A CHANNELIZATION MODEL OF LANDSCAPE EVOLUTION

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ABSTRACT. The geomorphic evolution of many landscapes is fundamentally determined by the evolution of the river channels and their interactions with hillslopes. Consequently, models of landscape evolution ought to track the evolution of the channel geometry so as to quantify the rate of erosion of channel bottoms and to follow the changes in hillslope-channel coupling over time. Unfortunately, the spatial resolution required to describe channel morphology adequately is computationally impractical. It is also beyond the resolution of most digital elevation data. What is required is a parameterization scheme for approximating fine scale channel morphology at a coarse pixel scale. Such a parameterization is already implicitly employed in most models by assuming channel equilibrium, which ties the width and depth of a model channel to the square root of discharge through a pixel. Channel fluxes are thereby predictable, and a closed form of the governing equations is attained. In reality, mountain river channels do not take a simple equilibrium form and show great spatial variability and evident disequilibrium geometry. Since the time scales of changes in channel geometry, bedrock channel erosion, and hillslope response are all closely related, it is reasonable to infer that the spatio-temporal development of the landscape is determined by their interaction and that channel disequilibrium is a fundamental factor in the dynamics of landscape evolution. If this is the case, we need an alternative sub-grid scale parameterization that aggregates channel properties such as surface morphology, roughness, cross-sectional geometry, so that the time dependent behavior of these properties can be estimated at the coarse pixel scale. We introduce such a parameterization measure, which we term channelization, after extensive investigation of the pixel resolution dependence of topographic relief. We focus in particular on the effect of coarse graining on digital elevation data for derived measures such as channel slope and upstream area and demonstrate that we can approximately correct for this effect. We show that a very simple geomorphic model can be constructed around the channelization parameter and the resolution-invariant topographic measures. This model demonstrates that channel disequilibrium may play a significant role in mountain landscape dynamics. It also shows how geomorphic models in general could be modified to incorporate such sub-pixel scale complexities and to better model these dynamics.

INTRODUCTION

The principal aim of this paper is to highlight the role of channel complexity in the dynamics of landscape evolution (Beaumont, Kooi, and Willett 1998; Densmore and others, 1997; Giacometti, Maritan, and Banavar, 1995; Howard, 1994; Kooi and Beaumont, 1996; Tucker and Slingerland, 1996; Willgoose, Bras, and Rodriguez-Iturbe 1991a). An underlying theme is the issue of pixel resolution: in particular, the ways in which digital elevation and geomorphic model grids limit our ability to characterize channel morphology and the ways by which those limitations can be overcome. We examine a key assumption of existing geomorphic which defines the channel geometry and makes the model set of equations tractable but dynamically problematic. We show that this simplifying assumption is an example of a sub-grid scale parameterization; in other words it is a method for summarizing the sub-pixel details of channel morphology in such a way that a workable model of channel erosion, sediment transport, and hillslope response can be constructed. Unfortunately, this particular parameterization has the effect of freezing the channel geometry and of excluding the

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time dependent behavior of channel shape on the dynamics of landscape evolution. A less restrictive parameterization can be defined, as we shall show, one that can also encompass the effect of coarse pixel resolution. This scheme is implemented through a parameter we term \textit{channelization}, which is the outcome of a detailed scaling analysis of topographic measures such as slope and area, discussed in the first half of this study. This parameterization is used in the second half of the study as the basis for a prototype geomorphic model. Numerical simulations of this simple model demonstrate that the evolution of a model landscape to steady-state can incorporate time and space variable channel morphology. We demonstrate that the time-dependent response of such a model landscape to changing boundary conditions is critically dependent on the degree of channel disequilibrium and its propagation across a channel network.

