MULTIPLE SCALES IN RIVER BASIN MORPHOLOGY

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ABSTRACT. Existing data reveal that the steepness $S$ of many river basins with mountainous headwaters, defined as the ratio of total relief to total length of a basin, is related to the logarithm of basin size. The relief of these same river basins is observed to be non-linearly related to basin steepness, with maximal values of relief occurring in basins for which the value of $S$ is about 0.03 or, equivalently, in basins in which the average slope is about $2^\circ$. These systematic relationships between size, relief, and steepness of some basins, which span nine orders of magnitude of area, are interpreted in terms of an intermediate limit process bounded by well separated spatial scales. The lower-limiting scale corresponds to a threshold between channel formation and significant mass wastage, and the upper-limiting scale corresponds to the size of the largest landmass. If the observed relationships between basin size, relief, and steepness indeed reflect a limit process, then they are independent of the details of the physics contributing to landscape development and instead result from the constraints imposed by the bounding, spatial scales. Exceptions to the limiting relationships explored here are readily noted, however, and are interpreted to reflect regional geological and temporal controls. The delineation of the importance of multiple scales provides a potentially powerful basis for the examination of morphological relationships in river basins.

INTRODUCTION

River basins dominate the landscape, and the study of their morphology reveals important information about their formation, sediment yield, and fill. I examine here the quantitative relationship between the area and steepness of river basins which drain mountainous terrain in terms of the bounding and intermediate length scales imposed by geological process and form. The steepness of a river basin is considered in terms of the ratio of its total relief to its total length. This ratio is a dimensionless number that has found wide use in the comparison of basins (Schumm 1956, 1963) and other landscape features, such as alluvial fans (Melton 1965), which vary greatly in size. Its value represents the average slope of the longitudinal profile of a basin and thus represents the basin-wide magnitude of the gravitational component of landscape-forming processes.

In the next section I review ideas regarding the existence and separation of spatial scales in fluvial landscapes. This is followed by a simple analysis of physical systems bounded exclusively by two well-separated spatial scales. The results of the analysis are then used to examine the relationships observed between river basin area, steepness, and relief. Additional points of interest are explored in the discussion. These include the situation when an intermediate spatial scale is imposed by regional geological structure and the evaluation of a denudational time scale and its relationship to basin size. I conclude with a summary of important findings.

SEPARATION OF MULTIPLE, BOUNDING SCALES IN RIVER BASIN MORPHOLOGY

Montgomery and Dietrich (1992) found a simple relationship between the length $L$ of a drainage basin and its area $A$ given by

$$L = (3A)^{1/2}. \quad (1)$$

Shown in figure 1 are data from over 200 river basins of the world, including observations for 100 of the world’s largest, individual rivers (Allen 1997; Hovius 1998),
and summary measures from numerous, smaller drainage basins in many different
mountain belts representing a range of geological settings and ages (Hovius 1996;
Talling and others 1997). The scatter in the data about the approximate relationship
given by eq 1 and shown in figure 1 reflects differences in local climate and geology as
well as differences in the methodologies used to acquire the data. The agreement
between the observations and the relationship given by eq 1 for which $R^2 = 0.86$ is
nevertheless remarkably good. The implication is that there is universal similarity in
the plan shape of drainage basins, each of which displays an average width that is about
1/3 its length.

An important aspect of the basins considered in figure 1 is that they extend from
regional base level to topographic divides associated with greatest regional relief and,
accordingly, originate in mountainous settings dominated by landsliding and related
mass movements. Such settings have been termed “ridge-pole” basins by Hovius (1996)
and are distinguished from range-front basins, pre-alpine basins, and small, nested
catchments that do not extend to regional topographic divides. Many small catch-
ments, in particular, originate in low-relief hills in which channel initiation is con-
trolled solely by overland flow. Such settings are not considered here.

