantiated by the close correspondence of the curves and symbols in figures 107 and 108, which are consistent with the standard state requirement for unit activity of pure H₂O at all pressures and temperatures. The symbols in these figures represent high-pressure/temperature standard molal activity product constants for H₂O ($K_w$) derived from conductance data reported in the literature, but the curves were generated from calculated standard partial molal Gibbs free energies of dissociation ($\Delta G^\circ_r$) and the relation

$$\log K_w = -\frac{\Delta G^\circ_r}{2.303RT} = -\frac{\Delta G^\circ_{\text{OH}^-}}{2.303RT} - \frac{\Delta G^\circ_{\text{H}_2\text{O}}}{2.303RT}$$  (279)

Fig. 106. Standard partial molal heat capacity of MgSO₄ as a function of temperature (T) at 1 bar for $T < 100^\circ C$ and pressures corresponding to those along the liquid-vapor equilibrium curve for H₂O at $T \geq 100^\circ C$. The symbols correspond to the intercepts of the curves in figure 82B (see text).
using the heat capacity parameters for OH− given in table 16, together with revised values of the equation of state parameters for OH− (which replace those given by Helgeson and Kirkham, 1976) to evaluate eq (276). The value of $\omega_{\text{OH}^-}$ and the thermodynamic/electrostatic properties of H$_2$O employed in the calculations were computed from equations summarized elsewhere (Helgeson and Kirkham, 1974a, 1976), but the standard partial molal properties of OH− at the reference pressure and temperature were taken from CODATA (1978).

The values of $c_1$, $c_2$, and $\theta$ for OH− in table 16, as well as the values of $a_1$, $a_2$, $a_3$, and $a_4$ given below for the hydroxyl ion were generated from a convergent series of regression calculations involving fits of eq (276) to values of $\Delta G^\circ_{\text{OH}^-}$ computed from the experimental data shown in figures 107 and 108, together with fits of eq (254) to finite difference derivatives of these values. This procedure was adopted in lieu of fitting eq (273) to the low-temperature heat capacity data shown in figure 109 in order to

Fig. 107. Calculated (curves) and experimental (symbols) logarithm of the standard molal activity product constant of H$_2$O ($K_w$) as a function of temperature at constant pressure (labeled in kb). Saturation (H$_2$O) refers to the liquid-vapor equilibrium curve for H$_2$O.
Fig. 108. Calculated (curves) and experimental (symbols) logarithm of the standard molal activity product constant of H₂O (Kₐ) as a function of pressure at constant temperature (labeled in °C). Saturation (H₂O) refers to the liquid-vapor equilibrium curve for H₂O.

Fig. 109. Calculated (curve) and experimental (symbols) conventional standard partial molal heat capacity of OH⁻ as a function of temperature at pressures corresponding to those along the liquid-vapor equilibrium curve for H₂O (see text).

Fig. 110. Calculated (curve) and experimental (symbols) standard partial molal enthalpy of dissociation for H₂O as a function of temperature at pressures corresponding to those along the liquid-vapor equilibrium curve for H₂O (see text).
insure adequate representation of log $K_w$ at high pressures and temperatures. Figures 107 and 108 leave little doubt that the regression calculations afford close approximation of the experimental values of log $K_w$ for H$_2$O reported by Quist (1970), Sweeton, Mesmer, and Baes (1974), Whitfield (1972), and Linov and Kryukov (1972). In contrast, the regression curve representing log $K_w$ for liquid H$_2$O in equilibrium with the vapor phase (fig. 107) differs considerably from the experimental values given by Fisher and Barnes (1972) for temperatures $> 250^\circ$C. The latter data appear to be inconsistent also with the other experimental values of log $K_w$ shown in figures 107 and 108. It should perhaps be emphasized in this regard that the apparent discrepancies between the symbols and curves in figure 108 are within the experimental uncertainty reported by Quist (1970), which is $\pm 0.3$ to 0.5 log units. Although the discrepancies at temperatures $\leq 150^\circ$C in figure 107 are greater than the corresponding experimental uncertainties, they are nevertheless small enough to be considered negligible in geochemical calculations.

It can be seen in figure 109 that the values of $C_{P,OH}^\circ$ computed above are in reasonably close agreement with those generated from experimental data reported in the literature. Similar agreement is apparent in figures 110 and 111A, where $\Delta \Pi^\circ$ and $\Delta v^\circ$ for the dissociation of H$_2$O are plotted as functions of temperature at pressures corresponding to the liquid-vapor equilibrium curve for H$_2$O. However, the corresponding values of $\Delta C_{P,r}^\circ$ computed in the present study for temperatures $> 150^\circ$C (fig. 111B) differ significantly from those calculated from power function fits of log $K_w$ by Sweeton, Mesmer, and Baes (1974).

The values of $c_i$ and $c_s$ for all the ions other than OH$^-$ in table 16 were generated by fitting eq (273) to conventional standard partial molal

![Fig. 111. Calculated (curve) and experimental (symbols) standard partial molal volume and heat capacity of dissociation for H$_2$O as a function of temperature at pressures corresponding to those along the liquid-vapor equilibrium curve for H$_2$O.](image-url)
heat capacities computed from the values of $C^o_{p,k}$ in table 15 and the relation

$$C^o_{p,j} = C^o_{p,j,abs} - Z_j C^o_{p,\Pi,abs}$$

(280)

where $C^o_{p,j}$ refers to the absolute standard partial molal heat capacity of the $j$th ion. The values of $a_z$, $a_x$, $\theta$, $\omega$, and $X$ specified in the calculations were taken from Helgeson and Kirkham (1974a, 1976). In principle, $c_i$ and $c_v$ coefficients for ionic species can also be calculated directly from corresponding coefficients obtained by regression of $C^o_p$ data for electrolytes. However, this procedure may lead to significant errors in computed values of conventional standard partial molal properties, owing to magnification of the consequences of uncertainties inherent in fit parameters obtained by nonlinear regression of experimental data (see discussion by Helgeson and Kirkham, 1976).

Values of $C^o_p$ computed from eq (273) using the parameters shown in table 16, together with the values of $a_z$ and $a_x$ employed in the regression calculations are listed in table 17. The values given in the table are also plotted as curves in figure 112, where it can be seen that $C^o_p$ for all the ions maximizes as a function of temperature at pressures corresponding to those along the vapor-liquid equilibrium curve for $H_2O$. Note, however, that the extrema in the curves representing monovalent anions are less pronounced than those exhibited by the curves for cations. Nevertheless,

<table>
<thead>
<tr>
<th>Ion</th>
<th>25</th>
<th>50</th>
<th>75</th>
<th>100</th>
<th>125</th>
<th>150</th>
<th>175</th>
<th>200</th>
</tr>
</thead>
<tbody>
<tr>
<td>Na$^+$</td>
<td>10.0</td>
<td>13.4</td>
<td>15.2</td>
<td>15.9</td>
<td>15.9</td>
<td>15.2</td>
<td>13.5</td>
<td>10.2</td>
</tr>
<tr>
<td>K$^+$</td>
<td>3.0</td>
<td>5.8</td>
<td>7.1</td>
<td>7.7</td>
<td>7.8</td>
<td>7.4</td>
<td>6.4</td>
<td>4.5</td>
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<tr>
<td>Cs$^+$</td>
<td>-5.5</td>
<td>0.3</td>
<td>3.8</td>
<td>6.1</td>
<td>7.5</td>
<td>8.4</td>
<td>8.8</td>
<td>8.5</td>
</tr>
<tr>
<td>Li$^+$</td>
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<td>16.4</td>
<td>16.7</td>
<td>16.2</td>
<td>14.9</td>
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<td>Ba$^{++}$</td>
<td>-11.2</td>
<td>-2.9</td>
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<td>-0.8</td>
<td>-4.1</td>
<td>-10.2</td>
<td>-20.9</td>
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<td>Cl$^-$</td>
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<td>-29.8</td>
<td>-32.6</td>
<td>-37.1</td>
<td>-43.8</td>
<td>-54.0</td>
<td>-70.8</td>
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<tr>
<td>Br$^-$</td>
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<td>-29.2</td>
<td>-31.2</td>
<td>-34.9</td>
<td>-40.7</td>
<td>-50.0</td>
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<td>F$^-$</td>
<td>-28.2</td>
<td>-24.8</td>
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<td>-43.9</td>
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<td>-41.1</td>
<td>-50.0</td>
<td>-64.7</td>
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<td>-19.5</td>
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<td>-24.3</td>
<td>-31.2</td>
<td>-42.5</td>
<td>-61.7</td>
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<tr>
<td>SO$_4^{2-}$</td>
<td>-68.5</td>
<td>-57.6</td>
<td>-57.3</td>
<td>-62.2</td>
<td>-71.4</td>
<td>-85.7</td>
<td>-107.7</td>
<td>-144.2</td>
</tr>
</tbody>
</table>

$^a$cal mole$^{-1}$($^oK)^{-1}$. 
all the curves are similar in configuration to those for the electrolytes in figures 108 through 105. Additivity values of $C^\circ_P$ for NaBr computed from eq (270) and the regression parameters for Na$^+$ and Br$^-$ are plotted on the ordinate of figure 113, where it can be seen that (except at $5^\circ$C) they are closely consistent with independent calorimetric data reported by Tanner and Lamb (1978). The slopes of the linear curves in figure 113 (which correspond to $\nu_{b_2}/4$ for NaBr) are plotted as a function of temperature in figure 114. Values of $C^\circ_{P,v}NaCl$ computed in the present study can be compared in figure 115 with those calculated by Silvester and Pitzer (1977) and Pitzer and others (1979) for temperatures to $275^\circ$C along the liquid-vapor equilibrium curve for H$_2$O, all of which are significantly less negative than the corresponding values of $C^\circ_P$ generated below from calorimetric data reported by Puchkov, Styazhkin, and Federov (1976). The $25^\circ$C values of $C^\circ_{P,j}$ in table 17 compare favorably with those calculated by Desnoyers and others (1976).

Although in certain instances the conventional standard partial molal heat capacities in table 17 yield additivity values of $C^\circ_{P,k}$ that differ (as a result of regression vagaries) by as much as 1.8 cal mole$^{-1}$ $(^\circ$K)$^{-1}$ from

![Fig. 112. Calculated conventional standard partial molal heat capacities of aqueous ions as a function of temperature at pressures corresponding to those along the liquid-vapor equilibrium curve for H$_2$O (see text and table 17).](image-url)
Fig. 113. \( \rho^* \) (eq 238) computed (assuming \( I = I \)) from apparent molal heat capacities at 1 bar reported by Tanner and Lamb (1978). The values represented by the open triangles on the ordinates of the diagrams correspond to values of \( C^*_P \) generated independently in the present study (see text).
those of corresponding electrolytes in table 15, the bulk of the discrepancies are ≤ 0.1 cal mole⁻¹ (°K)⁻¹. Differences of this order of magnitude are small enough to be considered insignificant in most geochemical calculations.

The equations derived above can also be used to represent the thermodynamic properties of neutral complexes at high pressures and temperatures. The validity of this conclusion has already been demonstrated in the case of aqueous silica (Walther and Helgeson, 1977, 1979), and another example is shown in figures 116 and 117 where experimental and calculated values of log K_{NaCl}° (where NaCl° stands for the aqueous NaCl molecule) are plotted against temperature and pressure. The values of log K represented by the filled circles in figure 116 correspond to the intercepts of the curves shown in figure 118, which were generated from eqs (180) and (181) using experimental osmotic coefficient data reported by Liu and Lindsay (1972). The requirement for both the slopes and intercepts of the curves shown in figure 118 to be smooth functions of temperature led to the interpretation of the data represented by the curves in the figures and the values of b_{v,NaCl}° and log K_{NaCl}° represented by the filled circles in figures 116 and 119. The open symbols in figure 119 represent values of b_{v,NaCl}° generated by fitting a combined statement of eqs (29), (103), and (198) to the experimental data represented by the symbols in figure 49 (see above). It can be deduced from figure 119 that the latter values are consistent with those derived from the high-temperature vapor pressure data represented by the symbols in figure 118. The minimum in the curve shown in figure 119 is similar to, but more pronounced than, that exhibited by b_{v,CO₂(aq)} in NaCl solutions as a function of temperature at pressures corresponding to liquid-vapor equilibrium (Helgeson

![Graph](image)

**Fig. 114.** Temperature dependence (at 1 bar) of the negative equivalents of the slopes of the linear curves for NaBr in figures 66 and 113 (see text and eq 238).

![Graph](image)

**Fig. 115.** Comparative plot of various computed values of the standard partial molal heat capacity of NaCl as a function of temperature (T) at 1 bar for T < 100°C and pressures corresponding to those along the liquid-vapor equilibrium curve for H₂O at T ≥ 100°C.
and James, 1968). In contrast, $b_{\gamma, \text{SiO}_2(aq)}$ in NaNO$_3$ solutions apparently decreases monotonically with increasing temperature, at least to 500°C (Marshall, 1980b).

The curves shown in figures 116 and 117 were generated using values of $a_1$, $a_2$, $a_3$, $c_1$, $c_2$, $\omega$, and $\theta$ for the aqueous NaCl molecule that were obtained by regression of the experimental data shown in the figures using an approach similar to that described above for OH$^-$. The close agreement of the curves and the bulk of the symbols in figure 117 leaves little doubt that eqs (254) and (273) afford close approximation of the thermodynamic behavior of NaCl$^\circ$ at high temperatures and pressures. The cause of the systematic discrepancies between the calculated and experimental values of log $K_{\text{NaCl}}$ at temperatures and pressures from $\sim$ 400°C and $\sim$ 500 bars to $\sim$ 500°C and 3 kb in figure 117 is not clear. It may well be due to experimental error, but it could also result from errors in the equation of state parameters caused by insufficient regression control at the boundaries of the fit region. In any event, these discrepancies, together with those apparent in figure 116 reduce the status of the parameters given for NaCl$^\circ$ in table 18 to first approximations. Under no circumstances should they be used for temperatures and pressures outside the fit region in figures 116 and 117.

---

**Fig. 116.** Calculated (curve) and experimental (symbols) logarithm of the standard molal dissociation constant of NaCl as a function of temperature at pressures corresponding to those along the liquid-vapor equilibrium curve for H$_2$O.
The opposite configurations of the curves shown in figures 116 and 119 suggest that the degree of ion association in NaCl solutions may maximize as a function of concentration at high temperatures. This behavior is exhibited by ZnSO₄ and other electrolytes at 25°C (Davies, 1962; Pitzer, in press), as well as LiCl, LiBr, KCl, and CsCl solutions at high pressures and temperatures (Hwang, Lidemann, and Hartmann, 1970). The latter electrolytes are “completely” dissociated in both concentrated and dilute solutions. At intermediate concentrations the electrolytes are highly associated. The fact that the second derivatives of the two curves in figures 116 and 119 are of opposite sign contributes significantly to the high uncertainties attending both theoretical prediction and experimental documentation of the thermodynamic properties of concentrated electrolyte solutions at high temperatures and low pressures (Helgeson, in press).

Fig. 117. Calculated (curves) and experimental (symbols) logarithm of the standard molal dissociation constant of NaCl as a function of pressure at constant temperature (labeled in °C). The symbols represent experimental data reported by Eigen and Wicke (1954), Pearson, Copeland, and Beuson (1963a), Quist and Marshall (1968a), and Millero (1970). Saturation (H₂O) refers to the liquid-vapor equilibrium curve for H₂O.
Fig. 118. Graphic representation of eq (181) for NaCl at various temperatures (indicated in °C) and pressures corresponding to those along the liquid-vapor equilibrium curves. The symbols represent experimental data reported by Liu and Lindsay (1972) (see text).

Fig. 119. Extended term parameter for the activity coefficient of NaCl° in NaCl solutions as a function of temperature at pressures corresponding to those along the liquid-vapor equilibrium curve for H₂O.
Fig. 120. Correlation of the heat capacity collapse parameter ($c_2$) with the corresponding compressibility parameter ($a_4$) in eq (273). The values of $c_2$ were generated in the present study (see text and tables 14 and 15), but those of $a_4$ were taken from Helgeson and Kirkham (1976).
**Correlation and estimation algorithms.** — A number of correlations among thermodynamic properties, effective electrostatic and crystal radii, and equation of state parameters for ionic species have been summarized elsewhere (Helgeson and Kirkham, 1976), which can be used in conjunction with those described below to estimate values of $c_2$ and $c_4$ for aqueous species. For example, it can be seen in figures 120 and 121 that $c_2$ can be regarded as a linear function of $a_4$, and that $a_4$ can also be expressed as a function of $a_4$. These observations permit calculation of $c_2$ for aqueous species from compressibility data. The equation corresponding to the curves shown in figure 120 can be written as

$$c_2 = a + b a_4$$  \hspace{1cm} (281)

where $a = -0.7$ and $b = -1.60 \times 10^5$ for monovalent ions and 1:1 electrolytes, but $a = -2.90$ and $b = -2.30 \times 10^5$ for divalent ions and 1:2 or 2:1 electrolytes. Eq (281) together with data and algorithms given by Helgeson and Kirkham (1976) were used in the present study to estimate values of $c_2$ for a large number of aqueous species for which insufficient heat capacity data are available to permit retrieval of $c_1$ and $c_2$ from regression calculations. These estimates were then combined with eq (270) and values

\[\text{Fig. 121. Correlation of } a_4 \text{ with } a_4 \text{ in eq (254). The values of the parameters (table 18) are those given by Helgeson and Kirkham (1976).}\]
of $\omega$, $\theta$, and $X$ to calculate estimates of $c_i$ from corresponding values of $C_{o,P,T}$. In certain instances where no heat capacity data are available, estimates of $C_{o,P,T}$ were used to calculate $c_i$. The latter estimates were generated from correlation algorithms represented by the linear curves in figures 122 through 124, which are consistent with

$$C_{o,P,T} = a + bS_{o}$$

(282)

where $a$ and $b$ stand for the following parameters:

<table>
<thead>
<tr>
<th></th>
<th>a</th>
<th>b</th>
</tr>
</thead>
<tbody>
<tr>
<td>Monovalent cations (curve a, fig. 122):</td>
<td>18</td>
<td>-0.67</td>
</tr>
<tr>
<td>Alkaline earth cations (curve b, fig. 122):</td>
<td>-10</td>
<td>-0.2</td>
</tr>
<tr>
<td>Transition metal cations (curve d, fig. 122):</td>
<td>7.2</td>
<td>0.55</td>
</tr>
<tr>
<td>Trivalent cations (curve c, fig. 122):</td>
<td>-48</td>
<td>-0.7</td>
</tr>
<tr>
<td>Monovalent anions (curve e, fig. 122):</td>
<td>-30.5</td>
<td>0</td>
</tr>
<tr>
<td>Oxygenated anions (fig. 123):</td>
<td>-76</td>
<td>1.5</td>
</tr>
<tr>
<td>Acid oxygenated anions (fig. 124):</td>
<td>-164.5</td>
<td>6.5</td>
</tr>
</tbody>
</table>

Fig. 122. Correlation of the conventional standard partial molal heat capacities and entropies of aqueous species at 25°C and 1 bar (tables 3, 13, and 18).
Similar estimates can be made for polar neutral species by taking account of other correlations, such as those discussed by Cobble (1958a, b, and c) and Helgeson (1969). One of these is shown in figure 125, which is consistent with

$$S^\circ_n = 105 - 205(r_{x,+},n + r_{x,-},n)^{-1}$$  (283)

where $r_{x,+},n$ and $r_{x,-},n$ refer to the crystal radii of the cation and anion in the $n$th neutral molecule.