\textit{Pixel size and the problem of channel resolution}.—Surface topography is difficult to resolve accurately either when modelling or when working with digital elevation models (DEMs). Digital elevation data are usually resolved to no better than 30 to 50 m, and the means of DEM construction and their consequent vertical precision vary widely. As a result, such data cannot generally resolve channels, much less channel geometries. The same difficulty arises in numerical models of landscape evolution. To simulate the development of topography on the scale of an orogen or perhaps just on the scale of a large catchment, we need to sacrifice any ability to resolve channel topography: model grid cells are typically of the order of 10 to 100 m or more in size. This limitation is remediated to some degree through the use of adaptive meshing methods such as those of Braun and Sambridge (1997). The resolution problem remains though, because even sophisticated meshing schemes do not allow models to resolve to the sub-meter scale necessary to characterize channel morphology. This inability to characterize channels fully compromises our attempts to study channel evolution and the disequilibrium between hillslopes and channels. These disequilibrium properties are directly linked to the evolution of the landscape as a whole. Consequently, there is a pressing need for a method to deal with the limiting effects of topographic resolution.

The effect of pixel resolution is illustrated in figures 1 and 2. Ideally, digital elevation data and model grids would be sufficiently high resolution (1A) to resolve the detailed morphology of channels. The fluxes of water and sediment over the surface could be calculated (1A-C) with enough precision to permit the meaningful estimation of the rate of energy dissipation across the channel bed. Consequently, rates of bedrock erosion through abrasion, plucking, and other mechanisms could be calculated and related to the results of field studies of these processes. No assumption about channel width and depth would need to be made because the channel geometry would be resolved. At present, geomorphic model grids at such a high resolution are computationally impractical, unless the grid is restricted to a very small catchment, and the quantitative form of the geomorphic processes that operate at this scale remains a matter of dispute. Digital elevation data at this resolution is extremely limited and will remain so for the foreseeable future. So, if we are to relate field observations of bedrock channel processes to long-term landscape evolution, we will require a parameterization scheme to span the great differences in their temporal and spatial scales.

A representation of a more typical geomorphic model grid is shown in figure 2. Pixels of the order of 100 m to 1 km in size are used to resolve the surface elevation and the pattern of flows across the surface, both on the hillslopes and in the channels. Such coarse pixels in general cannot distinguish channels from hillslopes and certainly cannot resolve channel morphology. This limitation is clarified if we adapt the pixel grid into a mesh of nodes (pixel centers) and links (pixel-to-pixel flow connections). The pattern of flows between pixels can be delineated, and the approximate discharge through each link can be calculated. However the channel flux, which is the ratio of
discharge to channel width, cannot be estimated directly at each link, because the percentage of each pixel over which channel flow is distributed is not resolvable. A predictive, sub-pixel scale parameterization of channel geometry is therefore required for each link. As is discussed in the next section, the typical method is to freeze the channel geometry so that channel width and depth are proportional to peak discharge. In this study we intend to demonstrate that other parameterizations are possible and indeed desirable. A more flexible description of sub-grid scale properties allows for geomorphic models that include the dynamic nature of channel geometry and its connection to landscape evolution as a whole. Furthermore, it could incorporate an adjustment for the effects of coarse pixel resolution discussed above. To address the latter issue we study in detail the effects of pixel resolution on the estimation of surface flow properties. We undertake a scaling analysis of digital elevation model data and examine the resolution bias on distributions of measures such as channel slope and flow accumulation (a proxy for peak discharge). Before discussing the scaling analysis, we consider first an implicit parameterization of channel properties used in most geomorphic models.

Channel equilibrium and sub-grid scale parameterization.—A close scrutiny of the majority of surface process models published in recent years (Beaumont, Kooi, and Willett, 1998; Densmore and others, 1997; Giacometti, Maritan, and Banavar, 1995; Howard, 1994; Kooi and Beaumont, 1996; Tucker and Slingerland, 1996; Willgoose, Bras, and Rodriguez-Iturbe, 1991a) reveals a common assumption. A set of coupled equations is written to describe the relationships between properties such as channel width, depth and slope, hydraulic radius, bed roughness, mean flow velocity, bankfull
discharge, and others. Some of these equations have a strong theoretical basis, others are based on model fits to observations made at gauging stations. Almost all the equations originate in studies performed on alluvial rivers, and very few are appropriate for modelling processes in the bedrock rivers that predominate in mountains (Burbank and others, 1996; Howard, 1994; Howard and Kerby, 1983; Kelsey, 1988; Montgomery and others, 1996; Seidl and Dietrich, 1992; Seidl, Dietrich, and Kirchner, 1994; Sklar and Dietrich, 1998; Tinkler and Wohl, 1998; Weisсл and Seidl, 1997, 1998). These equations cannot be written in a closed form without extra assumptions. By this we mean that there is no unique expression relating, for example, the channel slope to peak discharge, unless an extra equation is provided in order to simplify the equation set.