Some of the very largest basins considered in this analysis are developed on diverse
geological terrain but individually may be so large as to be effectively homogeneous in
underlying structure. Smaller basins in strictly mountainous settings occur on relatively
homogeneous bedrock. Thus it is likely that there are no obvious intermediate spatial
scales introduced by local geological structure in the river basins represented in figure
1. These basins span a size range that extends over nine orders of magnitude from a
minimal area, $A_{\text{min}}$, associated with processes of mass wastage and channel formation
(Horton 1945) to a maximal area, $A_{\text{max}}$, set by the size of the largest, existing land mass.
In the extreme, $A_{\text{max}}$ is limited by the size of the Earth itself. These preliminary
observations and assertions underlie the general analysis developed here regarding the
separation of multiple, bounding spatial scales. In the discussion I extend this view and
consider the likelihood that local conditions in some basins impose an intermediate
length scale or reflect a markedly unsteady process.

Fig. 1. River basin length $L$ as a function of basin area $A$. Data from Hovius (1996), Allen (1997), and
Talling and others (1997).
The slope of the terrain in a river basin is a direct measure of the gravitational component of processes of erosion and mass transport. Under uniform hydrologic conditions, the observed similarity in plan geometry of river basins indicated in figure 1 reflects in part the constraint that average slopes in the down-valley and cross-valley directions of individual catchments with mountainous headwaters, and hence average rates of sediment erosion and transport, must be proportional. The degree of proportionality reflects among other things the ratio of discharges in mainstem and distributary channels. If this proportionality were not the case, then head-water and lateral divides would wear down and retreat at grossly different rates. Such a scenario is unlikely, but, of course, this view is open to debate. To address the idea that aspects of the morphology of a river basin are constrained by its average slope, observations of basin steepness are reconciled with the limits to basin size $A_{\min}$ and $A_{\max}$ introduced above. These limits are evaluated as follows.

It is observed that channels tend to begin at a distance downslope of a drainage divide for which there is sufficient overland flow to support a channel (Horton 1945; Montgomery and Dietrich 1988, 1989, 1992; Dietrich and others 1992). For given local conditions of climate and geology, in other words, there is a minimum area, $A_{\min}$, below which the landscape is controlled primarily by processes of mass wastage, soil creep, and overland sheet flow rather than channel transport. Values of $A_{\min}$ are typically much less than 1 km$^2$ (Montgomery and Dietrich 1988). As a river basin with a mountainous divide approaches $A_{\min}$ in size, its steepness must therefore approach a critical gradient associated with the onset of significant landslides and avalanches. In basins with relatively uniform and planar slopes, this critical gradient corresponds to the tangent of the friction angle which is applicable at the landscape scale. This angle is in the range 15° to 30° (Schmidt and Montgomery 1995), which corresponds to a critical slope or steepness associated with $A_{\min}$ in the range 0.25 to 0.6.

In drainage basins much larger than $A_{\min}$, there is a predominance of the processes of river channel incision, migration, and alluviation. An upper limit to the areal extent of basins $A_{\max}$ is imposed, at the other extreme, by the size of the largest land mass. Thus, as the area $A$ of a river basin greatly exceeds $A_{\min}$ and approaches $A_{\max}$, its average gradient reflects the constraints imposed by transport processes in a well-developed and extensive network of river channels. The minimal value of steepness required for mass transport in river basins, for the purposes of the analysis given here, is negligibly small.