Although considerable uncertainty attends the estimation procedure described above, it can be seen in figures 103 through 105 that many of the dashed curves generated from such estimates are in close agreement with the solid regression curves. In fact, that for CsI coincides with the solid curve. In other cases, discrepancies of the order of 5 or 6 cal mole$^{-1}$ (°K)$^{-1}$ occur between the dashed and solid curves. Nevertheless, errors in $C^\circ_{p,i}$ of this order of magnitude commonly introduce corresponding errors in computed values of $\Delta C^\circ$ at high pressures and temperatures that are within the overall uncertainty attending geochemical calculations. Consequently even these rough estimates are useful. Estimated values of $C^\circ_{p,j}$ as a function of temperature for a number of aqueous species are shown in figure 126.

The equation of state parameters for aqueous species given by Helgeson and Kirkham (1976) and those derived above from regression calculations and estimation algorithms are summarized in table 18, together with thermodynamic data for the species at 298.15°K and 1 bar. It can be seen by comparing figures 103 through 105 and 112 with figure 126 that the curves generated from the estimated parameters given in table 18 exhibit the same configuration as those derived from the regression calculations.
Summary of the conventional standard partial molal thermodynamic properties of aqueous species designated by the subscript $j$ at 25°C and

<table>
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<tr>
<th>Species</th>
<th>$\Delta f_{\text{a}}^{\circ}$</th>
<th>$\Delta H_{\text{a}}^{\circ}$</th>
<th>$\Delta G_{\text{a}}^{\circ}$</th>
<th>$\Delta c_{\text{b}}^{\circ}$</th>
<th>$\Delta c_{\text{b}}^{\circ}$</th>
<th>$\Delta G_{\text{b}}^{\circ}$</th>
<th>$\Delta I_{\text{a}}^{\circ}$</th>
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<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
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<td>-69,946$^{bb}$</td>
<td>-66,552$^{z}$</td>
<td>2.7</td>
<td>14.4</td>
<td>-1.2$^k$</td>
<td>27.298$^k$</td>
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<tr>
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<td>-57,433$^{z}$</td>
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<td>10.0</td>
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<td>35.715$^k$</td>
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</tr>
<tr>
<td>K$^+$</td>
<td>-67,576$^{bb}$</td>
<td>-60,270$^{z}$</td>
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<td>3.0</td>
<td>8.7$^k$</td>
<td>18.468$^k$</td>
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</tr>
<tr>
<td>Cs$^+$</td>
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<td>-61,672$^{z}$</td>
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<td>-5.5</td>
<td>21.2$^k$</td>
<td>29.961$^k$</td>
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</tr>
<tr>
<td>NH$_4^+$</td>
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<td>-31,850$^{z}$</td>
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<td>17.8$^k$</td>
<td>34.04$^k$</td>
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<td>Ag$^+$</td>
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<td>25,275$^{z}$</td>
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<td>20.07$^b$</td>
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<td>Cu$^+$</td>
<td>11,950$^{dd}$</td>
<td>17,132$^{aa}$</td>
<td>9.7</td>
<td>11.5$^h$</td>
<td>-10.5$^n$</td>
<td>27.14$^n$</td>
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<tr>
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<td>48,175$^{aa}$</td>
<td>26.5</td>
<td>0.2$^h$</td>
<td>13.0$^o$</td>
<td>16.02$^o$</td>
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<tr>
<td>Mg$^{++}$</td>
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<td>-111,563$^{aa,gg}$</td>
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<td>-4.3</td>
<td>-21.3$^k$</td>
<td>39.025$^k$</td>
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<td>-129,804$^{z}$</td>
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<td>-130,458$^{aa}$</td>
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<td>-128,502$^{aa}$</td>
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<td>-36,661$^{z}$</td>
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<tr>
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<td>15,700$^{z}$</td>
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<td>-27.8$^k$</td>
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<td>40,669$^{z}$</td>
<td>-8.7</td>
<td>2.4$^h$</td>
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<td>-21,302$^{aa}$</td>
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<td>-52,724$^{aa}$</td>
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<td>45.59$^k$</td>
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<td>-11,601$^{aa}$</td>
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<td>-127,008$^{cc}$</td>
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<td>3.9</td>
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<td>65.27$^k$</td>
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<td>98,913$^{aa}$</td>
<td>-51.2</td>
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<td>-41.3$^k$</td>
<td>44.73$^k$</td>
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<td>-80,151$^{z}$</td>
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<td>-28.2</td>
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<td>0.8814$^k$</td>
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<td>-39,933$^{z}$</td>
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<td>-30.2</td>
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<td>-4.838$^k$</td>
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<td>-29,039$^{z}$</td>
<td>19.8</td>
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<td>6.4161$^k$</td>
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<tr>
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<td>-13,599$^{z}$</td>
<td>25.5</td>
<td>-29.0</td>
<td>36.3$^k$</td>
<td>1.3667$^k$</td>
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<tr>
<td>OH$^-$</td>
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<td>-54,977$^{z}$</td>
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<td>-33.0</td>
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<tr>
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<td>-4,218$^{aa}$</td>
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<td>-49,530$^{bb,gg}$</td>
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<td>-164,898$^{ff}$</td>
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<td>15.40$^k$</td>
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<td>-177,915$^{bb}$</td>
<td>-217,400$^{z}$</td>
<td>4.5</td>
<td>-68.5</td>
<td>14.1$^k$</td>
<td>1.3418$^k$</td>
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<tr>
<td>NaCl$^-$</td>
<td>-92,740$^{k}$</td>
<td>-99,873$^{k}$</td>
<td>15.0$^k$</td>
<td>43.8$^k$</td>
<td>13.5$^k$</td>
<td>63.6309$^k$</td>
<td></td>
</tr>
<tr>
<td>SiO$_2^{(aq)}$</td>
<td>-193,190$^{k}$</td>
<td>-210,726$^{k}$</td>
<td>14.0$^k$</td>
<td>-25.0$^k$</td>
<td>13.6$^k$</td>
<td>48.5169$^k$</td>
<td></td>
</tr>
</tbody>
</table>
1 bar and the parameters required to calculate the corresponding properties at high pressures and temperatures from eqs (254) and (273) through (276)*

<table>
<thead>
<tr>
<th>Species</th>
<th>$b_{2,2}\text{L}^{-1}$</th>
<th>$a_{d,2}\text{L}^{-2}$</th>
<th>$a_{d,2}\text{L}^{-1}\times10^{5}$</th>
<th>$a_{d,2}\text{L}^{-1}\times10^{6}$</th>
<th>$a_{d,2}\text{L}^{-2}\times10^{5}$</th>
<th>$c_{f,2}\text{L}^{-1}$</th>
<th>$\omega_{2,2}\text{L}^{-2}\times10^{5}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>H$^+$</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>Li$^+$</td>
<td>-1.2342</td>
<td>0.004L</td>
<td>3.62L</td>
<td>-0.043L</td>
<td>0.223L</td>
<td>0.0</td>
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</tr>
<tr>
<td>Na$^+$</td>
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<td>0.191L</td>
<td>-4.782L</td>
<td>-5.628L</td>
<td>3.6560L</td>
<td>215.17L</td>
<td>0.3306L</td>
</tr>
<tr>
<td>K$^+$</td>
<td>-2.9265</td>
<td>0.310L</td>
<td>0.752L</td>
<td>-1.9460L</td>
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<td>0.0</td>
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<tr>
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<td>6.06L</td>
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<tr>
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<td>-0.31L</td>
<td>0.39L</td>
<td>264.6L</td>
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<td>271.4L</td>
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</tr>
<tr>
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<td>-2.49L</td>
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<td>-1.50L</td>
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<td>0.1800L</td>
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<td>0.0704L</td>
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<td>0.1768L</td>
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<tr>
<td>Sr$^{++}$</td>
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<td>-0.344L</td>
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<td>0.0903L</td>
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<tr>
<td>Ba$^{++}$</td>
<td>-6.4428</td>
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<td>0.9851L</td>
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<tr>
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<td>-0.20L</td>
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<td>1.0788L</td>
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<td>5.5L</td>
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<td>257.0L</td>
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<td>-0.9L</td>
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<tr>
<td>OH$^-$</td>
<td>-6.2074L</td>
<td>0.0515L</td>
<td>-0.0998L</td>
<td>-1.9845L</td>
<td>1.0812L</td>
<td>253.52L</td>
<td>1.7246L</td>
</tr>
<tr>
<td>H$^+$</td>
<td>-2.271L</td>
<td>0.651L</td>
<td>-6.90L</td>
<td>-1.291L</td>
<td>0.983L</td>
<td>252.19L</td>
<td>1.4110L</td>
</tr>
<tr>
<td>NO$_3^-$</td>
<td>-17.72L</td>
<td>1.2438L</td>
<td>-32.602L</td>
<td>-18.275L</td>
<td>10.6342L</td>
<td>185.29L</td>
<td>1.1295L</td>
</tr>
<tr>
<td>HCO$_3^-$</td>
<td>-3.00L</td>
<td>0.788L</td>
<td>-8.225L</td>
<td>-2.678L</td>
<td>1.4350L</td>
<td>236.87L</td>
<td>1.3293L</td>
</tr>
<tr>
<td>SO$_4^{2-}$</td>
<td>-4.6940</td>
<td>0.5825L</td>
<td>4.1576L</td>
<td>-0.6501L</td>
<td>0.7125L</td>
<td>263.03L</td>
<td>3.1857L</td>
</tr>
<tr>
<td>NaG$^+$</td>
<td>-1.00L</td>
<td>4.6779L</td>
<td>267.2577L</td>
<td>-47.102L</td>
<td>-243.5009L</td>
<td>256.25L</td>
<td>1.1381L</td>
</tr>
<tr>
<td>SiO$_2$</td>
<td>-15.7891</td>
<td>0.0211L</td>
<td>4.9341L</td>
<td>-6.8259L</td>
<td>0.2979L</td>
<td>233.00L</td>
<td>0.1291L</td>
</tr>
</tbody>
</table>

*The $S_1$ and $S_2$ parameters shown in the table for NH$_4^+$, H$, $NO_{3}^-$, and HCO$_3^-$ together with $S_1$, $S_2$, $S_3$, $S_4$, $S_5$, $S_6$, $S_7$, and $S_8$ for Pb$^{++}$, Zn$^{++}$, Cu$^{++}$, Hg$^{++}$, Fe$^{++}$, Ag$^+$, Cu$^+$, and Au$^+$ and Mn$^{++}$ correspond to revised estimates of earlier parameters used to generate the curves for these species in Figures 7.3 and 7.4 of Helgeson (in press). The revised estimates do not affect the general configurations of the curves, but they alter slightly their positions. As a consequence, the curves in the lower diagram of Figure 7.3 of Helgeson (in press) are not the same as the corresponding curves in Figure 126 of the present communication.

Table 18 notes continued on p. 1436.
Table 18 and the equations derived in the preceding pages have been combined with thermodynamic data and equations for minerals (Helgeson and others, 1978) and electrostatic/thermodynamic equations and data for H₂O (Helgeson and Kirkham, 1974a; Delany and Helgeson, 1978) in computer program SUPCRT, which can be used to calculate equilibrium phase relations in a wide variety of hydrothermal systems at high pressures and temperatures. Prediction and analysis of the thermodynamic behavior of aqueous species under these conditions affords insight into the causal processes responsible for many of the chemical and mineralogic changes observed in rocks. Copies of the program and data file, together with a program description can be obtained at cost from the senior author before

Notes for table 18 (continued)
July 1, 1982. The program description, program (which is written in FORTRAN IV for CDC 6400, 6600, and 7600 computers), and data file are stored on magnetic tape (7 track, 800 binary bits inch⁻¹, 132 characters record⁻¹, 1 record block⁻¹).

High pressure/temperature predictions.—Prediction of the thermodynamic behavior of aqueous species at high pressures and temperatures with eqs (273) through (276) generated the curves shown for Mg²⁺, Na⁺, Cl⁻, and SO₄⁻⁻ in figures 127 through 130. The configurations of these curves are representative of those for other monovalent and divalent cations and anions. The electrostatic properties of H₂O employed in the calculations are those given by Helgeson and Kirkham (1974a).

It can be seen in figure 127 that the calculations discussed above indicate that C°ₚ as a function of temperature at constant pressure exhibits a double inflexion. The maxima in the isobars arise from opposing contributions by local collapse of the solvent structure and ion solvation to C°ₚ, which become less negative and more negative, respectively, with increasing temperature. The high-temperature minima are a consequence of the large negative values of (∂ ln e°/∂T)ₚ in the critical region, where (∂ ln e°/∂T)ₚ increases dramatically from −∞ at the critical point with either increasing pressure and/or temperature (Helgeson and Kirkham, 1974a). As expected, both extrema tend to dampen with

![Graph](image-url)

Fig. 125. Correlation of the standard partial molal entropies of neutral aqueous species at 25°C and 1 bar (Wagman and others, 1968, 1969) with the reciprocal sum of the crystal radii of the ionic constituents of the species taken from Sienko and Plane (1963).
increasing pressure. Note that the calculations indicate that $C_P^o$ also maximizes with increasing pressure at constant temperature. However, it should be emphasized in this regard that the positions and configurations of the curves shown in figures 127 through 130 are more uncertain for temperatures $\leq$ 100°C at pressures $> 1$ kb, temperatures $\geq$ 500°C at all pressures, and 5 kb at all temperatures. The greater uncertainties in these pressure/temperature regions (which are indicated by the dashed curves in figs. 127 through 130) arise from uncertainties in calculated values of the dielectric constant of $H_2O$ in these regions (Helgeson and Kirkham, 1974a).

Fig. 126. Estimated conventional standard partial molal heat capacities of aqueous ions as a function of temperature at pressures corresponding to those along the liquid-vapor equilibrium curve for $H_2O$ (see text). The curves were generated from eq (273) using parameters given in table 18.
Fig. 127. Conventional standard partial molal heat capacities of aqueous ions at high pressure (labeled in kb) and temperature (labeled in °C) computed from eq (273) using thermodynamic data and parameters taken from table 18 and the electrostatic properties of the solvent given by Helgeson and Kirkham (1974a). The dashed curves are more uncertain than their solid counterparts.
Fig. 128. Conventional standard partial molal entropies of aqueous ions at high pressure (labeled in kb) and temperature (labeled in °C) computed from eq (274) using thermodynamic data and parameters taken from table 18 and the electrostatic properties of the solvent given by Helgeson and Kirkham (1974a). The dashed curves are more uncertain than their solid counterparts.
Fig. 129. Conventional apparent standard partial molal enthalpies of formation of aqueous ions at high pressure (labeled in kb) and temperature (labeled in °C) computed from eq (275) using thermodynamic data and parameters taken from table 18 and the electrostatic properties of the solvent given by Helgeson and Kirkham (1974a). The dashed curves are more uncertain than their solid counterparts.
Fig. 130. Conventional apparent standard partial molal Gibbs free energy of formation of aqueous ions at high pressure (labeled in kb) and temperature (labeled in °C) computed from eq (276) using thermodynamic data and parameters taken from table 18 and the electrostatic properties of the solvent given by Helgeson and Kirkham (1974a). The dashed curves are more uncertain than their solid counterparts.
As a consequence of different degrees of influence on $C_{P,j}$ for cations and anions by the electrostatic properties of the solvent in different regions of pressure/temperature space, the curves in figure 128 representing $S^\circ$ for Cl$^-$ and SO$_4^{2-}$ as a function of temperature at constant pressure exhibit a "synclinal" configuration which is not apparent in the curves representing $S^\circ$ for Na$^+$ and Mg$^{++}$. Note in figure 129 that the same observation holds for $\Delta H^\circ$, which, like $S^\circ$ and $C_{P}$, approaches negative infinity at the critical point of H$_2$O.