The extra equation generally chosen specifies that the cross-sectional channel geometry must take an equilibrium form; generally, channel width is taken to be linearly proportional to channel depth (Smith and Bretherton, 1972). Observational data for alluvial channel geometries corroborate this assumption to some extent, although the considerable scatter in such regressions suggests that significant deviations from equilibrium are common. In bedrock rivers, this assumption is difficult to justify, because equilibrium channel form is rare if not meaningless. The rate of erosion of a bedrock channel is the same as the rate of channel deepening, so a model assumption of channel equilibrium imposes equal rates of model channel margin degradation and model hillslope response. In other words, a very tight coupling is imposed between the model rate of channel erosion and the model rate of hillslope erosion. Real mountain landscapes are apparently much more loosely coupled.

The discussion in the preceding section emphasized the connection between assumptions about channel geometry and the need in geomorphic models to approxi-
mate channel features that cannot be resolved at the model grid scale. Clearly, an assumption of channel equilibrium is an elegant but restrictive solution to this problem. The next section takes a step back and examines in depth the effect of grid resolution on our ability to quantify surface flow properties. Insights gained from this analysis are subsequently used to develop a fresh approach to the channel parameterization problem, and a geomorphic model is built to incorporate this new approach.

**Scaling Analysis of Relief**

A well-established approach to topographic analysis is to process a digital elevation model, or DEM, and map properties such as topographic slope (gradient) and upstream (accumulation) area. Slope and area are generally plotted in log-log form in order to characterize a power-law relation between these properties (Montgomery and Dietrich, 1994; Willgoose, Bras, and Rodriguez-Iturbe, 1991b). The resulting power-law is considered to be evidence of an underlying power-law dependence of channel slope on peak annual discharge. Despite the enormous scatter in such plots, this power-law dependence is correlated with empirical sediment transport relations obtained by linear regression of stream gauge data in log-log form. In the stronger interpretations of these data, the power-law relation between slope and area is thought to indicate a “stream power law.” Deviations from a power-law trend have been considered a deviation from this “law.” In the context of steady-state landscapes these deviations have been interpreted as deviations from steady-state.

Our topographic analysis also focuses on patterns of flow and slope estimated on a DEM, but we take a probability approach that treats explicitly the variation of these properties around any inferred power-law trends. This approach is summarized in figure 3. The scatter plot in figure 3A, top right, is a rotated form of the traditional slope-area plot that is typically obtained from DEM analysis. There are, however, some differences. We obtain the topographic slope, $S$, by following flow patterns determined by the DEM and taking the surface gradient along the estimated direction of local flow. These slopes are different from slopes estimated by local finite difference approximations and are more closely related to the gradient experienced by flows on the surface; the differences are, however, slight. The upstream area, $A$, is written as the area drained per unit contour length or pixel width and is estimated using a multiple outflow routing algorithm similar to that of Quinn and others (1991), after first performing pit removal using a method analogous to that of Garbrecht and Martz (1997) and Martz and Garbrecht (1998).

The values of slope and area at each pixel are very broadly scattered on this log-log axis graph. The frequency of observed values is expressed as a probability density, shown in this graph by the overlaid contours. This density was estimated using a simple non-parametric method, called kernel density estimation, which involves first projecting the scattered points onto a fine binning mesh and second smoothing out the bin densities by convolution with some kernel. In this case, we employed a Gaussian kernel to obtain the final bivariate probability density, which means that we assumed, for simplicity, that the dispersion in pixel values of both slope and area is log-normally distributed. In practice the choice of error kernel is moot, as long as the sample size is large and the variance of the error kernel is small relative to the data ranges, as was the case here.