The reasoning outlined above suggests that the relationship between area and steepness is different from the strictly geometrical relationship described by eq 1. The degree to which basin steepness is dependent on basin area, in other words, reflects the constraints imposed by the processes of landscape erosion and sediment transport and the spatial limits to size. I employ a well-established analysis of such a system, a preview of which is as follows. The existence of limits to basin size suggests that scaling of a mean dynamic variable, such as steepness, with basin area can be considered in terms of either of the dimensionless parameters $\eta \equiv A/A_{\min}$ and $\Gamma \equiv A/A_{\max}$. If the ratio $A_{+} \equiv A_{\max}/A_{\min}$ is large enough, then there is a region of overlap in which neither of the limiting scales $A_{\min}$ or $A_{\max}$ is uniquely important. For the river basins represented in figure 1, $A_{+}$ is of the order of $10^8$ or greater. Clearly the upper and lower limits to basin size are well separated, and under such conditions the existence of a wide range of spatial scales in individual basins of intermediate size or larger becomes possible. A schematic representation of this situation is shown in figure 2, in which $\Gamma$ is shown as a function of $A_{+}$. In the following section the matching regime in river basins for which $A_{+}$ is large and $A_{\min} \ll A \ll A_{\max}$ is considered in more detail.
In the previous section, distinct limits to the size of river basins were identified. These limits are determined by the dynamics of channel formation in steep terrain and the size of the largest existing land mass. Such limits impose real constraints on the functional relationships between basin size, relief, and slope. To examine these relationships, the transition between the regimes of river-basin dynamics associated with the two limiting spatial scales \( A_{\min} \) and \( A_{\max} \) can be considered in terms of the relief ratio \( S \), defined as the ratio of total topographic relief \( H \) to total length \( L \) of a basin. This ratio and related measures are used to describe the steepness or ruggedness of a catchment (Melton 1965). Other variables could be considered, but, as noted in the previous section, variables associated with slope are likely to be key measures that relate landscape process and form. The steepness or relief ratio \( S \) is among the most general measures of slope for which data are readily available. Accordingly, I focus on this variable.

As mentioned in the previous section, if the ratio \( A_{\ast} = A_{\max}/A_{\min} \) is sufficiently large then there exists the possibility that there is no uniquely dominant spatial scale in a landscape. Formally, this situation exists if the mathematical limits \( \eta \to \infty \) and \( \Gamma \to 0 \) can be taken simultaneously. Specifically, if \( \eta \) can be described by the functional form \( \eta \sim A_{\ast}^{-\alpha} \), then \( \Gamma \sim A_{\ast}^{\alpha-1} \), where the tilde (\( \sim \)) implies direct proportionality and is read “goes as”. If \( A_{\ast} \) is sufficiently large and \( 0 < \alpha < 1 \), moreover, there exists a range of relationships for which \( \eta \to \infty \) and \( \Gamma \to 0 \) simultaneously, as shown in figure 2. Such relationships are called intermediate limit processes. There are formal guidelines for the analysis of limit processes at steady state, but among the simplest methods used to match different regimes in a region of overlap for such processes is one first proposed in river basin morphology.
by Millikan (1939) as a rationale for some relationships observed in turbulent, channel flow. A general description of this approach, which is essentially one of dimensional analysis and is applicable to a wide range of physical processes, is given in more detail in Panton (1984). In applying such an analysis to river basin morphology, one asserts that there exist relationships $S = f(\eta)$ for $\eta$ in the range $0 < \eta < \infty$ and for which $G \ll 1$ and $S = r(G)$ for $G \to 1$ but $\eta \gg 1$. The functions $f(\eta)$ and $F(G)$ are not known in detail but are understood to be constrained by the processes of mass transport which are characteristic of the respective endmember regimes, as discussed above.

Within the transitional regime $A_{\text{min}} \ll A \ll A_{\text{max}}$, the functions $F(G)$ and $f(\eta)$ are assumed to be continuous and smooth. Smoothness is ensured by equating the derivative of $S$ with respect to $A$ for each function. Accordingly, $A_{\text{min}}^{-1} \frac{\partial f}{\partial \eta} = A_{\text{max}}^{-1} \frac{\partial F}{\partial \Gamma}$ and, upon multiplying both sides of this equality by $A$, one obtains the equivalent statement $\eta \frac{\partial f}{\partial \eta} = \Gamma \frac{\partial F}{\partial \Gamma}$. The left side of this equation is solely a function of $\eta$, and the right side is solely a function of $G$. If there are no other length scales, both sides must be equal to a dimensionless constant, say, $-\lambda$. This last statement is simply a mathematical necessity. Upon introduction of the constant $-\lambda$ and performing the required integrations, one obtains the equations