It can be seen in figure 130 that increasing pressure at constant temperature has little effect on $\Delta C^\circ$ for Na$^+$ and Mg$^{++}$ at pressures $> 2$ kb, which is not true for any pressure and temperature in the case of Cl$^-$ and SO$_4^{2-}$. As a consequence, $\Delta C^\circ$ for Cl$^-$ and SO$_4^{2-}$ minimizes with increasing temperature at constant pressure, while $\Delta C^\circ$ for Na$^+$ and Mg$^{++}$ changes monotonically. Contributions by ion solvation and local collapse of the solvent structure to $\Delta C^\circ$ are both negative. Because $S^\circ$ is positive for Na$^+$ and negative for Mg$^{++}$ throughout the pressure/temperature region considered in figures 127 through 130, $\Delta C^\circ_{Na^+}$ increases and $\Delta C^\circ_{Mg^{++}}$ decreases with increasing temperature at constant pressure. As a consequence, the solubilities of magnesian compounds would be expected to decrease relative to those of sodium minerals with increasing temperature at all pressures, which is in general agreement with geologic observations.

Standard molal enthalpies of hydration (see footnote 12) of aqueous species at 300°C and 86 bars computed with the aid of the equations and parameters summarized above using thermodynamic data and equations for gaseous species taken from Wagman and others (1969) and Helgeson (1969) compare favorably with those generated from calorimetric measurements by Cobble and Murray (1977). Similarly, activity/composition relations and stability fields for minerals on activity diagrams generated with the aid of the equations and parameters derived in the present study afford close approximation of both experimental and geologic phase relations at high pressures and temperatures (Helgeson and others, 1978; Walther and Helgeson, 1980; Bird and Helgeson, 1981). These observations underscore the general validity and applicability of the theoretical equations, parameters, and standard partial molal properties of aqueous species summarized above.

**Extended Term Parameters for Single Electrolytes**

Because the short-range interaction contribution to $\log \gamma_{\pm}$ as a function of pressure and temperature incorporates the ionic strength dependence of departures from ideality caused by collapse of the local solvent structure about the ions in solution, it would be reasonable to expect expressions of the form of eqs (254), (258), and (273) to describe the pressure and temperature dependence of $b_{\gamma,k}$, $b_{\kappa,k}$, and $b_{\delta,k}$, respectively. However, for this to be the case the reciprocals of the dielectric constants of electrolyte solutions ($\varepsilon^{-1}$) at a given ionic strength must be linearly related to the reciprocal of the dielectric constant of the solvent ($\varepsilon^\circ_{-1}$) as a func-
tion of pressure and temperature. Although the paucity of reliable experimental dielectric constant data for electrolyte solutions at high pressures and temperatures precludes a definitive conclusion in this regard, it appears from the correspondence of the curves and symbols in figure 131 that at least at low temperatures, $\varepsilon^{-1}$ can be regarded as a linear function of $\varepsilon^0$ at constant ionic strength. Although obvious discrepancies between the symbols and curves are apparent in figure 131, most of these can be attributed to experimental uncertainty, which increases substantially with increasing temperature. For example, it can be seen that the trend of the experimental values of $\varepsilon^{-1}$ for NaCl reported by Hasted, Ritson, and

Fig. 131. Correlation of the reciprocal of the dielectric constant of electrolyte solutions ($\varepsilon^{-1}$) with that of the solvent ($\varepsilon^0$) as a function of temperature at 1 bar and constant composition (see caption of fig. 3).
Fig. 132. Calculated (curves) and experimental (symbols) extended term parameters for the apparent molal heat capacities of electrolytes as a function of temperature (T) at 1 bar for T < 100°C and pressures corresponding to liquid-vapor equilibrium at T ≥ 100°C. The symbols represent the negative equivalents of the slopes of the linear curves shown for the electrolytes in figures 65 through 82, but (with the exception of the dashed curve for MgSO₄) the curves were generated from eq (291) (see text). The different symbols represent experimental data taken from the various sources cited in the caption of figure 65.
Collie (1948) from 0° to 40°C is inconsistent with the corresponding trend of the data reported by Hasted and Roderick (1958). It should perhaps be emphasized in this regard that the solid curves in figure 131 are consistent with one another and with the equations and parameters derived below. The dashed curves merely connect data points.

The bulk of the experimental data shown in figure 131 can be described by

\[
\frac{1}{\varepsilon_{p,T}} - \frac{1}{\varepsilon^{0}_{p,T}} = k \left( \frac{1}{\varepsilon^{0}_{p,T}} - \frac{1}{\varepsilon^{0}_{p,T}} \right)
\]

where \(\varepsilon_{p,T}\) and \(\varepsilon^{0}_{p,T}\) denote the dielectric constant of the electrolyte solution and the solvent, respectively, at the subscripted pressure and temperature, and \(k\) stands for a pressure/temperature-independent coefficient char-

![Diagram showing the apparent molal heat capacities of electrolytes as a function of temperature (T) at 1 bar for T < 100°C and pressures corresponding to liquid-vapor equilibrium at T ≥ 100°C. The symbols represent the negative equivalents of the slopes of the linear curves shown for the electrolytes in figures 65 through 82, but the curves were generated from equation (291) (see text). The different symbols represent experimental data taken from the various sources cited in the caption of figure 65.](image)
acteristic of both the electrolyte and the ionic strength of the solution. Note that it follows from eqs (144) and (284) that we can write

\[ k = 1 + \sum_k d_{s,k} y_k I \]  

(285)

where \( d_{s,k} \) represents an electrostatic parameter characteristic of the \( k \)th electrolyte, which is independent of ionic strength as well as pressure and

Fig. 134. Calculated (curves) and experimental (symbols) extended term parameters for the apparent molal volumes of electrolytes as a function of temperature (T) at 1 bar for \( T < 100^\circ \text{C} \) and pressures corresponding to liquid-vapor equilibrium at \( T \geq 100^\circ \text{C} \). The symbols represent the slopes of the linear curves shown for the electrolytes at 1 and 20 bars in figures 83 through 93, but the curves were generated from eq (288) (see text). The different symbols represent experimental data taken from the various sources cited in the caption of figure 83.
Eqs (284) and (285) are consistent with

$$\frac{1}{\varepsilon_{P,T}} = \frac{1}{\varepsilon_{0,P,T}} + \sum_k d_{s,k} y_k \frac{y_k}{}$$

(286)

where \(d_{s,k}\) corresponds to an electrostatic parameter analogous to \(\hat{d}_{s,k}\).

Hence,

$$b_k = d_{s,k} + \frac{\hat{d}_{s,k}}{\varepsilon_{P,T}}$$

(287)

Fig. 135. Calculated (curves) and experimental (symbols) extended term parameters for the apparent molal volumes of electrolytes as a function of temperature \(T\) at 1 bar for \(T < 100^\circ\text{C}\) and pressures corresponding to liquid-vapor equilibrium at \(T \geqslant 100^\circ\text{C}\). The symbols represent the slopes of the linear curves shown for the electrolytes at 1 and 20 bars in figures 83 through 93, but the curves were generated from eq (288) (see text). The different symbols represent experimental data taken from the various sources cited in the caption of figure 83.
Empirical evidence supporting the validity of the hypothesis that the temperature and pressure dependence of $b_v$ should be consistent with the functional form of eqs (254) and (273) can be adduced by comparative analysis of the distribution of the symbols shown in figures 132 through 136 with those in analogous figures depicting the standard partial molal properties of electrolytes (see below). The symbols in figures 132 through 136 represent the slopes of the curves shown for the electrolytes in figures 65 through 98.

It can be seen in figures 132 and 133 that the temperature dependence of $b_{T,k}$ is identical in form to that of $\overline{c}_{p,k}$ in figures 103 through 105. Similar comparison of figures 134 through 136 with those representing the

![Fig. 136. Calculated (curves) and experimental (symbols) extended term parameters for the apparent molal compressibilities of aqueous electrolytes as a function of temperature (T) at 1 bar for T < 100°C and pressures corresponding to liquid-vapor equilibrium for T ≥ 100°C. The symbols represent the slopes of the linear curves shown for NaCl, Na$_2$SO$_4$, MgCl$_2$, and MgSO$_4$ in figures 95, 98, and 99, but the curves were generated from eq (289) (see text). The different symbols represent experimental data reported by Allam (ms) and Millero and others (1974).](image1)

![Fig. 137. Correlation of the extended term parameters for the apparent molal volumes and compressibilities of aqueous electrolytes as a function of temperature at 1 bar. The symbols represent experimental data cited in the captions of figures 83 and 98, but the curves were generated from equations given in the text.](image2)
temperature dependence of the standard partial molal volumes and compressibilities of electrolytes in figures 17 through 19, 26, 27, and 32 through 36 in Helgeson and Kirkham (1976) leaves little doubt that \( b_{v,k} \) and \( b_{c,k} \) vary with increasing temperature in the same manner as \( -\bar{V}_k^o \) and \( -\bar{K}_k^o \), respectively. In fact, all the curves in these various figures have the same absolute configuration with extrema in the vicinity of \( \sim 100^\circ \text{C} \). These observations suggest that the linear correlation of \( b_{c,k} \) with \( b_{v,k} \) for electrolytes with a common cation at 25°C and 1 bar (which follows from the correlations shown in figs. 100 through 102) also holds at higher temperatures. This conclusion is supported by the fact that the relation of \( b_{c,k} \) to \( b_{v,k} \) for a given electrolyte as a function of temperature in figure 137 is identical to that of \( \bar{K}_k^o \) to \( \bar{V}_k^o \), which has been shown (Helgeson and Kirkham, 1976) to be consistent with eqs (254) and (258).

The observations and equations summarized above suggest that

\[
\nu_k b_{v,k}/2, -\nu_k b_{c,k}/2, \text{ and } -\nu_k b_{j,k}/2 \text{ (which correspond to the extended term contributions to } (\bar{V}_k - \bar{V}_k^o)/I, -(\bar{K}_k - \bar{K}_k^o)/I, \text{ and } j_k/I, \text{ respectively) for a given electrolyte can be represented by extended terms analogs of eqs (254), (258), and (270). These analogs can be written as}
\]

\[
\nu_k b_{v,k}/2 = \hat{d}_{s,k} + \hat{d}_{s,k} P + \frac{\hat{d}_{s,k} T + \hat{d}_{s,k} P T}{T - \theta_k} - \omega_k \hat{d}_{s,k} Q , \tag{288}
\]

\[
\nu_k b_{c,k}/2 = \hat{d}_{s,k} + \frac{\hat{d}_{s,k} T}{T - \theta_k} - \omega_k \hat{d}_{s,k} N \tag{289}
\]

and

\[
-\nu_k b_{j,k} / 2 = \hat{c}_{s,k} + \frac{\hat{c}_{s,k} T}{T - \theta_k} + \omega_k \hat{d}_{s,k} T X \tag{290}
\]

where \( \hat{d}_{s,k}, \hat{d}_{s,k}, \hat{d}_{s,k}, \hat{d}_{s,k}, \hat{c}_{s,k}, \text{ and } \hat{c}_{s,k} \) designate parameters analogous to \( a_{s,k}, a_{s,k}, a_{s,k}, c_{s,k}, c_{s,k}, \text{ and } c_{s,k}. \) The last terms in these equations represent the electrostatic contribution by ion solvation to \( \nu_k b_{v,k}/2, -\nu_k b_{c,k}/2 \) and \( -\nu_k b_{j,k}/2. \)

It follows from eqs (288) and (290) that we can now write the extended term analogs of eqs (273), (275), and (276) as

\[
-\nu_k b_{j,k} / 2 = \hat{c}_{s,k} + \frac{\hat{c}_{s,k} T}{T - \theta_k} - \frac{\theta_k T (2d_{s,k} (P - P_r) + d_{s,k} (P^2 - P_{r}^2))}{(T - \theta_k)^2} + \omega_k \hat{d}_{s,k} T X \tag{291}
\]

\[
-\nu_k b_{h,k} / 2 = -\nu_k b_{h,k} / 2 + (\hat{c}_{s,k} + \hat{c}_{s,k}) (T - T_r) + \hat{c}_{s,k} \theta_k \ln \left( \frac{T - \theta_k}{T_r - \theta_k} \right) + \hat{d}_{s,k} (P - P_r) + \hat{d}_{s,k} (P^2 - P_{r}^2)/2 + \frac{2d_{s,k} T (P - P_r) + d_{s,k} T (P^2 - P_{r}^2)}{2(T - \theta_k)} + \frac{2\theta_k d_{s,k} T (P - P_r) + \theta_k d_{s,k} T (P^2 - P_{r}^2)}{2(T - \theta_k)^2} + \omega_k \hat{d}_{s,k} (TY_{P,T} - T_r Y_{P,T} - Z_{P,T} + \nu_r T_r) \tag{292}
\]
and
\[- \nu_k b_{s,k,p,r,T_r} / 2 = - \nu_k b_{s,k,p,r,T_r} / 2 - \nu_k b_{s,k,p,r,T_r} (T - T_r)
\]
\[+ \epsilon_{1,k} (T \ln(T \ln(T/T_r)) - T + T_r)
\]
\[+ \epsilon_{2,k} \left( T - T_r - (T - \theta_k) \ln \left( \frac{T - \theta_k}{T_r - \theta_k} \right) \right)
\]
\[+ \frac{2(\delta_{1,k}(T - \theta_k) + \delta_{2,k} T) (P - P_r) + (\delta_{2,k}(T - \theta_k) + \delta_{4,k} T) (P^2 - P_r^2)}{2(T - \theta_k)}
\]
\[- \omega_k \delta_{2,k} (Z_{k,T} - Z_{k,T_r} - Y_{k,T_r} (T - T_r)) \]

where
\[b_{s,k,p,r,T_r} = -2(2.303) \cdot R T_s b_{y,k,p,r,T_r} \]

and
\[b_{g,k,p,r,T_r} = \frac{b_{h,k,p,r,T_r} - b_{g,p,r,T_r}}{T_r} \]

Regression calculations.—Fits of eqs (288) through (290) to the data represented by the symbols in figures 132 through 136 generated most of the values of \(\epsilon_{1,k}, \epsilon_{2,k}, \delta_{1,k}, \delta_{2,k}, \delta_{4,k}, \delta_{5,k}\) and \(\delta_{s,k}\) given in tables 19 through

**Table 19**

Regression coefficients for calculating extended term parameters for activity and osmotic coefficients and the relative apparent and partial molal properties of the kth aqueous electrolyte from eqs (288) through (295)

<table>
<thead>
<tr>
<th>Solute</th>
<th>(\epsilon_{1,k})</th>
<th>(\epsilon_{2,k})</th>
<th>(\theta_k)</th>
<th>(\omega_k)</th>
<th>(\delta_{5,k})</th>
</tr>
</thead>
<tbody>
<tr>
<td>HCl</td>
<td>0.4534</td>
<td>0.2980</td>
<td>246.02</td>
<td>1.4560</td>
<td>-0.162</td>
</tr>
<tr>
<td>LiCl</td>
<td>-17.0174</td>
<td>3.2366</td>
<td>246.49</td>
<td>1.9421</td>
<td>-0.303</td>
</tr>
<tr>
<td>NaCl</td>
<td>-12.1943</td>
<td>5.0130</td>
<td>228.58</td>
<td>1.7865</td>
<td>-0.206</td>
</tr>
<tr>
<td>KCl</td>
<td>-7.6178</td>
<td>2.4973</td>
<td>240.20</td>
<td>1.6487</td>
<td>-0.262</td>
</tr>
<tr>
<td>CsCl</td>
<td>-14.5437</td>
<td>5.3483</td>
<td>231.08</td>
<td>1.5534</td>
<td>-0.229</td>
</tr>
<tr>
<td>MgCl(_2)</td>
<td>-8.3421</td>
<td>1.8230</td>
<td>247.32</td>
<td>4.4492</td>
<td>-0.068</td>
</tr>
<tr>
<td>CaCl(_2)</td>
<td>-0.9301</td>
<td>0.7870</td>
<td>252.00</td>
<td>4.1485</td>
<td>-0.074</td>
</tr>
<tr>
<td>HBr</td>
<td>-3.8680</td>
<td>2.0470</td>
<td>206.75</td>
<td>1.3858</td>
<td>-0.162</td>
</tr>
<tr>
<td>LiBr</td>
<td>-32.5401</td>
<td>9.1978</td>
<td>209.09</td>
<td>1.8719</td>
<td>-0.303</td>
</tr>
<tr>
<td>CsI</td>
<td>-36.1803</td>
<td>17.0634</td>
<td>198.67</td>
<td>1.3908</td>
<td>-0.229</td>
</tr>
<tr>
<td>KF</td>
<td>-6.4343</td>
<td>1.6708</td>
<td>259.66</td>
<td>1.9797</td>
<td>-0.262</td>
</tr>
<tr>
<td>Na(_2)SO(_4)</td>
<td>-3.5235</td>
<td>5.5248</td>
<td>239.39</td>
<td>3.8468</td>
<td>-0.137</td>
</tr>
</tbody>
</table>

a: cal kg mole\(^{-2}\) (°K\(^{-1}\)). \(b_0\): K. \(c\): cal mole\(^{-1}\). \(d\): kg mole\(^{-1}\).
21 and those of $b_{f,k}$, $b_{v,k}$, and $b_{k,k}$ listed in tables 22 through 24, respectively, which (with the exception of MgSO$_4$) are represented by the curves in figures 132 through 136. Provision for $C_p - C_{sat}$ and the fact that figures 89 through 91 apply to 20 bars were omitted from the calculations, as were the slight pressure differences attending compositional variation and increasing temperature to 200°C along the liquid-vapor equilibrium curves for the electrolytes. It can be shown that these omissions introduce negligible uncertainties in the fit coefficients obtained from the regression calculations. The values of $\omega_k$, $\theta_k$, Q, N, and X employed in the calculations were taken from Helgeson and Kirkham (1974a, 1976). Owing to