The first point to observe is that we are treating the slope-area data as a joint, bivariate probability density, rather than simply attempting to fit a power-law model by linear regression of the log values. A maximum likelihood model is nevertheless plotted on (A) as a linear trend following the highest local probability density, but for guidance only. What we wish rather to emphasize here is that the variation around this model is enormous and must be accounted for.
The axes on graph 3A are log values of stream power and stream ratio, which we define as \( \psi = AS \) and \( \varphi = A/S \) respectively. The parameter \( A \) here is area per pixel width, which is equivalent to upstream area drained per unit contour length when estimated using a multiple outflow algorithm (Wolock and McCabe, 1995). This graph therefore has some useful properties:

1. The diagonal axis \( \log(\psi) = \log(\varphi) \) is the area axis \( \log(A) \).
2. The diagonal axis \( \log(\psi) = -\log(\varphi) \) is the slope axis \( \log(S) \).
3. The horizontal axis parameter \( \log(\varphi) \) is exactly equivalent to the topographic index \( \chi = \log(\varphi) = \log(A/S) \) of Beven and Kirkby (1979).

4. We can switch between slope versus area and stream power versus stream ratio by a simple 45° rotation of axes.

5. The pdf of topographic index, or stream ratio as we call it here, is seen to be related directly to the slope-area properties of the DEM — specifically, the pdf of topographic index is the marginalization of the rotated joint density of log slope and log area.

The adjacent graphs 3B and C illustrate the marginal densities of log stream power and stream ratio. A marginal density is a probability density function (pdf) obtained by integrating out a number of components of variation of a joint density. In this case, we integrate out the stream power component of the bivariate density to obtain the univariate stream ratio pdf; marginalization of stream power is performed by integrating out the stream ratio variation. While marginalization is a basic element of probability theory, it is worth explaining in detail because it links clearly two radically different model uses of the same data. In TOPMODEL (Beven, 1997; Beven and Kirkby, 1979; Quinn and others, 1991, Quinn, Beven, and Lamb, 1995), the variability of the topographic index is employed directly in the model treatment and is the means by which the geomorphology of a catchment is summarized; the joint variation of slope with area is not considered. Conversely, in “stream-power law” models, the variability is entirely ignored, while the systematic joint dependence of slope on area alone is employed. A reconciliation of these mutually exclusive approaches would be welcome.

The remaining insets, displayed in 3D, are the result of marginalizations diagonally. These marginal pdfs are of log slope and log area. The pdfs in all the graphs in figure 3 are calculated using natural log values of the ordinate parameters, and the probability densities are correspondingly of the log values. There is a significant difference between the probability density of a log parameter and the log probability density of that parameter. This difference is often ignored when interpreting log-log scatter plots and when considering properties such as the mean or variance of pdfs of logged parameters. Expressed mathematically, the density of some parameter \( x \) is related to the density of its logged value by the transformation:

\[
p(y) = f_{y|x}(y)p(x) = \left| \frac{dx}{dy} \right| p(x)
\]

where \( f_{y|x} \) is the Jacobian for the coordinate shift from \( x \) to \( y \). In this case,

\[
y = \log(x) \Rightarrow p[\log(x)] = p(y) = xp(x).
\]

In other words, the marginal densities shown in figure 3 are scaled forms of the raw densities. The statistics of these pdfs are correspondingly distorted. So, care must be taken when interpreting such graphs, unless of course the parameter in which we are interested is, in fact, the log value. This is generally the case in TOPMODEL applications, where the log value of \( A/S \) is assumed to relate directly to the baseflow (Sivapalan, Beven, and Wood, 1987). In other cases, log values are considered only for ease of visualization.

**Digital elevation model of Taiwan.**—The data source for our topographic study is a high resolution DEM of Taiwan. The Central Range of Taiwan may be in a steady-state (Suppe, 1981, 1984). This orogen (Chai, 1972) exhibits denudation rates in excess of 5mm/yr and sediment yields exceeding \( 10^4 \)t/km² for some catchments (Li, 1976; Milliman and Syvitski, 1992; Hovius and others, 2000). Analysis of the relief of the
Central Range is therefore an ideal constraint on a geomorphic model designed to simulate steady-state landscapes.