$$f(\eta) = -\lambda \ln(\eta) + S_{cr}, \quad (2A)$$

and

$$F(G) = -\lambda \ln(G) + S_0. \quad (2B)$$

In eq 2A the value of $S_{cr}$ corresponds to the critical slope required for the onset of significant mass wastage as $\eta$ approaches unity, as explained in the previous section. The value of $S_0$ introduced in eq 2B, on the other hand, corresponds to the minimal gradient required for mass transport in very large river basins. The value of $S_0$ is assumed to be very much less than $S_{cr}$ and thus, for the purposes of this analysis, to be negligibly small. Equating eqs 2A and B to ensure continuity and assuming $S_0$ to be very small thus requires that $A_{\text{min}} \approx A_{\text{max}}$ if $A_{\text{min}}$ be much larger than $A_{\text{max}},$ that there be no intermediate spatial scales, and that no rescaling of basin steepness $S$ is required in the hypothetical limit $A_{\text{max}} \to 0$. The result is otherwise independent of the details of the processes contributing to the forms of $f(\eta)$ and $F(G)$.

Eqs 2A and B provide a basis for examining the relationship between the steepness $S$ and the relief $H$ of a basin. Substitution of the definition $S = H/L$ and eq 1 into eq 2B, introduction of the terms $L_{\text{max}} = (3A_{\text{max}})^{1/2}$ and $L_{\text{min}} = (3A_{\text{min}})^{1/2}$, and rearrangement of the result yield the expression

$$H = L_{\text{max}}S \exp(-S/(2\lambda)). \quad (3A)$$

Because $2\lambda = S_{cr}/\ln(L_{\text{max}}/L_{\text{min}})$ as indicated above, eq 3a can be expressed equivalently as

$$H = L_{\text{max}}S(L_{\text{max}}/L_{\text{min}})^{S/S_{cr}}. \quad (3B)$$

Eqs 3A and B indicate that the relationship between basin relief and steepness in a matching regime is non-linear. An estimate of the basin steepness associated with the greatest possible relief $H_{\text{max}}$ in such a regime can be determined by taking the
derivative of $H$ with respect to $S$, setting this expression to zero, and solving for $S$. From such a calculation, the coefficient $\lambda$ introduced in eq 2 is found to be proportional to one half the average gradient associated with $H_{\text{max}} = 2\lambda e^{-1}L_{\text{max}}$. Thus, in this view, maximal relief on the Earth’s surface is determined solely by the values of the bounding length scales $L_{\text{max}}$ and $L_{\text{min}}$ and the limiting slope $S_{cr}$ required for the onset of significant mass wastage.

A value of $\lambda \approx 10^{-2}$, as estimated above, indicates that maximal topographic relief should occur in river basins for which the average slope is about $1^\circ$ to $2^\circ$. For $L_{\text{max}}$ reasonably taken to be of the order of $10^3$ km (and imposed by the size of existing land masses on a finite Earth), $H_{\text{max}}$ should therefore be of the order of 10 km. In very large basins which are less steep than $2\lambda$, $S$ is expected to be approximately linearly related to relief $H$. In basins steeper than $2\lambda$, on the other hand, $S$ is non-linearly related to basin size and relief but is ultimately constrained by the critical value $S_{cr}$. These quantitative predictions of river basin morphology are straightforwardly derived from eqs 2A and B and 3A and B and an approximate knowledge of the limiting value of $S_{cr}$ and of the spatial scales $L_{\text{min}}$ and $L_{\text{max}}$ (or, equivalently, $A_{\text{max}}$ and $A_{\text{min}}$) that constrain catchment geometry. In the following section, the relationships given by eqs 2A and B and 3A and B are compared with observations from natural river basins with mountainous headwaters.