<table>
<thead>
<tr>
<th>Solute</th>
<th>$\hat{a}_{1,k} \times 10^2$</th>
<th>$\hat{a}_{2,k} \times 10^2$</th>
<th>$\frac{b}{k}$</th>
<th>$\frac{c}{k} \times 10^{-5}$</th>
<th>$\frac{d}{k}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>HCl</td>
<td>-4.7746</td>
<td>0.8673</td>
<td>246.02</td>
<td>1.4560</td>
<td>-0.162</td>
</tr>
<tr>
<td>LiCl</td>
<td>-7.9087</td>
<td>1.1828</td>
<td>246.49</td>
<td>1.9421</td>
<td>-0.303</td>
</tr>
<tr>
<td>NaCl</td>
<td>-7.0809</td>
<td>2.1723</td>
<td>228.58</td>
<td>1.7865</td>
<td>-0.206</td>
</tr>
<tr>
<td>KCl</td>
<td>-3.6895</td>
<td>1.3218</td>
<td>240.20</td>
<td>1.6487</td>
<td>-0.262</td>
</tr>
<tr>
<td>CsCl</td>
<td>-2.0202</td>
<td>1.0613</td>
<td>231.08</td>
<td>1.5534</td>
<td>-0.229</td>
</tr>
<tr>
<td>NH$_4$Cl</td>
<td>-2.6961</td>
<td>0.6359</td>
<td>234.68</td>
<td>1.6351</td>
<td>-0.099</td>
</tr>
<tr>
<td>KBr</td>
<td>-8.4187</td>
<td>2.7757</td>
<td>219.27</td>
<td>1.5785</td>
<td>-0.262</td>
</tr>
<tr>
<td>KF</td>
<td>1.1050</td>
<td>0.3766</td>
<td>259.66</td>
<td>1.9797</td>
<td>-0.262</td>
</tr>
<tr>
<td>KI</td>
<td>-12.6597</td>
<td>4.5868</td>
<td>204.46</td>
<td>1.4861</td>
<td>-0.262</td>
</tr>
<tr>
<td>NaHCO$_3$</td>
<td>-3.2200</td>
<td>1.1594</td>
<td>226.95</td>
<td>1.6599</td>
<td>-0.206</td>
</tr>
<tr>
<td>KNO$_3$</td>
<td>-8.3133</td>
<td>3.4659</td>
<td>203.02</td>
<td>1.3222</td>
<td>-0.262</td>
</tr>
<tr>
<td>NaHS</td>
<td>-2.7498</td>
<td>0.9954</td>
<td>237.21</td>
<td>1.7716</td>
<td>-0.206</td>
</tr>
<tr>
<td>NH$_4$ClO$_4$</td>
<td>-2.2478</td>
<td>1.6117</td>
<td>71.37</td>
<td>1.1803</td>
<td>-0.099</td>
</tr>
<tr>
<td>MgCl$_2$</td>
<td>-3.2343</td>
<td>0.8303</td>
<td>247.32</td>
<td>4.4492</td>
<td>-0.068</td>
</tr>
<tr>
<td>CaCl$_2$</td>
<td>0.4665</td>
<td>0.4058</td>
<td>252.00</td>
<td>4.1485</td>
<td>-0.074</td>
</tr>
<tr>
<td>SrCl$_2$</td>
<td>-0.2140</td>
<td>0.4925</td>
<td>256.90</td>
<td>4.0482</td>
<td>-0.074</td>
</tr>
<tr>
<td>BaCl$_2$</td>
<td>2.1425</td>
<td>0.7341</td>
<td>245.52</td>
<td>3.8970</td>
<td>-0.065</td>
</tr>
<tr>
<td>Na$_2$SO$_4$</td>
<td>13.6043</td>
<td>1.2777</td>
<td>239.39</td>
<td>3.8468</td>
<td>-0.137</td>
</tr>
<tr>
<td>K$_2$SO$_4$</td>
<td>18.3867</td>
<td>1.2427</td>
<td>250.97</td>
<td>3.5711</td>
<td>-0.175</td>
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</tbody>
</table>

$^a$cal kg mole$^{-2}$ bar$^{-1}$. $^b$K. $^c$cal mole$^{-1}$. $^d$kg mole$^{-1}$. 
prediction, thermodynamic behavior of aqueous electrolytes: IV. 1453

insufficient data, the dashed curve for MgSO₄ in figure 132 merely connects the data points, which are not distributed over a wide enough temperature range to permit definitive calculation of \( \varepsilon_{1,MgSO_4} \) and \( \varepsilon_{2,MgSO_4} \) by regression.

It can be deduced from figures 132 through 136 that eqs (288) through (290) afford close representation of all the extended term parameters represented by the symbols. Note in figure 138 that the values of \( \psi_k \Delta_{s.k} / \nu_{i,k} \) obtained by regression of \( \nu b_3 / 2 \) (figs. 132 and 133) with eq (290) are nearly identical to those for the same electrolytes retrieved from the fits of eq (288) to \( \nu b_4 / 2 \) (figs. 134 and 135). It can also be seen in figure 138 that \( \psi_k \Delta_{s,k} / \nu_{i,k} \) is essentially the same for all electrolytes with a common cation. These observations, together with others summarized below, strongly support the theoretical model responsible for eqs (288) through (293).

It follows from the equations, data, and calculations summarized above that the extended term contributions to \( \bar{V} - \bar{V}^o \), \( \bar{K} - \bar{K}^o \), and \( \bar{C}_p - \)

<table>
<thead>
<tr>
<th>Solute</th>
<th>( \bar{a}_{2,k} \times 10^{-5} )</th>
<th>( \bar{a}_{1,k} \times 10^{-5} )</th>
<th>( \bar{c}_{k} )</th>
<th>( \bar{d}_{k} \times 10^{-5} )</th>
<th>( \bar{e}_{k} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>HCl</td>
<td>2.06(^d)</td>
<td>-0.334(^c)</td>
<td>246.02</td>
<td>1.4560</td>
<td>-0.162</td>
</tr>
<tr>
<td>LiCl</td>
<td>3.61(^d)</td>
<td>-0.529(^c)</td>
<td>246.49</td>
<td>1.9421</td>
<td>-0.303</td>
</tr>
<tr>
<td>NaCl</td>
<td>4.3391(^b)</td>
<td>-1.3217(^b)</td>
<td>228.58</td>
<td>1.7865</td>
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</tr>
<tr>
<td>KCl</td>
<td>2.12(^d)</td>
<td>-0.614(^c)</td>
<td>240.20</td>
<td>1.6487</td>
<td>-0.262</td>
</tr>
<tr>
<td>MgCl₂</td>
<td>0.8232(^b)</td>
<td>-0.4441(^b)</td>
<td>247.32</td>
<td>4.4492</td>
<td>-0.068</td>
</tr>
<tr>
<td>CaCl₂</td>
<td>-1.46(^c)</td>
<td>-0.050(^c)</td>
<td>252.00</td>
<td>4.1485</td>
<td>-0.074</td>
</tr>
<tr>
<td>Na₂SO₄</td>
<td>-13.1620(^b)</td>
<td>-0.5209(^b)</td>
<td>239.39</td>
<td>3.8468</td>
<td>-0.137</td>
</tr>
</tbody>
</table>

\(^a\)Cal kg mole\(^{-2}\) bar\(^{-2}\). \(^b\)Generated by regression of experimental values of \( b_k \) shown in figure 136 with equation (289). \(^c\)Estimated from equation (296) and values of \( \bar{a}_{2,k} \) taken from table 20. \(^d\)Calculated from equation (289) using the values of \( \bar{a}_{4,k} \), \( \bar{c}_{k} \), \( \bar{d}_{k} \), and \( \bar{e}_{k} \) shown above together with values of \( b_k \) taken from table 12 and \( N = 2.24 \times 10^{-10} \) bar\(^{-2}\) (Helgeson and Kirkham, 1974a). \(^e\)Computed as described in footnote \( d \) assuming \( b_k, CaCl₂ = b_k, MgCl₂ = 6.9 \times 10^{-4} \) cm\(^3\) kg mole\(^{-2}\) bar\(^{-1}\) at 25°C and 1 bar. \( f \)Cal mole\(^{-1}\). \( g \)Cal mole\(^{-1}\).
$C^o_P$ as a function of temperature oppose those by $\bar{V}^o$, $\bar{K}^o$, and $C^o_P$. In contrast to $(\partial^2 b_v/\partial T^2)_P$, $(\partial^2 b_{v,\alpha}/\partial T^2)_P$, and $-(\partial^2 b_J/\partial T^2)_P$, which are positive, $(\partial^2 \bar{V}^o/\partial T^2)_P$, $(\partial^2 \bar{K}^o/\partial T^2)_P$, and $(\partial^2 C^o_P/\partial T^2)_P$ are negative. Hence $b_v$, $b_{v,\alpha}$ and $-b_J$ approach $\infty$ at the critical point of H$_2$O, where $\bar{V}^o$, $\bar{K}^o$, and $C^o_P$ approach $-\infty$. Similarly, the various Debye-Hückel and ion association contributions to $\bar{V} - \bar{V}^o$, $\bar{K} - \bar{K}^o$, and $C_P - C^o_P$ approach $\infty$ or $-\infty$ (depending on the contribution) at the critical point of H$_2$O. As a consequence, the densities and heat capacities of aqueous electrolyte solutions (and therefore the apparent molal and partial molal properties of the components of the solutions) at high temperatures and low pressures may exhibit dramatic and multiple inflections and extrema as a function of concentration and/or temperature and pressure.

Because all the regression calculations described above led to essentially the same values of $\psi_k \hat{a}_{5,5,v}/\nu_{i,k}$ for electrolytes with the same cation (fig. 138), a single optimized value of this parameter was adopted for each such group of electrolytes. These values (tables 19 and 20) were used together with values of $\hat{a}_{5,5}$ taken from table 4 to compute corresponding values of $\hat{a}_{5,v}$ from eq (287). The results of these calculations are summarized in table 25, which at least in principle can be used together with

![Fig. 138. Correlation of $\hat{a}_5$ values derived by regression of extended term parameters for the apparent molal heat capacities of aqueous electrolytes as a function of temperature in figures 132 and 133 with those generated by regression of the corresponding apparent molal volume parameters in figures 134 and 135 (see text).](image-url)
eqs (149), (174), (286), and (287) to calculate values of $b_k$, $b_k$, $b_{hl}$ and $\epsilon$ at high pressures and temperatures from the dielectric constant data for H$_2$O given by Helgeson and Kirkham (1974a) and calculated values of $b_{\gamma}$ like those in table 26 (see below). Estimates of $b_k$ computed from eq (287) using values of $d_s$ and $d_{\epsilon}$ taken from tables 20 and 25 were used to generate the solid linear curves for temperatures other than 25°C in figures 5, 6, 10, 14, and 131. It can be seen that these curves are in close agreement with the bulk of the experimental data shown in the figures for RbCl, NaCl, NaBr, MgCl$_2$, and H$_2$SO$_4$, but not with those for other than dilute solutions of BaCl$_2$ and HCl in figure 14. The cause of the discrepancies at higher concentrations in the latter two cases remains unclear.

As might be expected, large uncertainties attend calculation of values of $b_k$, $b_k$, $b_{hl}$, and $\epsilon$ at high pressures and temperatures from eqs (149), (174), (286), and (287) using calculated values of $d_s$ and $b_{\gamma}$ generated by regression of low-temperature data to obtain values of $d_s$, $d_{\epsilon}$, $d_{\gamma}$, $d_{\gamma}$, and $\epsilon$. The uncertainties arise in part from the fact that small errors in $d_s$ resulting from regression vagaries are magnified in high-temperature values of $b_k$ computed from eq (287). As a consequence, calculated values of $b_k$ for a series of electrolytes at a given high temperature may be inconsistent with the cross differentiation requirement represented by eq (148).

**Table 22**

Computed values of the extended term parameter$^a$ for calculating the relative apparent and partial molal heat capacities of aqueous electrolytes from eqs (224) and (229) at 1 bar for temperatures $< 100^\circ$C and pressures corresponding to liquid-vapor equilibrium at temperatures $\geq 100^\circ$C

<table>
<thead>
<tr>
<th>Solute</th>
<th>$0$</th>
<th>$25$</th>
<th>$50$</th>
<th>$75$</th>
<th>$100$</th>
<th>$125$</th>
<th>$150$</th>
<th>$175$</th>
<th>$200$</th>
</tr>
</thead>
<tbody>
<tr>
<td>HCl</td>
<td>$-5.3$</td>
<td>$-4.1$</td>
<td>$-4.2$</td>
<td>$-4.4$</td>
<td>$-4.9$</td>
<td>$-5.6$</td>
<td>$-6.7$</td>
<td>$-8.3$</td>
<td>$-11.1$</td>
</tr>
<tr>
<td>LiCl</td>
<td>$-20.6$</td>
<td>$-11.3$</td>
<td>$-2.9$</td>
<td>$-1.3$</td>
<td>$-1.3$</td>
<td>$-2.4$</td>
<td>$-4.5$</td>
<td>$-8.2$</td>
<td>$-14.7$</td>
</tr>
<tr>
<td>NaCl</td>
<td>$-21.3$</td>
<td>$-12.8$</td>
<td>$-8.8$</td>
<td>$-7.0$</td>
<td>$-6.2$</td>
<td>$-6.4$</td>
<td>$-7.3$</td>
<td>$-9.3$</td>
<td>$-13.1$</td>
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<tr>
<td>KCl</td>
<td>$-16.4$</td>
<td>$-9.3$</td>
<td>$-6.7$</td>
<td>$-5.8$</td>
<td>$-5.8$</td>
<td>$-6.6$</td>
<td>$-8.2$</td>
<td>$-11.0$</td>
<td>$-15.7$</td>
</tr>
<tr>
<td>CsCl</td>
<td>$-22.9$</td>
<td>$-12.6$</td>
<td>$-8.0$</td>
<td>$-5.8$</td>
<td>$-4.8$</td>
<td>$-4.8$</td>
<td>$-5.6$</td>
<td>$-7.4$</td>
<td>$-11.0$</td>
</tr>
<tr>
<td>MgCl$_2$</td>
<td>$-8.9$</td>
<td>$-3.5$</td>
<td>$-1.8$</td>
<td>$-1.1$</td>
<td>$-1.1$</td>
<td>$-1.4$</td>
<td>$-2.1$</td>
<td>$-3.3$</td>
<td>$-5.5$</td>
</tr>
<tr>
<td>CaCl$_2$</td>
<td>$-7.7$</td>
<td>$-4.7$</td>
<td>$-3.9$</td>
<td>$-3.8$</td>
<td>$-4.1$</td>
<td>$-4.6$</td>
<td>$-5.5$</td>
<td>$-6.9$</td>
<td>$-9.2$</td>
</tr>
<tr>
<td>HBr</td>
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<td>$-4.9$</td>
<td>$-4.2$</td>
<td>$-3.9$</td>
<td>$-4.1$</td>
<td>$-4.5$</td>
<td>$-5.4$</td>
<td>$-6.8$</td>
<td>$-9.3$</td>
</tr>
<tr>
<td>LiBr</td>
<td>$-11.0$</td>
<td>$-3.6$</td>
<td>$0.5$</td>
<td>$2.5$</td>
<td>$3.1$</td>
<td>$2.7$</td>
<td>$1.1$</td>
<td>$-2.1$</td>
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</tr>
<tr>
<td>CsI</td>
<td>$-28.8$</td>
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<td>$-11.5$</td>
<td>$-7.5$</td>
<td>$-5.1$</td>
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<td>$-3.4$</td>
<td>$-4.2$</td>
<td>$-6.7$</td>
</tr>
<tr>
<td>KF</td>
<td>$-31.5$</td>
<td>$-11.4$</td>
<td>$-7.6$</td>
<td>$-6.6$</td>
<td>$-6.8$</td>
<td>$-8.0$</td>
<td>$-10.0$</td>
<td>$-13.4$</td>
<td>$-19.2$</td>
</tr>
<tr>
<td>Na$_2$SO$_4$</td>
<td>$-30.1$</td>
<td>$-19.7$</td>
<td>$-15.6$</td>
<td>$-13.7$</td>
<td>$-13.2$</td>
<td>$-13.4$</td>
<td>$-14.4$</td>
<td>$-16.3$</td>
<td>$-20.0$</td>
</tr>
</tbody>
</table>

$^a$Cal kg mole$^{-2}$ (°K)$^{-1}$. $^b$Owing to derivation of these values by regression of $b_{\gamma,k}$ as a function of temperature, they differ slightly from those generated from $J_{\gamma,k}$ as a function of $\bar{I}$ in table 10.
For example, it can be shown that such inconsistencies occur among calculated values of \( \tilde{b}_k \) for HCl, KCl, NaCl, LiCl, MgCl_2, and CaCl_2 at temperatures \( \geq 100^\circ \text{C} \). Errors in computed values of \( b_k \) and \( b_{y,k} \) at high pressures and temperatures are magnified in values of \( b_{ii} \) generated from eq (174) to the extent of \( \omega_k \psi_k / 2 (2.303) \nu_{i,k} \nu_{l,k} \) and \( \nu_k \psi_k / 2 \nu_{i,k} \nu_{l,k} \), respectively. Similarly, uncertainties in computed values of \( b_k \) are magnified in calculated estimates of \( \varepsilon \) at high pressures and temperatures by \( \varepsilon^2 \). It follows that computed values of \( b_{kr} \), \( b_{kr} \), \( b_{ii} \), and \( \varepsilon \) at high pressures and temperatures from eqs (149), (174), (286), and (287) should be regarded as provisional approximations.