The horizontal precision is 40 m per pixel, with a vertical precision that varies across the DEM but is nominally about 20 m. It was constructed by mosaicking existing contour maps converted to digital form using a mixture of methods and is of a similar quality to the United States Geological Survey 30 m resolution, 7.5 min quadrangle DEMs currently available for the contiguous United States. The source DEM was provided on a local, non-standard UTM projection, which was subsequently re-projected onto UTM zone N51. One region in particular was chosen for this study: an area covering the Hoping catchment, located at the northeastern end of the Central Range. The chosen grid was of 1024 × 1024 pixels or 40.96 × 40.96 km. This catchment drains directly into the Hoping Basin through a very small fan delta. Elevations range from 0 to 3590 m above sealevel. This watershed, as with others draining the east of the Central Range, is very steep, and erosion is rapid: hillslope erosion processes are dominated by bedrock landsliding, and bedrock channels are common (Hovius and others, 2000). Intramontane storage of colluvial and alluvial sediments is minor relative to the fluxes of sediment out of the catchment each year: sediment yields for catchments along the eastern Central Range are all of the order of ~10,000 t/km²/yr (Milliman and Syvitski, 1992; Water Resources Planning Commission, 1972-1997). The bulk of the erosion occurs during the annual monsoon, when one to three typhoons typically strike the island, principally from the western Pacific (Water Resources Planning Commission, 1989).

A multi-resolution topographic analysis.—The resolution of digital elevation data is insufficient to resolve channel morphology, but it has been argued that the crossover between hillslope and channel regimes can be identified on DEMs. Slope-area plots typically show a strong clustering around a peak, which in some cases marks a switchover between a linear relation, \( S \sim A \), and a negative power-law trend, \( S \sim A^{-n} \). The inference is that, even at pixel resolutions of \( \sigma = 10 \) to 100 m, the slope and upstream area values approximated on the pixel grid are determined by the dominant process within each pixel. The linear component is thought to reflect pixels dominated by diffusive hillslope processes; the power-law component is considered an indication of pixels dominated by channel transport processes. More complex models identify intermediate components of the slope-area relation and associate these with surface wash or mass-wasting. The two-component, hillslope-channel model is illustrated in figure 3A, where a line is plotted following the maximum likelihood (peak probability density) of the slope-area joint distribution.

The main purpose of our topographic study is to establish a means of rescaling DEM-derived parameters so that their distributions are independent of DEM resolution (Wolock, 1998). These scale-corrected parameters will be a much more reliable description of the state of the landscape than resolution-dependent parameters. Furthermore, such rescalable parameters will be ideal candidates for model variables that describe the state of an evolving landscape.

The method that we use is very simple. A smoothing function is chosen to mimic the effect of a loss of resolution through blurring. The DEM grid is smoothed by convolution with this function, and a coarser resolution DEM is the result. For simplicity we have chosen to use a Gaussian smoothing function; the Gaussian function is the Green’s function for the diffusion equation (the fundamental solution for the spreading of an initial spike input). The effective resolution of the smoothed grid is set by the standard deviation \( \sigma \) of the Gaussian.

The scaling effect of subsampling is not examined empirically in this analysis because we wish to concentrate specifically on the following problem. Ideally, we would like to be able to take a coarse pixel, coarse resolution DEM \((L_c, \sigma_c)\), to interpolate this
onto a fine pixel, coarse resolution grid \((L_f < L_c, \sigma_f)\), and then to predict the surface morphology at a high resolution on this grid \((L_f < L_c, \sigma_f < \sigma_c)\). In practice, of course, the second step will only be possible in a probabilistic sense. In order to find out how, our method is to perform the operation empirically in reverse, we take a fine pixel,

Fig. 4. Multi-resolution DEM analysis of Hoping catchment: topographic index, or log stream ratio, is shown as the color attribute on shaded topography. The resolutions illustrated here are: (A) \(\sigma = 117\) m, (B) \(\sigma = 342\) m, and (C) \(\sigma = 1\) km. The red lines are a 10' interval latitude and longitude mesh; the region shown in each figure is the same 32.45 km \(\times\) 32.45 km square, spanning most of the Hoping watershed.