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Shown in figure 3 is the relationship between basin area $A$ and steepness $S$ for the same rivers represented in figure 1 with some exceptions. Data represented in figure 1 and not shown in figure 3 include observations from mountain belts underlain by growth folds (for example, Kettleman Hills, western United States; Talling and others 1997). Such settings are considered in more detail in the discussion. In figure 3, as elsewhere in this analysis, steepness $S$ is represented on the horizontal axis to facilitate comparison with area-slope plots conventionally employed by geomorphologists (Montgomery and Dietrich 1988, 1992; Dietrich and others 1992). No causality is necessarily
Logarithmic axes are required to accommodate the wide range in values encountered. Returning to figure 3, I note that in basins with areas much less than about 1 km$^2$, $S$ approaches a value of no more than about 0.6. This value corresponds to an average gradient of about 30° and reflects the constraint on basin geometry imposed by the onset of significant mass wastage in small, steep catchments. Also shown in figure 3 is the logarithmic relationship given by eq 2. The decreasing trend in area with increasing steepness is well described by such a relationship, with calculations from a least-squares regression of the form given by eq 2 yielding a value of $l = 0.015$ ± $3 \times 10^{-4}$ (± 1 standard error). This value of $l$ coincides with the order-of-magnitude estimate given above.

Shown in figure 4 is a plot of basin relief $H$ as a function of basin steepness $S$ for the same rivers represented in figure 3. Note that maximal relief is of the order of 10 km and occurs in basins for which $S$ is of the order of $10^{-2}$. In less steep basins, $S$ is approximately linearly related to $H$. In smaller and steeper basins, $S$ asymptotically approaches $S_c$, basin length approaches $L_{min}$, and $H$ diminishes accordingly. This functional behavior is captured by eq 3, as discussed above, for which a non-linear, least-squares regression yields $L_{max} = 800$ km ± 45 km for $l = 0.015$. The regression relationship is indicated in figure 4 by the solid line.

There is considerable scatter in the data shown in figures 3 and 4. As mentioned above, this scatter reflects the variability associated with local conditions and different methodologies. It nevertheless appears that the analysis described here can be used to interpret the relationships between the size, total relief, and steepness of basins in a matching regime bounded by $A_{min}$ and $A_{max}$. In particular, the analysis captures a transitional behavior proposed by others employing different criteria (Tarboton and others 1992). Importantly, the analysis of river basin morphology as an intermediate limit process anticipates the non-linear relationship between basin relief and slope revealed in figure 4. Because the catchments considered here are ridge-pole basins in
areas of relatively uniform geology, I propose that the analytical relationships between basin area, relief, and steepness given by eqs 2A and B and 3A and B and indicated in figures 3 and 4 represent matching-limit envelopes for river basin morphology. These enveloping relationships reflect the interaction of multiple spatial scales in a very general way.

**DISCUSSION**

The analysis described in the previous sections provides a rationale for the relationships observed between basin size, relief, and steepness for rivers extending to regional, mountainous divides. Briefly stated, the steepness of ridge-pole river basins is proposed to result from an intermediate limit process that spans wide ranges of basin size and dynamic regime. At one extreme are small, steep basins in which processes of mass wastage and channel formation are relatively important. At the other extreme, constrained in size by the size of existing land masses, are large, low-lying basins dominated by river-channel and diffusional, slope-driven processes. The assertion that these regimes overlap in a smooth and continuous fashion in the absence of intermediate length scales leads to the prediction that, within the range of overlap, basin steepness is related to the logarithm of basin size. This prediction is based on the mathematical properties of a simple limit process and is independent of the details of the underlying physics of the process. A rearrangement of the expression of the relationship between basin size and steepness indicates that there is a non-linear relationship between basin relief and steepness. The results, described by eqs 2A and B and 3A and B and shown in figures 3 and 4, provide general envelopes for river basin morphology. Such relationships indicate that maximal values of overall relief in ridge-pole basins are associated with terrains in which the average slope, or steepness, is about 2°.

I emphasize that the relationships between basin size, relief, and steepness shown in figures 3 and 4 are essentially empirical. The analysis described here simply provides a mathematical rationale for these relationships. Thus, while an inverse relationship between area and slope is not unexpected (as it embodies, effectively, a scaling between $L^2$ and $L^{-1}$), only a quantitative analysis of an intermediate limit process as it is described here and given in eqs 2A and B and 3A and B anticipate the observed strongly non-linear relationship between basin relief and steepness.