### Table 23

Computed values of the extended term parameter\(^a\) for calculating the relative apparent and partial molal volumes of aqueous electrolytes from eqs (222) and (227) at 1 bar for temperatures \(< 100^\circ \text{C} \) and pressures corresponding to liquid-vapor equilibrium at temperatures \( \geq 100^\circ \text{C} \)

<table>
<thead>
<tr>
<th>Solute</th>
<th>0</th>
<th>25</th>
<th>50</th>
<th>75</th>
<th>100</th>
<th>125</th>
<th>150</th>
<th>175</th>
<th>200</th>
</tr>
</thead>
<tbody>
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<td>HCl</td>
<td>2.2</td>
<td>0.7</td>
<td>0.2</td>
<td>0.1</td>
<td>0.2</td>
<td>0.4</td>
<td>0.9</td>
<td>1.6</td>
<td>2.8</td>
</tr>
<tr>
<td>LiCl</td>
<td>3.1</td>
<td>1.0</td>
<td>0.5</td>
<td>0.6</td>
<td>1.0</td>
<td>1.7</td>
<td>2.9</td>
<td>4.8</td>
<td>7.7</td>
</tr>
<tr>
<td>NaCl</td>
<td>3.4</td>
<td>1.8</td>
<td>1.2</td>
<td>1.0</td>
<td>1.1</td>
<td>1.5</td>
<td>2.2</td>
<td>3.3</td>
<td>5.0</td>
</tr>
<tr>
<td>KCl</td>
<td>4.0</td>
<td>2.4</td>
<td>1.9</td>
<td>1.8</td>
<td>2.1</td>
<td>2.6</td>
<td>3.4</td>
<td>4.8</td>
<td>6.9</td>
</tr>
<tr>
<td>CsCl</td>
<td>2.8</td>
<td>2.0</td>
<td>1.8</td>
<td>1.8</td>
<td>2.0</td>
<td>2.5</td>
<td>3.2</td>
<td>4.3</td>
<td>6.0</td>
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<td>0.3</td>
<td>0.4</td>
<td>0.5</td>
<td>0.9</td>
<td>1.4</td>
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</tr>
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<td>MgCl₂</td>
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<td>0.7</td>
<td>1.0</td>
<td>1.4</td>
<td>2.0</td>
<td>3.0</td>
</tr>
<tr>
<td>CaCl₂</td>
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<td>1.6</td>
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<td>3.6</td>
</tr>
<tr>
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<td>1.9</td>
<td>2.0</td>
<td>2.2</td>
<td>2.5</td>
<td>3.1</td>
<td>3.9</td>
</tr>
<tr>
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<td>1.3</td>
<td>1.1</td>
<td>1.3</td>
<td>1.7</td>
<td>2.4</td>
<td>3.7</td>
<td>5.6</td>
</tr>
<tr>
<td>KF</td>
<td>4.8</td>
<td>3.0</td>
<td>2.8</td>
<td>3.0</td>
<td>3.5</td>
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<td>5.3</td>
<td>7.0</td>
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<td>1.1</td>
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<td>0.8</td>
<td>1.1</td>
<td>1.8</td>
<td>2.9</td>
<td>4.7</td>
</tr>
<tr>
<td>KNO₃</td>
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<td>1.4</td>
<td>1.7</td>
<td>2.3</td>
<td>3.3</td>
<td>4.9</td>
</tr>
<tr>
<td>K₂SO₄</td>
<td>10.3</td>
<td>8.4</td>
<td>7.9</td>
<td>7.9</td>
<td>8.2</td>
<td>8.7</td>
<td>9.6</td>
<td>10.9</td>
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<td>6.5</td>
<td>6.2</td>
<td>6.2</td>
<td>6.5</td>
<td>6.9</td>
<td>7.6</td>
<td>8.8</td>
<td>10.5</td>
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<td>NaHCO₃</td>
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<td>1.3</td>
<td>1.3</td>
<td>1.5</td>
<td>1.9</td>
<td>2.6</td>
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</tr>
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<td>1.8</td>
<td>1.5</td>
<td>1.5</td>
<td>1.7</td>
<td>2.2</td>
<td>2.9</td>
<td>4.1</td>
<td>5.9</td>
</tr>
<tr>
<td>NH₄ClO₄</td>
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<td>0.2</td>
<td>0.3</td>
<td>0.3</td>
<td>0.5</td>
<td>0.6</td>
<td>0.9</td>
<td>1.3</td>
<td>1.8</td>
</tr>
</tbody>
</table>

\(^a\text{cm}^3 \text{kg mole}^{-2}\). \(^b\)Owing to the 19 bar difference in pressure, these values differ slightly from those for 25°C and 1 bar in Table 11.
**Table 24**

Computed values of the extended term parameter\(^a\) for calculating the relative apparent and partial molal compressibilities of aqueous electrolytes from eqs (226) and (231) at 1 bar for temperatures < 100°C and pressures corresponding to liquid-vapor equilibrium at temperatures \(\geq 100°C\)

<table>
<thead>
<tr>
<th>Solute</th>
<th>Temperature, °C</th>
</tr>
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<tbody>
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<td></td>
<td>0</td>
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<tr>
<td>HCl</td>
<td>6.8</td>
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<tr>
<td>LiCl</td>
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<tr>
<td>MgCl(_2)</td>
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<td>CaCl(_2)</td>
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<tr>
<td>Na(_2)SO(_4)</td>
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\(^a\)(cm\(^3\)/kg mole\(^{-2}\)/bar\(^{-1}\)) \(\times 10^4\). \(^b\)As a consequence of differences in the experimental data reported by Allam (ms) and Millero and others (1974), these values differ slightly from those in table 12.

---

**Table 25**

Summary of electrostatic parameters for calculating the dielectric constant and short-range interaction parameters for aqueous electrolytes at high temperatures and pressures — see text

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<th>(\tilde{\varepsilon}) (\times 10^3)</th>
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<tr>
<td>CaCl(_2)</td>
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</table>

\(\tilde{\varepsilon}\) (kg mole\(^{-1}\)).

---

**Table 26**

Predicted values of the extended term parameter (b\(_x\))\(^a\) for computing osmotic and mean ionic activity coefficients of aqueous electrolytes at 1 bar for temperatures < 100°C and pressures corresponding to liquid-vapor equilibrium at temperatures \(\geq 100°C\) — see text

<table>
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<tr>
<th>Solute</th>
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<td>CaCl(_2)</td>
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\(^a\)(kg mole\(^{-1}\)) \(\times 10^2\).
Regression of the experimental values of $b_{r,k}$ represented by the symbols for HBr, HCl, and CaCl$_2$ in figure 139 with eqs (293) through (295) after setting $\hat{a}_{s,k}$, $\hat{a}_{r,k}$, $\hat{a}_{s,k}$, and $\hat{a}_{s,k}$ to zero generated the curves shown for these electrolytes in figure 139 and the values of $\hat{a}_{s,k}$ for CaCl$_2$ and $\hat{c}_{l,k}$ and $\hat{c}_{r,k}$ for HCl, HBr, and CaCl$_2$ in table 19. Because only a small pressure variation (< 15 bars) is associated with increasing temperature from 0° to 200°C along the liquid-vapor equilibrium curves for the electrolytes, this procedure yields reliable values of $\hat{a}_{s,k}$, $\hat{c}_{l,k}$, and $\hat{c}_{r,k}$. The values of $\nu b_{s,k}/2$ employed in the calculations were generated from eq (295) using data given in tables 5 and 9. It can be seen in figure 139 that the regression calculations yield values of $\nu b_{r,k}/2$ that are in close agreement with independent calorimetric data at low temperatures.

Figure 139. Calculated extended term parameters for activity coefficients and apparent molal heat capacities of electrolytes as a function of temperature (T) at 1 bar for $T < 100^\circ$C and pressures corresponding to liquid-vapor equilibrium at $T \geq 100^\circ$C (see text). The symbols in the upper two diagrams correspond to the slopes of the solid linear curves for HBr, HCl, and CaCl$_2$ in figures 19 and 26 (eq 178) and twice the slopes of the linear curves for CaCl$_2$ in figure 46 (eq 195). The symbols in the lower two diagrams correspond to the negative equivalents of the slopes of the linear curves for HBr, HCl, and CaCl$_2$ in figures 65, 69, 70, and 79 (eq 238). The curves in all four of the diagrams were derived by regression of the $b_r$ values with eqs (295) through (295) (see text).
Correlation and estimation of compressibility parameters.—It can be seen in figure 140 that the regression values of $a_{\alpha,k}$ and $\tilde{a}_{\alpha,k}$ derived above correlate with each other in the same manner as $a_{\gamma,k}$ and $\tilde{a}_{\gamma,k}$. This observation, together with others adduced above strongly supports the general applicability of the equation of state derived by Helgeson and Kirkham (1976). Because the equation of the curve in figure 140 is identical to that given by Helgeson and Kirkham (1976) for $a_{\gamma,k}$ as a function of $a_{\gamma,k}$, it follows that

$$\tilde{a}_{\gamma,k} = 2 \times 10^{-6} - 6.16 \times 10^{-4} \tilde{a}_{\gamma,k}. \quad (296)$$

Eq (296) was used to generate the estimated compressibility parameters shown in table 21. Note that in the absence of experimental compressibility data, values of $\tilde{a}_{\gamma,k}$ and $\tilde{a}_{\gamma,k}$ can be estimated from $\tilde{\gamma}_{\gamma,k}^0$ and $\tilde{a}_{\gamma,k}$ by simultaneous evaluation of eqs (246), (247), (289), and (296).

Discussion.—In contrast to the close agreement of the symbols and curves in figures 132 and 133, it can be deduced from figure 141 that the values of $b_2$ for NaCl in table 22 are consistent with calorimetric data reported by Puchkov, Styzhkin, and Federov (1976) only at temperatures from 25° to 125°C. In addition, the intercepts of the curves in figure 141 differ significantly from the values of $\tilde{C}_{P,NaCl}^0$ given in table 16 and plotted as symbols on the ordinates of the diagrams in figure 141. The latter discrepancies increase markedly at higher temperatures, and none of the data reported by Puchkov, Styzhkin, and Federov (1976) for 150° to 350°C is compatible with values of $b_{2,NaCl}$ computed from equation (291). In contrast, it can be deduced from figure 142 that predicted values

![Fig. 140. Correlation of $a_i$ with $a_i$ (Helgeson and Kirkham, 1976) and $\tilde{a}_i$ with $\tilde{a}_i$ (tables 20 and 21) (see text).](image-url)
Fig. 141. $\rho^*$ (eq 238) for NaCl computed (assuming $I = I$) from experimental heat capacity data reported by Puchkov, Styazhkin, and Fedorov (1976) for 1 bar at temperatures < 100°C and pressures corresponding to liquid-vapor equilibrium for temperatures $\geq 100°C$ (circles). The slopes of the solid curves correspond to $-1/2$ the values of $b_2$ for NaCl at the corresponding temperatures in table 22, which were generated independently from eq (291) using parameters given in tables 19 through 21. The dashed curves merely connect data points, but the symbols on the ordinates of the diagrams correspond to values of $C^*_v$ taken from table 16 (see text).
Fig. 142. $\rho^*_{ij}$ (eq 238) for NaCl computed (assuming $I = I$) from experimental heat capacity data reported by Lipphard, Jost, and Schneider (1977) at 20°C and 1 and 2 kb (circles). The symbols on the ordinates and the slopes of the linear curves correspond to values of $C^*_{ij}$ and $\rho \beta_{ij}$/4 generated independently from eqs (273) and (291) (see text).

Fig. 143. Predicted values of the extended term parameter for activity coefficients of aqueous electrolytes as a function of temperature (T) at 1 bar for $T < 100^\circ$C and pressures corresponding to liquid-vapor equilibrium for H$_2$O at $T \geq 100^\circ$C. The curves were generated from eqs (293) through (295) with $d_i = d_j = 0$ (see text).
Fig. 144. Extended term parameter for the apparent molal enthalpy of aqueous electrolytes (eq 228) as a function of temperature \(T\) at 1 bar for \(T < 100^\circ\text{C}\) and pressures corresponding to liquid-vapor equilibrium for \(\text{H}_2\text{O}\) at \(T \geq 100^\circ\text{C}\). The symbols shown above and those in figure 145 correspond to the negative equivalent of the slopes of the linear curves for the corresponding electrolytes in figures 50, 51, 52, 55, 59, and 60, but the curves were generated independently from eq (292).
Fig. 145. Extended term parameter for the apparent molal enthalpy of aqueous electrolytes (eq 228) as a function of temperature ($T$) at 1 bar for $T < 100^\circ$C and pressures corresponding to liquid-vapor equilibrium for $H_2O$ at $T \geq 100^\circ$C (see caption of fig. 144).

Fig. 146. Experimental (symbols) and independently predicted (linear curves) values of $\rho^*$ (eqs 177 and 178) for $I = I$ in NaCl and KCl solutions as a function of stoichiometric ionic strength at various temperatures (indicated in $^\circ$C) and 1 bar (see text). The symbols represent experimental data reported by Scatchard and Prentiss (1933) and Harned and Owen (1968).
of $b_{2,NaCl}$ at 25°C and 1 and 2 kb (which were used to generate the slopes of the curves shown in the figure) are consistent with the relative distribution of the bulk of the data reported by Liphard, Jost, and Schneider (1977). However, the intercepts of the curves in figure 142, which correspond to values of $C_{P,NaCl}^t$ computed from eq (273), are not. The large experimental uncertainty (represented by the size of the circles in fig. 142) renders the significance of the latter observation somewhat dubious. Similarly, the fact that the experimental data shown in figure 141 are inconsistent with those responsible for the symbols for NaCl in figure 103 casts

Fig. 147. Experimental (symbols) and independently predicted (linear curves) values of $P^*_e$ and $P^*_f$ (eqs 177, 178, 194, and 195) for $I = 1$ in NaCl solutions as a function of stoichiometric ionic strength at various temperatures (indicated in °C) and 1 bar for temperatures < 100°C and pressures corresponding to liquid-vapor equilibrium for temperatures ≥ 100°C (see text). The symbols and nonlinear curves represent experimental data reported by Gibbard and others (1974) and Liu and Lindsay (1972).
doubt on the reliability of the higher temperature data reported by Puchkov, Styazhin, and Federov (1976).

Predicted values of \( b_\gamma \) generated from eq (293) for pressures and temperatures corresponding to liquid-vapor equilibrium are listed in table 26 and plotted as curves in figure 143. It can be seen in this figure that all the curves except that for MgCl\(_2\) exhibit extrema, and that in every case \( b_\gamma \) becomes negative at high temperatures. The latter observation is consistent with decreasing ion solvation with increasing temperature above \( \sim 100^\circ \text{C} \). Although at first glance it might seem unreasonable for a curve representing \( b_\gamma \) as a function of temperature to exhibit a double inflection like that for LiBr in figure 143, such a configuration is inevitable if \( \nu b_{\text{II}}/2 \) maximizes as a function of temperature at values \( \geq 0 \). It can be seen in figures 132 and 133 that this is indeed the case for LiBr and nearly so for LiCl and MgCl\(_2\), all three of which exhibit seemingly anomalous changes in \( b_\gamma \) with increasing temperature. This behavior is consistent with the curves for these electrolytes in figures 144 and 145, which represent predicted values of \( \nu b_{\text{II}}/4 \) as a function of temperature at pressures corresponding to liquid-vapor equilibrium. The fact that the values of \( b_\gamma \) and \( b_{\text{II}} \) shown in figures 143 through 145 for CsI, CsCl, LiBr, LiCl, KF, HBr, and Na\(_2\)SO\(_4\) were generated with \( d_2 = d_4 = 0 \) introduces negligible uncertainties in the calculated values at pressures corresponding to liquid-vapor equilibrium at temperatures \( \leq 300^\circ \text{C} \). The symbols in figures 144 and 145 correspond to values of \( \nu b_{\text{II}}/4 \) computed from data given in table 9 and the slopes of the linear curves shown in figures 50 through 60. It can be seen in figures 144 and 145 that the curves generated from the equations and parameters derived above for LiCl and NaCl at temperatures \( \geq 25^\circ \text{C} \)

---

Fig. 148. Experimental (symbols) and independently predicted (curves) values of \( \rho^*_\phi \) (eqs 194 and 195) for \( I = I \) in MgCl\(_2\) solutions as a function of stoichiometric ionic strength at various temperatures (indicated in \(^\circ \text{C}\)) and pressures corresponding to liquid-vapor equilibrium (see text). The symbols represent experimental data reported by Holmes, Baes, and Mesmer (1978).

Fig. 149. Experimental (symbols) and independently predicted (curves) values of \( \rho^*_\phi \) (eqs 194 and 195) for \( I = I \) in KCl solutions as a function of stoichiometric ionic strength at various temperatures (indicated in \(^\circ \text{C}\)) and pressures corresponding to liquid-vapor equilibrium (see text). The symbols represent experimental data reported by Holmes, Baes, and Mesmer (1978).
are in close agreement with the experimental data represented by the symbols. Similar agreement is apparent between the predicted (curves) and experimental (symbols) values of \( \rho^{*}_{\gamma, \text{NaCl}} \) at 0°C and \( I \leq 3.0 \) and \( \rho^{*}_{\gamma, \text{KCl}} \) at 30°C and 40°C in figure 146. Other predictions of \( \rho^{*}_{\gamma} \) and \( \rho^{*}_{\phi} \), which were generated from calculated values of \( b_{\gamma, \text{NaCl}}, b_{\gamma, \text{NaCl}}^{2}, \) and \( b_{\gamma, \text{KCl}} \) in table 26, are represented by the curves in figures 147 through 149. It can be deduced from these figures that the experimental data represented by the symbols are in close agreement with the independently predicted values of \( b_{\gamma} \) at all temperatures \( \leq 250°C \). The effect of ion association on \( \rho^{*}_{\gamma} \) is manifested in figure 147 by increasing departures from linearity of the distribution of data points toward lower values of \( \rho^{*}_{\gamma} \) with increasing stoichiometric ionic strength. Note, however, that at temperatures \( > 250°C \), the trend of Liu and Lindsay's (1972) data departs systematically in the opposite direction from the linear curves. The cause of these departures at high temperatures is not clear, but they cannot be attributed to specific interaction among the species in solution. Although the discrepancies could result from increasing solvation of the NaCl molecule with increasing temperature above \( \sim 250°C \), it seems highly unlikely that this is the case. A more reasonable explanation is probably lurking in the

### Table 27

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<th>( b_{\gamma} ) (mol kg(^{-1}))</th>
<th>Pressure, Kb</th>
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### Table 28

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questionable validity of a number of the assumptions made by Liu and Lindsay in calculating osmotic coefficients from their high-temperature vapor pressure data.