Fig. 5. Estimates of the joint probability density of log stream power \(\log(w_s)\) versus log stream ratio \(\log(c_s)\) (topographic index) from flow routing on DEMs at the resolutions shown in figure 4: (A) \(\sigma = 117\) m, (B) \(\sigma = 342\) m, and (C) \(\sigma = 1\) km. The contours and colors indicate probability density calculated using kernel density estimation. This is a method of binning which involves a convolution of the scattered point estimates \(\log(c_s), \log(w_s)\) with an error kernel that represents an estimate of the imprecision associated with each point location. The result is a smooth characterization of a pdf and is a significant improvement over simple binning.
high resolution grid \((L_f, \sigma_f)\) and examine the effect of losing detail when moving to a fine pixel, low resolution grid \((L_s, \sigma_s)\). Our hope is that distributions of topographic measures estimated at high resolution \((\sigma_f)\) can be quantitatively related to those estimated at a low resolution \((\sigma_s)\) in such a way that the operation can be reversed, and a prediction of fine scale detail can be made from coarse scale information.

A sequence of Gaussian approximations of the Hoping region was performed: the result was a set of DEMs with simulated resolutions between \(\sigma = 40 \text{ m}\) and \(\sigma = 1000 \text{ m}\) (fig. 4). DEM correction (pit removal) and flow routing were performed on each of these grids, and the slope and upstream area were estimated. Note that for multiple outflow routing algorithms (Freeman, 1991; Quinn, Chevalier, and Planchon, 1991; Tarboton, 1998) such as that applied here, the “area” value is a diffuse measure designed to estimate the average water flow across the pixel (given constant rainfall per unit area, zero water loss through infiltration and evapotranspiration, and neglecting inertial effects). Since we seek parameters that best describe the hydrological and geomorphological properties of the surface, this “flow” measure is more appropriate than accumulation area, which is the model flow for single outflow routing algorithms (Jenson and Domingue, 1988). In practice, the derived probability densities of stream power and stream ratio are largely insensitive to this choice (Wolock and McCabe, 1995).

Figure 4 illustrates the pattern of flow and the shape of the resolved topography at selected resolutions \(\sigma = 117 \text{ m}\), \(\sigma = 342 \text{ m}\), and \(\sigma = 1000 \text{ m}\). The success of the multiple outflow routing algorithm is demonstrated by the diffuse pattern of estimated flow on the smoothed surfaces; this feature is particularly clear in figure 4C. The color attribute indicates the log value of the stream ratio, \(\log (\varphi) = \log (A/S)\), also known as the topographic index \(\chi\). These results were used to estimate the joint density of stream power and stream ratio; example joint pdfs are illustrated in figure 5, at the same selected resolutions.

As was explained in the previous section, \(p[\log (\psi_{s}), \log (\varphi_{s})]\), the joint probability density of log values of stream power, \(\log (\psi_{s}) = \log (A_{s}/S_{s})\), and stream ratio, \(\log (\varphi_{s}) = \log (A_{s}/S_{s})\), is directly related to the joint density for log slope and area, \(p[\log (S_{s}), \log (A_{s})]\), by a simple rotation of axes by 45°. Therefore figure 5 is both a representation of the variations of stream power with stream ratio and of the variations of slope with area. The subscript \(s\) here indicates the resolution (length scale) of the DEM at which these values were estimated.

A key point to note in these analyses is that as the resolution decreases (as \(\sigma\) grows) the shape of the joint density changes. In particular, the two components of the \(\psi - \varphi\) dependence become clearer: in figure 5C the diffusive flow component (\(\varphi = \text{constant, vertical trend in peak probability, } S \sim A\)) and the negative power-law component (diagonal trend in peak probability, \(S \sim A^{-n}\)) become more distinct — refer to the schematic figure 3 if these components are not clear. This property is not surprising, because the diffuse character of the topography increases as the degree of smoothing increases. The separation of these two “flow” regimes in this analysis vindicates to some extent the hope that hillslope and channel morphologies may be identified on slope-area plots.