Given the empirical character of the correlations revealed here, additional study is nevertheless required to understand the physical processes that govern these relationships. The observation that maximal relief is associated with average slopes of about 2° (or, equivalently, in basins of about 500 km in length), for example, is likely to reflect constraints imposed by the rigidity of the crust underlying topography of a given wavelength (Turcotte and Schubert 1982). Large average elevations (and hence large maximal elevations) in very large basins, in other words, are inadequately supported by the lithosphere. Large relief in very small basins is associated with steep slopes and is thus reduced by mass wastage at the Earth’s surface. The greatest relief is thus associated with basins which are intermediate in size between these two extremes. Such an explanation can be described as an intermediate limit process.

The analysis described here provides a useful basis for the graphical examination of the dominant controls of river basin morphology. One implication of the results of this analysis is that the well-known inverse relationship between drainage area and slope within individual basins (Leopold and Miller 1956) may reflect a general condition that is independent of the details of the processes of landscape evolution. On the other hand, basin relief and steepness are clearly controlled by regional geology in some settings. Examples of such control are explored below. Care must be taken, therefore, in evaluating detailed theories of basin development that rely solely on agreement with an area-slope relationship for verification.
Additional issues of interest include situations in which an intermediate spatial scale is imposed by regional geological structure and the evaluation of a time scale associated with the denudation of existing relief. Each of these issues is briefly discussed in turn.

**Geological control and intermediate spatial scales.**—Regional geological structure is likely to control the morphology of many river basins. Shown in figure 5, for example, is the relationship between basin area and length for 37 individual catchments in the Kettleman Hills in the western United States (Talling and others 1997). The solid line indicates the quantitative relationship given by eq 1. Although the data for the river basins of the Kettleman Hills lie approximately within the scatter of the data shown in figure 1, these basins systematically deviate from the relationship given by eq 1 to exhibit a length scale of no less than about 2 km. This geometry is likely to reflect the regular and closely-spaced arrangement of folded layers of resistant, underlying rock (Woodring and others 1940). Such settings are not necessarily limited solely by the values of $A_{\text{min}}$ and $A_{\text{max}}$ but are instead constrained by an intermediate length scale $L_i$ imposed by regional geological structure. This is in contrast to the majority of basins represented in figure 1 and which are amenable to the analysis of the limit process described in the previous sections.

In basins dominated by an intermediate length scale $L_i$, the analysis described in the previous sections is inappropriate. In contrast to eq 3, for example, overall relief $H$ is by definition expected to be linearly related to basin steepness $S$ wherein $H = L_i S$. To demonstrate this, the relationship between $H$ and $S$ for the catchments in the Kettleman Hills is shown in figure 6 (triangles; Talling and others 1997). These basins exhibit a morphology with a plan length scale of about 2 km (compare fig. 5). As a result, the relationship between $H$ and $S$ in the Kettleman Hills does not conform to the matching limit function established in figure 4 and indicated in figure 6 by the dashed line.

The dominance of intermediate length scales in river basins owing to local structural control is likely to be common. At the other extreme in size from the
Kettleman hills basins, for example, are 15 trunk-stream basins of the Oregon coast range of the western United States (squares in fig. 6; Rhea 1993). These catchments drain an upland terrain bounded regionally by the large Willamette River to the east (the location of which itself is likely to indicate intermediate-scale geological control) and the Pacific Ocean to the west. Geographic confinement in this setting apparently imposes a length scale of about 100 km on basin size.

An additional demonstration of the difference between ridge-pole and range-front basins emerges in a comparison of 18 individual alpine and pre-alpine basins in northern Italy. The partitioning between alpine and pre-alpine settings here reflects the distinctions made by Guzzetti and others (1997) in their study of these basins. Alpine ridge-pole basins (open circles in fig. 6) conform to the matching limit function established in figure 4. In contrast, pre-alpine basins drain only the foothills in front of the mountain range and comprize moderately hilly terrain. Such basins (filled circles in fig. 6) exhibit an approximately linear relationship between $S$ and $H$ for which the imposed length scale is inferred to be about 10 km. Pre-alpine catchments thus exhibit a morphology apparently controlled by the spacing of adjacent ridge-pole basins and the distance to local base level established by the Po River. This distinction raises important questions regarding, among other things, the significance and variability of empirical relationships between the areas of alluvial fans and their source catchments (Hooke 1968). These questions will be explored in more detail elsewhere.