Calculated values of $b_\gamma$ for NaCl and MgCl$_2$ at temperatures and pressures to 600°C and 5 kb are given in tables 27 and 28 and plotted together with corresponding predictions of $b_H$, $b_J$, and $b_\gamma$ for these electrolytes in figures 150 and 151. Note in figure 150 that $b_{\gamma,\text{NaCl}}$ as a function of temperature at constant pressure exhibits extrema at all pressures $\leq 5$ kb, which is not true of $b_{\gamma,\text{MgCl}_2}$ in figure 151. Nevertheless, the isothermal pressure dependence of $b_\gamma$ is nearly identical for the two electrolytes, which is a consequence of the similarity of $b_{\gamma,\text{NaCl}}$ and $b_{\gamma,\text{MgCl}_2}$ as isobaric functions of temperature. As expected (see above) the configuration of the latter two functions in figures 150 and 151 approximates an inverted image of $\nabla^0_{\text{NaCl}}$ and $\nabla^0_{\text{MgCl}_2}$ as a function of temperature at constant pressure (Helgeson and Kirkham, 1976). Similarly, the isobaric dependence of $b_J$ and $b_H$ in figures 150 and 151 is similar to that of $C^0_p$ and $\Delta H^0$ for NaCl and MgCl$_2$. It can be seen in figures 150 and 151 that $b_\gamma$ changes only slightly with increasing temperature from $\sim 100^\circ$ to $500^\circ$ C at pressures $\geq 3$ kb. However, at high temperatures and low pressures, $b_\gamma$ is a highly sensitive function of both variables.

Fig. 150. Predicted (eqs 288 and 291 through 295) values of $b_\gamma$, $b_J$, $b_H$, and $b_\gamma$ for NaCl as a function of temperature at high pressures (indicated in kb). Saturation (H$_2$O) refers to the liquid-vapor equilibrium curve for H$_2$O. The dashed curves are more uncertain than their solid counterparts.
Activity and Osmotic Coefficients

Calculated osmotic and mean ionic activity coefficients generated from the equations and parameters summarized above are represented by the curves plotted against or specified for a given value of \( I \) in figures 152 through 159 at pressures corresponding to liquid-vapor equilibrium. The symbols in these figures represent experimental osmotic and stoichiometric mean activity coefficients, which are plotted against, or refer to a given value of \( I \).

It can be seen in figures 152 through 159 that the curves generated in the present study are in close agreement with the bulk of the experimental data represented by the symbols. The departures at high temperatures in figure 157 of the predicted curves as a function of \( I \) from the experimental osmotic coefficients for MgCl\(_2\) solutions as a function of \( I \) are consistent with increasing ion association with increasing temperature. In contrast, the discrepancies between the curve and symbols for NaCl at 300°C and high ionic strengths in figures 154 and 157 cannot be attributed to ion association (see above). It should perhaps be emphasized in this regard that eq (178), which was used to generate the curves shown in figures 152 through 159, is not applicable to the dissociated fraction of

---

Fig. 151. Predicted (eqs 288 and 291 through 295) values of \( b_v \), \( b_y \), \( b_n \), and \( b_0 \) for MgCl\(_2\) as a function of temperature at high pressures (indicated in kb). Saturation (\( H_2O \)) refers to the liquid-vapor equilibrium curve for \( H_2O \). The dashed curves are more uncertain than their solid counterparts.
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partially associated 2:1 or 1:2 electrolytes. Because \( m_i \neq \nu_{i,k}m_k \), \( m_i \neq \nu_{i,k}m_k \), or \( m_i \neq m_i \neq I/\psi_k \) in these cases, eq (168) must be used together with dissociation constants to calculate \( \bar{\gamma}_+ \) as a function of \( I \). In contrast, eq (173) can be used to compute \( \bar{\gamma}_\pm \) for the dissociated fraction of 1:1 or 2:2 electrolytes in which significant ion association is limited to a single ion pair. Under these circumstances, dissociation constants are needed only to calculate \( I \) as a function of \( I \).

It can be seen in figures 152 through 159 that increasing temperature decreases \( \bar{\gamma}_\pm \) to an increasing degree with increasing ionic strength. Note that the "synclinal" configurations of the curves in figures 152 through 154, 157, and 158 tend to dampen (and disappear if \( b_\gamma \) becomes negative)

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**Fig. 152.** Experimental stochiometric mean activity coefficients (symbols) and independently predicted mean ionic activity coefficients (curves) of electrolytes as a function of "true" (curves) and stochiometric (symbols) ionic strength at various temperatures (indicated in °C) and 1 bar for temperatures < 100°C and pressures corresponding to liquid-vapor equilibrium for temperatures \( \geq 100°C \) (see text).
with increasing temperature. At high temperatures and low pressures, $\tilde{\gamma}_\pm$ for all electrolytes becomes a monotonic function of $I$.

Calculated values of $b_{y,NaCl}$ and $b_{y,MgCl_2}$ at pressures and temperatures to 5 kb and 500°C are given in tables 27 and 28. The parameters shown in table 27 were used to generate the curves shown in figures 160 through 162. Note in figure 160 that the monotonic character of the high-temperature curves in figure 154 disappears with increasing pressure above $\sim 2$ kb. At higher pressures, all the curves are “synclinal.” The pressure dependence of $\tilde{\gamma}_{\pm,NaCl}$ at constant $I$ and temperature is depicted in figure 161, where it is apparent that $\tilde{\gamma}_{\pm,NaCl}$ at high ionic strengths increases sharply with increasing pressure at all temperatures. However, at low temperatures in dilute solutions, $\tilde{\gamma}_{\pm,NaCl}$ is hardly affected by increasing pressure.

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**Fig. 153.** Experimental stoichiometric mean activity coefficients (symbols) and independently predicted mean ionic activity coefficients (curves) of electrolytes as a function of “true” (curves) and stoichiometric (symbols) ionic strength at various temperatures (indicated in °C) and 1 bar for temperatures $< 100°C$ and pressures corresponding to liquid-vapor equilibrium for temperatures $\geq 100°C$ (see text).
It can be seen in figure 162 that the calculations suggest that $\tilde{\gamma}_{\pm, \text{NaCl}}$ minimizes as a function of temperature at high ionic strengths and constant pressure above $\sim$ 2 kb. However, the fact that the minima occur at the boundary of the dielectric constant fit region (Helgeson and Kirkham, 1974a) renders the significance of this observation somewhat moot. Note in figure 162 that $\tilde{\gamma}_{\pm, \text{NaCl}}$ is $< 0.1$ above $\sim 400^\circ\text{C}$ at 1 kb and high ionic strengths. The curves shown in figures 160 through 162 represent the dissociated fraction of the solute, which decreases with increasing temperature, but increases with increasing pressure at constant temperature. At high temperatures and low pressures the "true" ionic strength of NaCl solutions is highly sensitive to changes in either variable. It is in this "supercritical" region that departures from ideality become large and significant in geochemical processes. It is also in this region where the exothermic heats of solution of buried evaporite deposits may exceed a million cal mole$^{-1}$.

Geochemical approximations.—Owing to the inaccessibility and complex nature of geochemical processes at high pressures and temperatures,
prediction of the thermodynamic behavior of aqueous species involved in the processes necessarily requires simplifying approximations to provide for compositional variation in the aqueous phase. Approximations of this kind can be made without introducing undue uncertainties in calculated activity coefficients by taking account of fluid inclusion data and analyses of the compositions of interstitial fluids in geothermal reservoirs. These data suggest that in many cases the thermodynamic properties of natural hydrothermal solutions are similar to those of NaCl solutions. Consequently, despite the relatively large uncertainties involved (see above), calculated values of \( b_k \) and \( b_d \) for NaCl (\( b_{\text{NaCl}_{\text{aq}}} \), respectively) at high pressures and temperatures probably afford a reasonable basis for generating estimates of activity coefficients of other aqueous species in geochemical processes. Calculated values of these parameters are shown in tables 29 and 30 and plotted as curves in figure 168. It can be seen in this figure that increasing temperature is accompanied at all pressures by a dramatic decrease in \( b_{\text{NaCl}} \) which becomes negative at \(~170^\circ\text{C} < T < 240^\circ\text{C}\) at \( P \leq 5 \text{ kb} \). In contrast, \( b_{\text{NaCl}} \) increases with increasing pressure at constant temperature.

The configuration of the curves shown in figure 168A is consistent with decreasing ion solvation with increasing temperature and/or decreasing pressure. The fact that \( b_{\text{NaCl}} \) is negative at high temperatures is manifested in figure 164A by the negative slopes of the curves for temperatures \( \geq 200^\circ\text{C} \). The curves in figure 164A indicate that addition of NaCl to

![Fig. 155](image1.png)

![Fig. 156](image2.png)

Fig. 155. Experimental stoichiometric mean activity coefficients (symbols) and independently predicted mean ionic activity coefficients (curves) of HCl as a function of temperature at constant "true" (curves) and corresponding stoichiometric (symbols) ionic strengths at 1 bar for temperatures < 100°C and pressures corresponding to liquid-vapor equilibrium at temperatures \( \geq 100^\circ\text{C} \) (see text).

Fig. 156. Experimental stoichiometric mean activity coefficients (symbols) and independently predicted mean ionic activity coefficients (curves) of NaCl as a function of temperature at constant "true" (curves) and corresponding stoichiometric (symbols) ionic strengths at 1 bar for temperatures < 100°C and pressures corresponding to liquid-vapor equilibrium at temperatures \( \geq 100^\circ\text{C} \) (see text).
H₂O at high temperatures increases the dielectric constant of the solution, which is the opposite effect to that observed at low temperatures. The configurations of the curves in figures 168A and 164A can be attributed to disruption of the local solvent structure by NaCl only at temperatures < 170° to 240°C, depending on the pressure. At higher temperatures, addition of NaCl apparently stabilizes the local solvent structure and enhances transitory alignment of the H₂O dipoles. As a consequence, the dielectric constant of NaCl solutions varies only slightly as a function of pressure and concentration in the vicinity of 200°C. Note in figure 165 that increasing temperature from 25° to 500°C is accompanied by a decrease in the dielectric constant of a 3m NaCl solution of only ~ 30 units, compared to ~ 70 units for H₂O. Note also in figure 165 that addition of

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**Figure 157.** Experimental (symbols) and independently predicted (solid curves) osmotic coefficients of aqueous electrolytes as a function of “true” (curves) and stoichiometric (symbols) ionic strength at various temperatures (indicated in °C) and 1 bar for temperatures < 100°C and pressures corresponding to liquid-vapor equilibrium for temperatures ≥ 100°C (see text). The solid curves were generated from equations given in the text (assuming I = I), but the dashed curves merely connect experimental data points.
NaCl to H₂O reduces substantially the pressure dependence of the dielectric constant of the solution.

It can be seen in figure 163B that the short-range interaction parameter for NaCl (\(b_{Na^+Cl^-}\)) increases dramatically and becomes positive with increasing temperature at all pressures \(\leq 5\) kb. Note also that only a slight increase in \(b_{Na^+Cl^-}\) accompanies increasing pressure at temperatures above 60°C. At lower temperatures, \(b_{Na^+Cl^-}\) decreases slightly with increasing pressure. The positive values of \(b_{Na^+Cl^-}\) at high temperatures in figure 163B can be attributed to temporary capture of intervening solvent dipoles during collision of ionic species (as in case a of figure 16), which apparently dominates short-range interaction at temperatures \(\geq 90^\circ\) to 150°C, depending on the pressure. At lower temperatures, where the ions are highly solvated (and therefore "shielded"), short-range interaction is apparently too weak to cause capture of intervening solvent dipoles. Nevertheless, the collisions result in negative contributions to the relative partial molal Gibbs free energy of NaCl at low temperatures. In contrast, contributions

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Fig. 158. Experimental (symbols) and independently predicted (solid curves) osmotic coefficients of aqueous electrolytes as a function of "true" (curves) and stoichiometric (symbols) ionic strength at various temperatures (indicated in °C) and 1 bar for temperatures <100°C and pressures corresponding to liquid-vapor equilibrium for temperatures \(\geq 100^\circ\)C (see text). The curves were generated from equations given in the text assuming \(I = 1\).
by ion solvation to the relative partial molal Gibbs free energy of NaCl are negative only at high temperatures. The effects of short-range interaction and ion solvation on $\gamma_{\pm,NaCl}$ are thus reversed at high and low temperatures. Nevertheless, in both cases the concentration dependence of ion solvation dominates the behavior of $b_\gamma$ as a function of temperature.

As noted above, calculation of values of $b_\mu$ at high temperatures and pressures from parameters generated by regression of low-temperature data is accompanied by large uncertainties. Accordingly, it seems preferable for most purposes to estimate "average" relative values of these parameters in electrolyte solutions involved in geochemical processes by taking account of the correlation of $b_\mu$ with cation charge in figure 166. The open symbols shown in this figure represent the values of $b_\mu$ for Li$^+$Cl$^-$, Na$^+$Cl$^-$, K$^+$Cl$^-$, Rb$^+$Cl$^-$, Cs$^+$Cl$^-$, NH$_4$+Cl$^-$, Ag$^+$Cl$^-$, Au$^+$Cl$^-$, Cu$^+$Cl$^-$, Mg$^{++}$Cl$^-$, Ca$^{++}$Cl$^-$, Sr$^{++}$Cl$^-$, Ba$^{++}$Cl$^-$, Pb$^{++}$Cl$^-$, Zn$^{++}$Cl$^-$, Cu$^{++}$Cl$^-$, Cd$^{++}$Cl$^-$, Hg$^{++}$Cl$^-$, Fe$^{++}$Cl$^-$, Mn$^{++}$Cl$^-$, and Al$^{+++}$Cl$^-$ at 25°C and 1 bar in table 7. The hatched symbols in figure 166 represent highly uncertain values of $b_\mu$ for Li$^+$Cl$^-$, Na$^+$Cl$^-$, K$^+$Cl$^-$, Mg$^{++}$Cl$^-$, and Ca$^{++}$Cl$^-$ at 200°C and 15 bars computed from eqs (149), (174), and (287) using values of $d_s$, $a_0$, and $b_\gamma$ taken from tables 21, 25, and 26. It can be deduced from figure 166 that the isothermal distribution of

---

Fig. 159. Experimental (symbols) and independently predicted (curves) osmotic coefficients of aqueous electrolytes as a function of temperature at constant "true" (curves) and stoichiometric (symbols) ionic strength at 1 bar for temperature < 100°C and pressures corresponding to liquid-vapor equilibrium for temperatures $\geq$ 100°C (see text). The curves were generated assuming I = 1.
the symbols in the figure is consistent with linear correlation of the mean $b_{ii} \pm \sim 0.05$ for cations of the same charge with the charge of the cation. Note also that the slopes of the two curves in figure 166 are the same. The mean of the $b_{ii}$ values for monovalent cations at 25°C in figure 166 is $-0.099$, which is nearly identical to the value of $b_{Na^+Cl^-}$ in table 7 ($-0.096$).

Because the predominant ligand in most natural hydrothermal solutions is Cl$^-$, the observations summarized above suggest that values of $b_{ii}$ for aqueous species in these solutions can be estimated from

$$b_{ii} = b_{Na^+Cl^-} - 0.19(Z_i - 1)$$

(297)

Similarly, because the values of $b_{H^+Cl^-}$ and $b_{H^+NO_3^-}$ at 25°C in table 7 are close to zero, which is also true of calculated values of $b_{H^+Cl^-}$ at higher temperatures, $b_{ii}$ for short-range interaction of H$^+$ (or more explicitly, H$_3$O$^+$) with anions in hydrothermal solutions can be regarded as zero. Note also in table 7 that $b_{Li^+NO_3^-}$, $b_{Na^+OH^-}$, and $b_{K^+OH^-}$ fall within or just outside the range of values for monovalent cations at 25°C in figure 7. Although the same observation cannot be made for all the fluorides, bromides, and iodides in table 7, these ligands are commonly present in

Fig. 160. Predicted values of the mean ionic activity coefficient ($\bar{\gamma}_i$) of NaCl at high pressures (labeled in kb) and temperatures (labeled in °C) as a function of “true” ionic strength (see text). The dashed curves are more uncertain than their solid counterparts.
Table 29  
Predicted values of the solvation parameter for NaCl ($b_{\text{NaCl}}^a$) at high pressures and temperatures — see text. Sat refers to liquid-vapor equilibrium, and the parentheses indicate high uncertainty.

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$^a$ (kg mole$^{-1}$) x 10$^8$.  $^b$ Temperature, °C.

Table 30  
Predicted values of the short-range interaction parameter for NaCl ($b_{\text{Na}^-\text{Cl}^+}^a$) at high pressures and temperatures — see text. Sat refers to liquid-vapor equilibrium, and the parentheses indicate high uncertainty.

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$^a$ (kg mole$^{-1}$) x 10$^8$.  $^b$ Temperature, °C.