We can obtain a clearer picture of the resolution dependence of the surface properties by marginalizing the joint pdfs. The resulting marginal pdfs for log measures of slope, area, stream power, and stream ratio are shown in figure 6. The systematic resolution dependence of each pdf, in particular the mode (peak) and mean of each distribution, is clear in each case. The average slopes appear to increase as the resolution improves (as \(\sigma\) decreases); average drainage area (flow) decreases as resolution improves. Both stream ratio and stream power decrease with improving resolution; however, the resolution dependence of stream power is relatively weak.
Fig. 6. Marginal pdfs of (A) slope, (B) area, (C) stream power, and (D) stream ratio, for the Hoping catchment. The probability densities of the log measures are shown, on linear axes, for a range of DEM resolutions, from 40 m to 1 km.
Remember that the probability density (eq 2) of some log measure, \( p[\log(x)] \), is a scaled version of the corresponding probability density of the raw measure \( xp(x) \). Therefore a raw density, when plotted using a power-law \( x \) axis, will still differ from the density of the log value plotted on a linear axis. The statistics of the raw density and the log density are also systematically different. The effect of the implicit log transformation of probability densities is often ignored when topographic properties are plotted on log-log axes, and then power-law properties are assessed.

The resolution dependence of the stream ratio probability density is strong, but the distribution can be rescaled to a resolution-invariant form using the following method:

1. Identify the location of the mode of the distribution, which is the value of the stream ratio at which the density has a peak
   \[
   \bar{\varphi}_0 = \varphi_0 \sup \{ p(\varphi) \} 
   \]  
   (3)
2. Regress the modal stream ratio \( \bar{\varphi}_0 \) against resolution \( \sigma \) (see fig. 7)
3. Fit a suitable model to this resolution dependence, in this case
   \[
   \bar{\varphi}_0 = \bar{\varphi}_0 \left[ 1 + \left( \frac{\sigma}{\sigma_r} \right)^\gamma \right] 
   \]  
   (4)
4. Rescale the stream power axis using this calibration model
   \[
   \bar{\varphi}_0 = \bar{\varphi}_0 \left[ 1 + \left( \frac{\sigma}{\sigma_r} \right)^\gamma \right] 
   \]  
   (5)
5. Rescale the corresponding probability density using this calibration model and the pdf transform method (eq 2)
   \[
   p(\bar{\varphi}) = \left( \frac{\bar{\varphi}}{\bar{\varphi}_0} \right) p(\bar{\varphi}_0) 
   \]  
   (6)
If the recalibration model is correct, we will obtain,
\[
  p(\bar{\varphi}) \sim p(\varphi_0) \quad \text{for all } \sigma
\]  
   (7)
where the symbol \( \sim \), denotes “distributed as.” In this way, we will approximate the probability density of stream ratio at perfect DEM resolution from each rescaled, coarse resolution density.

The application of this method to the Hoping analysis is illustrated in figures 7 and 8. The multi-resolution estimated, modal values of the stream ratio \( \bar{\varphi}_\sigma \) are plotted (small circles) in figure 7 against resolution \( \sigma \). A non-linear regression fit of the scaling model (eq 4) is shown spanning the range of observed resolutions 40 to 1000 m and extrapolated downscale to \( \sigma \rightarrow 0 \). The model fit in this case took the parameters,
\[
\bar{\varphi}_0 = 148.5 
\]  
   (8)
\[
\sigma_r = 225.6 
\]  
   (9)
\[
\gamma = 1.70 
\]  
   (10)
The parameter \( \bar{\varphi}_0 \) is the asymptotic value of the modal stream ratio in the limit of perfect resolution \( \sigma \rightarrow 0 \). The parameter \( \sigma_r \) is a scale threshold below which the stream ratio density begins to converge to its asymptotic, perfect resolution limit; above this threshold the modal stream ratio grows as a power function of resolution \( \sigma \). It is an open question as to whether this threshold \( \sigma_r \) is a significant topographic measure. It
may represent a characteristic length scale of the topography, indicating a crossover either between different geomorphic processes or between different morphologies. Conversely, there is the less interesting possibility that this threshold relates more to the procedure of multi-resolution estimation than to anything physical. Nevertheless, this scaling equation is very important: it demonstrates that it is possible to recalibrate a stream ratio pdf estimated on a DEM of coarse resolution to that of a pdf estimated at a resolution close to ideal. The corresponding statistics of this pdf such as mean, variance, and skewness can easily be obtained from the recalibrated pdf. The stream ratio probability density can also be transformed to that of the log value or topographic index. Therefore this technique will be an invaluable tool for making scale corrections of maps of topographic index calculated on coarse resolution DEMs. Note, however, that this recalibration method does not solve the problem of variable grid spacing, which may require a further correction model to deal with subsampling artifacts.