In general, exceptions to the analysis presented here are likely to be numerous. Such exceptions are likely to be associated with pre-alpine, range-front and nested, low-lying catchments for which an intermediate, structural length scale dominates basin geometry, and in which mass-wastage processes do not necessarily dominate channel formation at the lower limit of basin size.
A denudational time scale.—Throughout this analysis it is assumed that the temporal
evolution of landscapes is irrelevant. Such a view is very simplistic. In this section, a
time scale associated with the denudation of existing relief is identified and correlated
to the size of large, individual river basins. The relevance of this denudational time
scale to the analysis of multiple spatial scales is also briefly discussed.

The argument considered here is an extension of ideas originally advanced by
Ahnert (1970) and is based on recent attempts to describe quantitatively the evolution
of topography (Willgoose 1994). Such efforts generally relate the rate of change of
elevation at a point in a basin to a balance between the effects of uplift, divergence of
overland and fluvial transport of sediment, and diffusional transport (for example, soil
creep and rain splash). The equation that describes such a balance introduces several
possible time scales for landscape evolution. One time scale that readily emerges and is
easily evaluated is the time \( t_d \) required for the denudation of the existing landscape
under current hydrological conditions (Willgoose 1994; Zhou and Stüwe 1994). The
significance of this time scale is as follows. To the degree to which \( t_d \) represents
the dominant time scale for landscape evolution and its values are large relative to basin
age, the steady state relationships invoked here are likely to be invalid (Ahnert 1970).
On the other hand, changes in rates of uplift that are short-lived relative to \( t_d \)
essentially constitute background noise in the time-averaged tectonic regime (Willgoose 1994).
Thus, one possibility is that the tendency of landscapes to evolve toward steady state is
sufficient to render the approach taken here and in similar analyses useful.

Estimates of the time scale \( t_d \) are calculated here as the quotient
\[
\frac{H_{av}}{d'},
\]
where \( H_{av} \) is the average elevation of a basin and \( d' \) is the rate of total denudation \( d \)
(corresponding to the sum of mechanical and dissolved loads) corrected for the effects of isostatic
compensation of the eroded mass. To make such corrections, the degree of isostatic
compensation \( C \) associated with two-dimensional, periodic topography of wavelength
\( 2L \) is evaluated following Turcotte and Schubert (1982). Hence,

\[
d' = (1 - C \rho_c / \rho_m)^{-1} d,
\]

where

\[
C = \{1 + (\pi L_e / L)^3\}^{-1},
\]

and \( \rho_c (= 2800 \text{ kg m}^{-3}) \) and \( \rho_m (= 3300 \text{ kg m}^{-3}) \) are the density of crustal and
underlying materials, respectively. The flexural length scale \( L_e = D / g (\rho_m - \rho_c)^{1/4} \) and
\( D = E T_e^3 / 12 (1 - \sigma^2) \), where \( E (= 10^{11} \text{ Pa}) \) is Young’s modulus, \( T_e (= 20 \text{ km}) \) is the
elastic thickness of the lithosphere, \( \sigma (= 1/2) \) is Poisson’s ratio, and \( g (= 9.8 \text{ m s}^{-2}) \) is
the gravitational acceleration. For the nominal values used here, \( L_e \) is about 65 km. It is
important to note that this length scale is not a physical constraint to basin morphology
as are the size of the largest land mass (corresponding approximately to \( L_{max} \)) and a
lower-limiting threshold (corresponding to \( L_{min} \)). \( L_e \) represents instead a parameter in
the mathematical description of the isostatic compensation of relief.