Fig. 161. Predicted values of the mean ionic activity coefficient ($\it{\gamma}_i$) of NaCl at high temperatures (labeled in °C) and "true" ionic strengths of 0.01, 0.1, 0.5, 2, and 4 as a function of pressure (see text). The dashed curves are more uncertain than their solid counterparts.
relatively minor concentrations in natural electrolyte solutions, which is also true of OH\(^-\), NO\(_3\)\(^-\), HSO\(_4\)\(^-\), SO\(_4\)\(^{2-}\), HCO\(_3\)\(^-\), and CO\(_3\)\(^{2-}\). It seems likely that \(b_{\Pi}\) representing short-range interaction of a monovalent cation with a divalent anion is comparable in magnitude to that representing corresponding interaction of a divalent cation with a monovalent anion. It thus appears that eq (297) can be used in a first approximation to estimate “average” values of \(b_{\Pi}\) for short-range interaction of all the cations and anions in electrolyte solutions involved in geochemical processes. In addition, if the curves shown in figure 163A are representative of those for most such solutions (which seems likely), \(b_k\) in eqs (165) and (166) for natural hydrothermal solutions can be replaced in a first approximation by \(b_{\text{NaCl}}\). Taking account of eqs (165), (166), and (297), it follows from these observations (and the high probability that the bulk of the positively charged species in natural electrolyte solutions are monovalent) that we can write for the \(j\)th positively or negatively charged aqueous species,

\[
\log \tilde{\gamma}_j \approx -\frac{A_j Z_j^2 \gamma^{1/2}}{\Lambda} + \Gamma_\gamma + (\omega_j)^{ab} b_{\text{NaCl}} + b_{\text{Na}^+\text{Cl}^-} - 0.19 (|Z_j| - 1) \Gamma, \tag{298}
\]

which should afford close approximation of individual ion activity coefficients in concentrated as well as dilute electrolyte solutions involved in geochemical processes at high temperatures and pressures. Values of \(\Gamma_\gamma\) can be approximated from \(m^* = 1\). Owing to the dearth of definitive data for neutral species, there is little alternative to assuming for the general case,

\[
\tilde{\gamma}_n = 1 \tag{299}
\]

in these solutions at high pressures and temperatures. Figure 119 notwithstanding, ample evidence suggests that the latter approximation introduces negligible error in geochemical calculations (Orville, 1963; Helgeson, 1969; Marshall, 1980b).

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**Fig. 162.** Predicted values of the mean ionic activity coefficient (\(\tilde{\gamma}_j\)) of NaCl at high pressures (labeled in kbar) and “true” ionic strengths of 0.01, 0.1, 0.5, 2, and 4 as a function of temperature (see text). The dashed curves are more uncertain than their solid counterparts.
Fig. 163. Predicted (tables 29 and 30) solvation (b) and short-range interaction parameters (b_{ij}) for NaCl as a function of temperature at high pressures (labeled in kb) (see text). SAT refers to liquid-vapor equilibrium.

Fig. 164. Predicted (assuming I = I) reciprocals of the dielectric constant (eqs 284 through 286) of NaCl solutions (\(\varepsilon^{-1}\)) at temperatures and pressures corresponding to liquid-vapor equilibrium as a function of stoichiometric ionic strength and the reciprocal of the dielectric constant of the solvent (\(\varepsilon^{\infty-1}\)).
Eq (298) permits calculation of mineral solubilities in hydrothermal solutions at high pressures and temperatures. Calculations of this kind can be carried out by first specifying trial values of $I$ and $m^*$ and then evaluating simultaneously (1) statements of the law of mass action for each homogeneous and heterogeneous reaction in the system, (2) conservation of mass and charge equations for the species in the aqueous phase, and (3) statements of eqs (298) and (299) for all aqueous species. New values of $I$ and $m^*$ can then be computed, and the procedure repeated until the successive approximations converge. The requisite equilibrium constants can be calculated using equations, parameters, and thermodynamic data summarized above, together with those of minerals, gases, and aqueous complexes (Helgeson, 1967, 1969; Helgeson and Kirkham, 1974a, 1976; Helgeson and others, 1978; Sillen and Martell, 1964, 1971; Smith and Martell, 1976; Hogfeldt, in press; Seward, in press). The values of $A_\gamma$, $B_\gamma$, $\hat{a}$, $\omega_{\gamma}$, $b_{Na^+Cl^-}$ and $b_{Na^+Cl^-}$ required for the calculations can be obtained from equations and tables given above and/or generated from equations and correlation algorithms (like those in figs. 1 and 2) given by Helgeson and Kirkham (1974b; 1976). Where values of $r_{e,j}$ cannot be computed with

![Fig. 165. Predicted (assuming $I = 1$) values of the dielectric constant of NaCl solutions as a function of temperature at constant pressure and "true" ionic strength corresponding to the labels on the curves. SAT refers to liquid-vapor equilibrium. The curves for H$_2$O represent values of $\varepsilon^*$ given by Helgeson and Kirkham (1974a), but those for NaCl solutions were generated from eq (286).](image-url)
confidence, they can be estimated by assuming equalities of $r_{e,j}$ for similar aqueous species.

CONCLUDING REMARKS

Although the approach taken above is based on sound principles, it is also somewhat unconventional; but then, so too is the geologic problem. Much remains to be done to refine and test further the many theoretical concepts, equations, calculations, and extrapolations summarized in the preceding pages. Although they are consistent with a myriad of experimental observations, the predictive equations and fit parameters were generated primarily from consideration of the thermodynamic/electrostatic behavior of electrolytes at low temperatures. Nevertheless, the fact that values of $r_{e,j}$, $\omega_j$, $\theta_j$, and $\hat{a}$ derived from standard state considerations are consistent with both standard and nonstandard-state properties of aqueous electrolytes insures internal consistency in the equations and lends considerable support to the generality of the approach. As more and better high pressure/temperature data become available, the theoretical concepts, as well as the equations and parameters will no doubt require revision and improvement; particularly the algorithms represented by eqs (124) and (142). In the meantime, the calculations summarized above should afford reasonable approximation of geologic reality.

Fig. 166. Short-range interaction parameters ($b_{11}$) for cation-chloride interaction but the hatched symbols correspond to computed values at 200°C and 15 bars (see as a function of cation charge. The open circles represent values taken from table 7, text).
in applications of high temperature/pressure solution chemistry to the study of geochemical processes.

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Note added in proof

Line 14 of footnote 1 in Helgeson (in press) should be amended to read "and the first two terms on the right side of the second identity in eq (128) should be multiplied by $Z_i$." In addition, NaHCO$_3$ at the bottom of figure 60 in Helgeson and Kirkham (1976) should read NaNO$_3$ and table 55 in the caption of the figure should read table 51.
APPENDIX A

Summary of Identities Among Partial Molar Properties of Components and Aqueous Species

It follows from equations derived in the text that

\[ \overline{G}_k = \overline{v}_k = \left( \frac{\partial G}{\partial m_k} \right)_{P,T,\hat{m}_k} = \sum_j v_{\downarrow, j} \overline{G}_j \]  

(A-1)

\[ \overline{v}_k = \left( \frac{\partial v}{\partial m_k} \right)_{P,T,\hat{m}_k} = \left( \frac{\partial G}{\partial P} \right)_{T,\hat{m}(k)} = \sum_j v_{\downarrow, j} \overline{v}_j \]  

(A-2)

\[ \overline{s}_k = \left( \frac{\partial s}{\partial m_k} \right)_{P,T,\hat{m}_k} = -\left( \frac{\partial G}{\partial T} \right)_{P,\hat{m}(k)} = \sum_j v_{\downarrow, j} \overline{s}_j \]  

(A-3)

\[ \overline{h}_k = \left( \frac{\partial h}{\partial m_k} \right)_{P,T,\hat{m}_k} = \overline{g}_k + T \overline{s}_k = \sum_j v_{\downarrow, j} \overline{h}_j \]  

(A-4)

\[ \overline{c}_{p,k} = \left( \frac{\partial c_p}{\partial m_k} \right)_{P,T,\hat{m}_k} = \left( \frac{\partial h}{\partial T} \right)_{P,\hat{m}(k)} = \sum_j v_{\downarrow, j} \overline{c}_{p,j} \]  

(A-5)

\[ \overline{f}_{\chi,k} = \left( \frac{\partial f_{\chi}}{\partial m_k} \right)_{P,T,\hat{m}_k} = \left( \frac{\partial v}{\partial T} \right)_{P,\hat{m}(k)} = \sum_j v_{\downarrow, j} \overline{f}_{\chi,j} \]  

(A-6)

and

\[ \overline{e}_k = \left( \frac{\partial e}{\partial m_k} \right)_{P,T,\hat{m}_k} = -\left( \frac{\partial v}{\partial P} \right)_{T,\hat{m}(k)} = \sum_j v_{\downarrow, j} \overline{e}_j \]  

(A-7)

where \( \hat{m}_k \) stands for constant molalities of all solute components other than the \( k \)th, \( m_k \) represents the molality of the \( k \)th component, \( v_{\downarrow, j} \) denotes the number of moles of the \( j \)th ion in one mole of the \( k \)th component, \( \hat{m}(k) \) refers to constant molalities of all the components of the solute, \( \overline{G}_k, \overline{G}_j, \overline{h}_k, \overline{h}_j, \overline{c}_{p,k}, \overline{c}_{p,j}, \overline{f}_{\chi,k}, \overline{f}_{\chi,j}, \overline{e}_k, \overline{e}_j, \overline{v}_k, \overline{v}_j, \) and \( \overline{e}_j \) refer to the partial molar Gibbs free energies, enthalpies, heat capacities,
Appendix A (continued)

expansibilities, and compressibilities of the \(k\)th solute component and the
\(j\)th aqueous ion, respectively, \(G, H, C_p, E_x, \) and \(\kappa\) stand for the exten-
sive analogs of these properties for the solution, and

\[
\bar{G}_j = \left( \frac{\partial G}{\partial m_j} \right)_{P,T,\hat{m}_j},
\]

\[
\bar{V}_j = \left( \frac{\partial V}{\partial m_j} \right)_{P,T,\hat{m}_j} = \left( \frac{\partial G}{\partial P} \right)_{T,\hat{m}_j} T, \hat{m}_j(k),
\]

\[
\bar{S}_j = \left( \frac{\partial S}{\partial m_j} \right)_{P,T,\hat{m}_j} = - \left( \frac{\partial G}{\partial T} \right)_{P,\hat{m}_j} T, \hat{m}_j(k),
\]

\[
\bar{H}_j = \left( \frac{\partial H}{\partial m_j} \right)_{P,T,\hat{m}_j} = \bar{G}_j + T \bar{S}_j,
\]

\[
\bar{C}_p,j = \left( \frac{\partial C_p}{\partial m_j} \right)_{P,T,\hat{m}_j} = \left( \frac{\partial H}{\partial T} \right)_{P,\hat{m}_j} T \left( \frac{\partial \bar{S}_j}{\partial T} \right)_{P,\hat{m}_j} T, \hat{m}_j(k),
\]

\[
\bar{E}_x,j = \left( \frac{\partial E_x}{\partial m_j} \right)_{P,T,\hat{m}_j} = \left( \frac{\partial \bar{V}_j}{\partial T} \right)_{P,\hat{m}_j} T, \hat{m}_j(k),
\]

and

\[
\bar{\kappa}_j = \left( \frac{\partial \kappa}{\partial m_j} \right)_{P,T,\hat{m}_j} = - \left( \frac{\partial \bar{V}_j}{\partial P} \right)_{T,\hat{m}_j} T, \hat{m}_j(k).
\]

where \(m_j\) stands for the molality of the \(j\)th ion and \(\hat{m}_j\) refers to constant
molalities of all other species in solution. The partial molal properties
of the \(j\)th complex in solution are given by

\[
\bar{G}_q = \left( \frac{\partial G}{\partial m_j} \right)_{P,T,\hat{m}_j} = \sum_j \bar{V}_j, \bar{G}_q,
\]

\[
\bar{V}_q = \left( \frac{\partial V}{\partial m_j} \right)_{P,T,\hat{m}_j} = \sum_j \bar{V}_j, \bar{V}_q.
\]
\[ \bar{\xi}_q = \left( \frac{\partial S}{\partial m_q} \right)_{p, T, \hat{\beta}_q} = \sum_j v_{j, q} \bar{\xi}_j \quad (A-17) \]

\[ \bar{\eta}_q = \left( \frac{\partial H}{\partial m_q} \right)_{p, T, \hat{\beta}_q} = \bar{\xi}_q + \gamma \bar{\xi}_q = \sum_j v_{j, q} \bar{\eta}_j, \quad (A-18) \]

\[ \bar{\zeta}_{p, q} = \left( \frac{\partial C_p}{\partial m_q} \right)_{p, T, \hat{\beta}_q} = \sum_j v_{j, q} \bar{\zeta}_{p, j}, \quad (A-19) \]

\[ \bar{\xi}_{x, q} = \left( \frac{\partial \xi}{\partial m_q} \right)_{p, T, \hat{\beta}_q} = \sum_j v_{j, q} \bar{\xi}_{x, j}, \quad (A-20) \]

and

\[ \bar{\xi}_q = \left( \frac{\partial K}{\partial m_q} \right)_{p, T, \hat{\beta}_q} = \sum_j v_{j, q} \bar{\xi}_j \quad (A-21) \]

where \( m_q \) stands for the molality of the \( q \)th complex, \( \hat{\beta}_q \) designates constant molalities of all aqueous species other than the \( q \)th, and \( v_{j, q} \) corresponds to the number of moles of the \( j \)th ion in one mole of the \( q \)th complex.

APPENDIX B

Summary of Relations Among Relative Partial Molal Properties of Components and Aqueous Species

Equations derived in the text lead to the following identities:

\[ \bar{G}_k^* - \bar{G}_k = v_k RT \ln (\gamma_{z, k} m_{k}) + RT \sum_j v_{j, k} \ln v_{j, k} \]

\[ = v_k RT \ln (\bar{G}_k^* - \bar{G}_k) + RT \sum_j v_{j, k} \ln (\frac{v_{j, k} m_{j}}{m_{z, j}}) \]

\[ = \sum_j v_{j, k} (\bar{G}_j - \bar{G}_k) \bar{G}_j \bar{G}_j^* \] \( \frac{m_{j}}{m_{z, j}} \), \( (B-1) \)

\[ \bar{V}_k^* - \bar{V}_k = \left( \frac{\partial \bar{G}_k^*}{\partial p} \right)_{T, \hat{\beta}_k} - \left( \frac{\partial \bar{G}_k}{\partial p} \right)_{T} = v_k RT \left( \frac{\partial \ln \gamma_{z, k}}{\partial p} \right)_{T, \hat{\beta}_k} \]

\[ = RT \left( \frac{\partial \ln \gamma_{z, k}}{\partial p} \right)_{T, \hat{\beta}_k} + \sum_j v_{j, k} \left( \frac{\partial \ln m_{j}}{\partial p} \right)_{T, \hat{\beta}_k} \]

\[ = \sum_j v_{j, k} (\bar{V}_j - \bar{V}_k) \bar{V}_j \bar{V}_j^* \] \( \frac{m_{j}}{m_{z, j}} \), \( (B-2) \)
\[ \bar{\bar{L}}_k = \bar{H}_k - \bar{H}^*_k = - \tau^2 \left( \frac{\partial (\bar{H}_k / T)}{\partial T} \right)_{\partial p} + \left( \frac{\partial (\bar{H}^*_k / T)}{\partial T} \right)_{\partial p} \]

\[ = - \gamma_k \left( \frac{\partial}{\partial T} \left( \frac{\partial \ln Y_{\tau,k}}{\partial T} \right) \right)_{\partial p, \pi(k)} \]

\[ = - \tau^2 \left( \nu_k \left( \frac{\partial}{\partial T} \left( \frac{\partial \ln \bar{Y}_{\tau,k}}{\partial T} \right) \right)_{\partial p, \pi(k)} + \sum_j \nu_{j,k} \frac{\partial}{\partial T} \left( \frac{\partial \ln m_j}{\partial T} \right)_{\partial p, \pi(k)} \right) \]

\[ = \sum_{j} \nu_{j,k} \left( \bar{Y}_{\tau,k} - \bar{H}^*_k \right) \]  

\[ (8-3) \]

\[ \bar{\bar{S}}_k - \bar{S}^*_k = \left( \frac{\partial \bar{H}_k}{\partial T} \right)_{\partial p, \pi(k)} - \left( \frac{\partial \bar{H}^*_k}{\partial T} \right)_{\partial p} - \frac{\left( \bar{H}_k - \bar{H}^*_k \right) - \left( \bar{H}_k - \bar{H}^*_k \right)}{\tau} \]

\[ = \sum_{j} \nu_{j,k} \left( \bar{S}_{\tau,k} - \bar{S}^*_k \right) \]  

\[ (8-4) \]

\[ \bar{\bar{J}}_k = \bar{C}_{\pi,k} - \bar{C}^*_{\pi,k} = \left( \frac{\partial}{\partial T} \right)_{\partial p, \pi(k)} \left( \frac{\partial \bar{S}_k}{\partial T} \right)_{\partial p, \pi(k)} \]

\[ = - \gamma_k \left( \frac{\partial}{\partial T} \left( \frac{\partial \ln Y_{\tau,k}}{\partial T} \right) \right)_{\partial p, \pi(k)} + \tau^2 \left( \frac{\partial}{\partial T^2} \left( \frac{\partial \ln Y_{\tau,k}}{\partial T} \right) \right)_{\partial p, \pi(k)} \]

\[ = - \tau^2 \left( \nu_k \left( \frac{\partial}{\partial T^2} \left( \frac{\partial \ln Y_{\tau,k}}{\partial T} \right) \right)_{\partial p, \pi(k)} + \sum_j \nu_{j,k} \left( \frac{\partial}{\partial T^2} \left( \frac{\partial \ln m_j}{\partial T} \right) \right)_{\partial p, \pi(k)} \right) \]

\[ = \sum_{j} \nu_{j,k} \left( \bar{J}_{\tau,k} - \bar{C}^*_{\pi,k} \right) \]  

\[ (8-5) \]

\[ \bar{E}_{x,k} - \bar{E}^*_{x,k} = \left( \frac{\partial \bar{S}_k}{\partial T} \right)_{\partial p, \pi(k)} \]

\[ = \nu_k \left( \frac{\partial}{\partial T} \left( \frac{\partial \ln Y_{\tau,k}}{\partial T} \right) \right)_{\partial p, \pi(k)} + \frac{\bar{V}_k - \bar{V}^*_k}{\tau} \]

\[ = - \tau \left( \nu_k \left( \frac{\partial}{\partial T} \left( \frac{\partial \ln Y_{\tau,k}}{\partial T} \right) \right)_{\partial p, \pi(k)} + \sum_j \nu_{j,k} \left( \frac{\partial}{\partial T} \left( \frac{\partial \ln m_j}{\partial T} \right) \right)_{\partial p, \pi(k)} \right) \]

\[ = \sum_{j} \nu_{j,k} \left( \bar{E}_{\tau,k} - \bar{E}^*_{\pi,k} \right) \]  

\[ (8-6) \]
and

\[ \bar{\kappa}_k - \bar{\kappa}_k^o = - \left( \frac{\partial \bar{V}_k}{\partial \rho_T} \right)_{T, \hat{n}(k)} - \left( \frac{\partial \bar{V}_k^o}{\partial \rho_T} \right)_{T} = - v_k \text{RT} \left( \frac{\partial^2 \ln \gamma_{z,k}}{\partial \rho_T^2} \right)_{T, \hat{n}(k)} \]

\[ = - \text{RT} \left( \frac{\gamma_{z,k}}{\rho_T^2} \right)_{T, \hat{n}(k)} + \sum_{i} v_{i,k} \left( \frac{\partial^2 \ln \gamma_{k}}{\partial \rho_T^2} \right)_{T, \hat{n}(k)} \]

\[ = \sum_{i} v_{i,k} \left( \bar{\kappa}_{i,k} - \bar{\kappa}_{i,k}^o \right), \quad (B-7) \]

where \( \hat{n}(k) \), \( v_{i,k} \), \( \bar{\kappa}_{i,k} \) and the other symbols are defined in Appendix A and

\[ \bar{G}_{i} - \bar{G}_{i}^o = \text{RT} \ln \left( \frac{\gamma_{i}m_{i}}{\gamma_{i}^o m_{i}} \right) = \text{RT} \ln \left( \frac{\gamma_{i}m_{i}}{\gamma_{i}^o m_{i}} \right) \quad (B-8) \]

\[ \bar{V}_i - \bar{V}_i^o = \left( \frac{\partial \bar{G}_i}{\partial \rho_T} \right)_{T, \hat{n}(k)} - \left( \frac{\partial \bar{G}_i^o}{\partial \rho_T} \right)_{T} = \text{RT} \left( \frac{\partial \ln \gamma_i}{\partial \rho_T} \right)_{T, \hat{n}(k)} \]

\[ = \text{RT} \left( \frac{\gamma_{i}}{\rho_T} \right)_{T, \hat{n}(k)} + \left( \frac{\partial \ln m_i}{\partial \rho_T} \right)_{T, \hat{n}(k)} \quad (B-9) \]

\[ \bar{L}_{i} \equiv \bar{H}_{i} - \bar{H}_{i}^o = - \text{RT}^2 \left( \frac{\partial \left( \frac{\bar{G}_{i}}{T} \right)}{\partial T} \right)_{P, \hat{n}(k)} - \left( \frac{\partial \left( \bar{G}_{i}^o \right)}{\partial T} \right)_{P} \]

\[ = - \text{RT}^2 \left( \frac{\gamma_{i}}{T} \right)_{P, \hat{n}(k)} \]

\[ = - \text{RT}^2 \left( \frac{\gamma_{i}}{T} \right)_{P, \hat{n}(k)} + \left( \frac{\gamma_{i} m_i}{T} \right)_{P, \hat{n}(k)} \quad (B-10) \]

\[ \bar{T}_{i} \equiv \bar{S}_{i} - \bar{S}_{i}^o = \left( \frac{\partial \bar{G}_{i}}{\partial T} \right)_{P, \hat{n}(k)} - \left( \frac{\partial \bar{G}_{i}^o}{\partial T} \right)_{P} \]

\[ = \frac{\left( \bar{H}_{i} - \bar{H}_{i}^o \right) - \left( \bar{G}_{i} - \bar{G}_{i}^o \right)}{T} \quad (B-11) \]
\[ \vec{J}_{\perp} = \vec{C}_{\perp} - \vec{C}_{\perp}^{\text{abs}} = \left( \frac{\partial C_{\perp}}{\partial T} \right)_{p, \bar{m}(k)} \]
\[ = T \left( \frac{\partial C}{\partial T} \right)_{p, \bar{m}(k)} - \left( \frac{\partial C_{\text{abs}}}{\partial T} \right)_{p, \bar{m}(k)} \]
\[ = -R \left( \frac{\partial \ln \gamma_{\perp}}{\partial T} \right)_{p, \bar{m}(k)} + T^2 \left( \frac{\partial^2 \ln \gamma_{\perp}}{\partial T^2} \right)_{p, \bar{m}(k)} \]
\[ = -R \left( \frac{\partial \ln \bar{y}_{\perp}}{\partial T} \right)_{p, \bar{m}(k)} + \left( \frac{\partial \ln m_{\perp}}{\partial T} \right)_{p, \bar{m}(k)} \]
\[ + T^2 \left( \frac{\partial^2 \ln \bar{y}_{\perp}}{\partial T^2} \right)_{p, \bar{m}(k)} + \left( \frac{\partial^2 \ln m_{\perp}}{\partial T^2} \right)_{p, \bar{m}(k)} \] \hspace{1cm} (B-12)

\[ \vec{E}_{\perp} - \vec{E}_{\perp}^{\text{abs}} = \left( \frac{\partial \bar{V}}{\partial T} \right)_{p, \bar{m}(k)} - \left( \frac{\partial \bar{V}_{\perp}^{\text{abs}}}{\partial T} \right)_{p, \bar{m}(k)} = RT \left( \frac{\partial \ln \gamma_{\perp}}{\partial P} \right)_{T, \bar{m}(k)} \bar{m}(k) \]
\[ = RT \left( \frac{\partial \ln \bar{y}_{\perp}}{\partial P} \right)_{T, \bar{m}(k)} + \left( \frac{\partial \ln m_{\perp}}{\partial P} \right)_{T, \bar{m}(k)} \] \hspace{1cm} (B-13)

and

\[ \vec{K}_{\perp} - \vec{K}_{\perp}^{\text{abs}} = -\left( \frac{\partial \bar{V}}{\partial P} \right)_{T, \bar{m}(k)} - \left( \frac{\partial \bar{V}_{\perp}^{\text{abs}}}{\partial P} \right)_{T, \bar{m}(k)} = -RT \left( \frac{\partial^2 \ln \gamma_{\perp}}{\partial P^2} \right)_{T, \bar{m}(k)} \]
\[ = -RT \left( \frac{\partial^2 \ln \bar{y}_{\perp}}{\partial P^2} \right)_{T, \bar{m}(k)} + \left( \frac{\partial^2 \ln m_{\perp}}{\partial P^2} \right)_{T, \bar{m}(k)} \] \hspace{1cm} (B-14)
APPENDIX C

The temperature/pressure dependent parameters in equations derived in the text for activity coefficients, osmotic coefficients, and apparent molal and relative partial molal properties are defined by the expressions summarized below.

Debye-Hückel Parameters

The electrostatic parameters in the Debye-Hückel equation are defined by

\[ A_Y \equiv \frac{(2\pi N^o)^{1/2} e^3 \rho^o^{1/2}}{2.302585(1000)^{1/2}(e^o kT)^{3/2}} \]

\[ = \frac{1.824829238 \times 10^6}{(e^o kT)^{3/2}} \rho^o^{1/2} \quad (C-1) \]

and (for \( \delta \) expressed in cm)

\[ B_Y \equiv \frac{8\pi N^o e^2 \rho^o}{1000 e^o kT}^{1/2} \]

\[ = \frac{50.29158649 \times 10^8}{(e^o kT)^{1/2}} \rho^o^{1/2} \quad (C-2) \]

where \( \pi = 3.14159265 \), \( N^o \) corresponds to Avogadro's number \( (6.02252 \times 10^{23} \text{ mole}^{-1}) \), \( e \) stands for the absolute electronic charge \( (4.80298 \times 10^{-10} \text{ esu}) \), \( \rho^o \) represents the density in g cm\(^{-3}\) and \( e^o \) the dielectric constant of H\(_2\)O. \( T \) designates temperature on the thermodynamic scale in °K, and \( k \) denotes Boltzmann's constant \( (1.38054 \times 10^{-16} \text{ erg (°K)}^{-1}) \). Values of \( A_Y \), \( B_Y \), and other Debye-Hückel parameters defined below are given in table 1 for temperatures up to 350°C at 1 bar and/or pressures corresponding to the vapor-liquid equilibrium curve for H\(_2\)O. The remaining Debye-Hückel parameters can be expressed in terms of \( A_Y \), \( B_Y \), or their partial derivatives by writing
\[ A_G \equiv -2(2.303)RT A_Y \quad (C-3) \]

\[ A_H \equiv -\left( \frac{\partial (A_G/T)}{\partial T} \right)_P T^2 = 2(2.303)RT^2 \left( \frac{\partial A_Y}{\partial T} \right)_P \quad (C-4) \]

\[ A_J \equiv \left( \frac{\partial A_H}{\partial T} \right)_P = 2(2.303)R \left( 2T \left( \frac{\partial A_Y}{\partial T} \right)_P + T^2 \left( \frac{\partial^2 A_Y}{\partial T^2} \right)_P \right) \quad (C-5) \]

\[ A_V \equiv \left( \frac{\partial A_J}{\partial P} \right)_T = -2(2.303)RT \left( \frac{\partial A_Y}{\partial P} \right)_T \quad (C-6) \]

\[ A_K \equiv \left( \frac{\partial A_V}{\partial P} \right)_T = -2(2.303)RT \left( \frac{\partial^2 A_Y}{\partial P^2} \right)_T \quad (C-7) \]

\[ A_{E_x} \equiv \left( \frac{\partial A_V}{\partial T} \right)_P = \frac{A_V}{T} - \left( \frac{\partial A_H}{\partial P} \right)_T = \frac{A_V}{T} - 2(2.303)RT \left( \frac{\partial \left( \frac{\partial A_Y}{\partial P} \right)}{\partial T} \right)_P \quad (C-8) \]

\[ B_H \equiv 2(2.303)RT^2 \left( \frac{\partial B_Y}{\partial T} \right)_P \quad (C-9) \]

\[ B_J \equiv \left( \frac{\partial B_H}{\partial T} \right)_P = 2(2.303)R \left( 2T \left( \frac{\partial B_Y}{\partial T} \right)_P + T^2 \left( \frac{\partial^2 B_Y}{\partial T^2} \right)_P \right) \quad (C-10) \]

\[ B_V \equiv 2(2.303)RT \left( \frac{\partial B_Y}{\partial P} \right)_T \quad (C-11) \]

\[ B_K \equiv \left( \frac{\partial B_V}{\partial P} \right)_T = 2(2.303)RT \left( \frac{\partial^2 B_Y}{\partial P^2} \right)_T \quad (C-12) \]

and

\[ B_{E_x} \equiv \left( \frac{\partial B_V}{\partial T} \right)_P = \frac{B_V}{T} + \left( \frac{\partial B_H}{\partial P} \right)_T = \frac{B_V}{T} + 2(2.303)RT \left( \frac{\partial \left( \frac{\partial B_Y}{\partial P} \right)}{\partial T} \right)_P \quad (C-13) \]
Solvation Parameters

\[ b_k = \frac{\varepsilon_k}{2.303RT} \]  
(C-14)

\[ b_{H,k} \equiv 2T^2 \left( \frac{\partial (\varepsilon_k)}{\partial T} \right)_p = 2(2.303)RT^2 \left( \frac{\partial b_k}{\partial T} \right)_p \]  
(C-15)

\[ b_{J,k} = \left( \frac{\partial b_{H,k}}{\partial T} \right)_p = 2(2.303)R \left( 2T \left( \frac{\partial b_k}{\partial T} \right)_p + T^2 \left( \frac{\partial^2 b_k}{\partial T^2} \right)_p \right) \]  
(C-16)

\[ b_{V,k} \equiv 2 \left( \frac{\varepsilon_k}{\partial P} \right)_T = 2(2.303)RT \left( \frac{\partial b_k}{\partial P} \right)_T \]  
(C-17)

\[ b_{K,k} = - \left( \frac{\partial b_{V,k}}{\partial P} \right)_T = - 2(2.303)RT \left( \frac{\partial^2 b_k}{\partial P^2} \right)_T \]  
(C-18)

and

\[ b_{K,x,k} = \left( \frac{\partial b_{V,k}}{\partial T} \right)_p = \frac{b_{V,k}}{T} + \frac{1}{T} \left( \frac{\partial b_{V,k}}{\partial P} \right)_T = \frac{b_{V,k}}{T} + \frac{1}{T} \left( \frac{\partial b_{V,k}}{\partial P} \right)_T \]  

\[ + 2(2.303)RT \left( \frac{\partial b_k}{\partial P} \right)_T \]  
(C-18a)

where \( k \) designates a component of the solute and \( \varepsilon_k \) is defined in the text.

Short-Range Interaction Parameters

If any two aqueous species are designated by the subscripts \( \hat{j} \) and \( \hat{k} \), short range interaction parameters for both charged and neutral species can be represented by

\[ b_{\hat{j},\hat{k}} = \frac{\hat{b}_{\hat{j},\hat{k}}}{2.303RT} \]  
(C-19)

\[ b_{H,\hat{j},\hat{k}} \equiv 2T^2 \left( \frac{\partial (\varepsilon_{\hat{j},\hat{k}})}{\partial T} \right)_p = 2(2.303)RT^2 \left( \frac{\partial b_{\hat{j},\hat{k}}}{\partial T} \right)_p \]  
(C-20)

\[ b_{J,\hat{j},\hat{k}} = \left( \frac{\partial b_{H,\hat{j},\hat{k}}}{\partial T} \right)_p = 2(2.303)R \left( 2T \left( \frac{\partial b_{\hat{j},\hat{k}}}{\partial T} \right)_p + T^2 \left( \frac{\partial^2 b_{\hat{j},\hat{k}}}{\partial T^2} \right)_p \right) \]  
(C-21)
APPENDIX C (continued)

\[ b_{V,j,j} = 2 \left( \frac{\partial b_{j,j}}{\partial p} \right)_T = 2(2.303)RT \left( \frac{\partial b_{j,j}}{\partial p} \right)_T \]  \hspace{1cm} (C-22)

\[ b_{\kappa,j,j} = - \left( \frac{\partial b_{V,j,j}}{\partial p} \right)_T = - 2(2.303)RT \left[ \frac{\partial^2 b_{j,j}}{\partial p^2} \right]_T \]  \hspace{1cm} (C-23)

and

\[ b_{\varepsilon \kappa, j,j} \equiv \left( \frac{\partial b_{V,j,j}}{\partial T} \right)_p = \frac{b_{V,j,j}}{T} + 2(2.303)RT \left( \frac{\partial^2 b_{j,j}}{\partial T^2} \right)_T \]  \hspace{1cm} (C-24)

where \( b_{j,j} \) is defined in the text.

**Extended Term Parameters for 'Completely' Dissociated Single Electrolytes**

It follows from equations derived in the text and those summarized above that we can write for the \( k \)th "completely" dissociated electrolyte consisting of the \( i \)th and \( j \)th cation and anion, respectively,

\[ b_{\gamma,k} = - \frac{b_{\gamma,k}}{2(2.303)RT} = \frac{\psi_k \omega_k b_{\gamma,k} + 2\psi_i \kappa_k \nu_i \kappa_k b_{i,k}}{\psi_i \kappa_k} \]  \hspace{1cm} (C-25)

\[ b_{H,k} = 2(2.303)RT^2 \left( \frac{\partial b_{\gamma,k}}{\partial T} \right)_p = \frac{\psi_k \omega_k b_{H,k} + 2\psi_i \kappa_k \nu_i \kappa_k b_{H,i,k}}{\psi_i \kappa_k} \]  \hspace{1cm} (C-26)

\[ b_{j,k} = 2(2.303)RT \left( 2 \left( \frac{\partial b_{\gamma,k}}{\partial T} \right)_p + T \left( \frac{\partial^2 b_{\gamma,k}}{\partial T^2} \right)_p \right) \]

\[ = \frac{\psi_k \omega_k b_{j,k} + 2\psi_i \kappa_k \nu_i \kappa_k b_{j,i,k}}{\psi_i \kappa_k} \]  \hspace{1cm} (C-27)

\[ b_{\nu,k} = 2(2.303)RT \left( \frac{\partial b_{\gamma,k}}{\partial p} \right)_T = \frac{\psi_k \omega_k b_{\nu,k} + 2\psi_i \kappa_k \nu_i \kappa_k b_{\nu,i,k}}{\psi_i \kappa_k} \]  \hspace{1cm} (C-28)

\[ b_{\kappa,k} = - 2(2.303)RT \left( \frac{\partial b_{\gamma,k}}{\partial T} \right)_p = \frac{\psi_k \omega_k b_{\kappa,k} + 2\psi_i \kappa_k \nu_i \kappa_k b_{\kappa,i,k}}{\psi_i \kappa_k} \]  \hspace{1cm} (C-29)

and

\[ b_{\varepsilon \kappa,k} \equiv 2(2.303)RT \left( \frac{\partial b_{\gamma,k}}{\partial T} \right)_p \frac{b_{\gamma,k}}{T} = \frac{\psi_k \omega_k b_{\varepsilon \kappa,k} + 2\psi_i \kappa_k \nu_i \kappa_k b_{\varepsilon \kappa,i,k}}{\psi_i \kappa_k} \]  \hspace{1cm} (C-30)

where \( b_{\gamma,k}, b_{\nu,k}, \) and \( b_{\kappa,k} \) are defined in the text.


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