The correction term \( C \) for the degree of isostatic compensation given in eq 5
reflects the assumption that isostatic compensation occurs instantaneously. Its value
becomes very small for a small basin for which \( L \ll L_e \). The topography of such a basin,
in other words, is fully supported by the rigidity of the Earth’s crust, and, as a result,
isostatic compensation for the eroded mass is negligible. In contrast, the topography of
a large basin for which \( L \gg L_e \) is inadequately supported by the crust and is essentially
supported instead by isostatic forces. Under such extreme conditions the effective rate
of relief reduction \( d' \) is reduced from the observed rate of mass removal \( d \) by a factor
\( (1 - \rho_c / \rho_m)^{-1} \).

This correction is based on a simple, two-dimensional model for topographic
loading of the crust. The values chosen for the various parameters in eqs 4 and 5,
moreover, are nominal values only. Accordingly, the resulting estimates of the time scale $t_d$ presented here are comparative only. Such estimates are nevertheless instructive. Shown in figure 7, for example, are estimates of $t_d$ as a function of basin size for 60 major rivers of the world. Values of $t_d$ range from $10^6$ yrs to in excess of $10^9$ yrs and increase with basin size.

The key point for the analysis here is that there exists at least one time scale associated with topographic adjustment to changes in the conditions that force landscape evolution. In relatively large river basins this time scale is well in excess of millions of years and is a function of, among other things, the size of individual basins. Because the values of $t_d$ calculated here correspond approximately to a time constant associated with the exponential decay of all existing relief (Ahnert 1970), they are likely to over-estimate the time actually required for landscape adjustment to changes in uplift. Nevertheless, given the large values of $t_d$, there are unquestionably settings in which the notion of a limit process at steady state is inappropriate. The degree to which landscapes are not in strict equilibrium is likely to contribute to the scatter observed in figures 1 to 5. Such conditions may also contribute to the dominance of intermediate spatial scales imposed by geological process and form which are time dependent and regional in scope.

**Conclusions**

Relationships between area, relief, and steepness of some river basins can be considered in terms of an overlap or matching regime between well-separated, bounding spatial scales. The lower-limiting scale reflects a threshold for channel formation controlled by landsliding in mountainous terrain. The upper-limiting scale reflects the size of the largest existing land mass or, in the extreme, the size of the Earth itself. A simple prediction for such a regime is that some dynamical variables that relate river-basin process and form are related to the logarithm of basin size. This is the case for the steepness of river basins that originate at regional topographic divides and overlie regions of relatively homogeneous geology. As an extension of this view, basin
relief is predicted and observed to be related to basin steepness in a non-linear way, with maximal values of relief to be of the order of 10 km and associated with values of steepness which are of order $10^{-2}$. If these relationships indeed reflect a limit process, then they are independent of the details of the physics underlying landscape development. Exceptions to the limiting relationships explored here are likely to be common and are proposed to reflect the introduction of intermediate spatial scales associated with regional geological process and form. Taken in total, the observations and analysis reported here have ramifications for landscape simulation models whose verification relies solely on comparison with relationships between basin size, relief, and slope. A time scale is associated with the denudational adjustment of existing relief of river basins in response to changes of basin uplift. For river basins with areas greater than $10^4 \text{ km}^2$, this time scale is in excess of millions of years and is a function of basin size. The magnitude of such an adjustment time implies that the assumption of steady state implicit to the analysis described here is not strictly valid. One possibility is that the tendency of landscapes to evolve toward steady state is sufficient to render this and similar analyses useful. On the other hand, the degree to which river basins are not in strict equilibrium is likely to contribute to the scatter observed in the morphological relationships delineated here. These complications notwithstanding, the analysis described here provides a new basis for examining the relationships that exist between the size, relief, and steepness of some river basins and that reflect the influence of multiple spatial scales.

ACKNOWLEDGMENTS

P. Friend, N. Hovius, H. Huppert, D. Montgomery, R. Slingerland, P. Talling, and three anonymous reviewers made helpful comments on earlier versions of the text. The Leverhulme Foundation and the Natural Environmental Research Council (UK) provided financial support. E. Dade provided patience, generosity, and laughter.

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