The scaling exponent in the recalibration model (eq 4) is probably related to the scaling properties of the topography itself. For example, if the Hoping topography exhibits any self-affine scaling (Huang and Turcotte, 1989; Weissel, Pratson, and Malinverno, 1994), there will be a power-law dependence of slope on resolution. The dependence on upstream area is less clear. Such issues merit further investigation, because an understanding of the origin of this scaling would provide a theoretical underpinning of the empirical recalibration method presented here.

Fig. 7. A regression of stream ratio $\bar{\phi}_\sigma$ against resolution $\sigma$. The multi-resolution estimated, modal values of the stream ratio are plotted as small circles. A non-linear regression fit of the scaling model (eq. 4) is shown spanning the range of observed resolutions 40 to 1000 m and extrapolated downscale to $\sigma \rightarrow 0$. The model fit in this case took the parameters, $\bar{\phi}_0 = 148.5$, $\sigma_r = 225.6$, and $\gamma = 1.70$. 
The task now is to adapt the results of the DEM topographic scaling analysis to the needs of geomorphic modeling: in particular, we need to link fine-scale topographic and channel network structure into a coarse-scale geomorphic model through some sort of sub-grid scale parameterization scheme. There is no need at this stage to develop a fully-fledged new landscape evolution model to accomplish this task. Nor do we need to design a properly calibrated, sub-grid scale parameterization of channel properties based explicitly on the scaling analysis of the previous section. Instead, it will be sufficient to present a prototype geomorphic model that incorporates some mechanism for modeling channel disequilibrium, links this property to the rates of channel and hillslope erosion, and does so through the use of parameters whose dependence on pixel resolution can be eliminated. We accomplish this task by basing a geomorphic model around the DEM-derived and scale-corrected measures discussed in the previous section and by formulating the governing equations with the model grid topology of nodes and links (fig. 2) explicitly in mind.

The DEM multi-resolution analysis has shown that (1) topographic resolution imposes a systematic bias on the distributions of derived measures such as stream

![Graphs showing recalibrated marginal probability densities of log stream ratio and stream ratio](image)
power and stream ratio (topographic index); (2) raw densities and log densities can be used in tandem to quantify this resolution bias; (3) the distribution of the stream ratio or stream ratio can be corrected for scale bias; (4) the distribution of stream power needs only weak scale correction, because its raw probability density is approximately resolution invariant. The stream ratio and stream power measures at coarse pixel resolution contain enough topographic information to permit a sub-pixel description of the topography and related drainage structure in a distributional sense. They do not, of course, provide explicit information on channel morphology. In the future, more sophisticated parameterization schemes may address that issue, and these would require the scaling analysis of very high resolution DEMs in order to produce scaleable measures of channel geometry. In this study we shall attempt only to introduce an ad hoc parameterization of sub-pixel channel structure that can be tied into a working geomorphic model.

The channelization index.—At this point it is worth emphasizing once again that a geomorphic model based on a coarse pixel grid cannot adequately resolve channel geometries or the spatial distribution of surface flows (fig. 2). This problem has typically been solved by assuming channel equilibrium to allow the formulation of a closed set of non-linear equations that describe sediment transport and bedrock erosion. If this assumption is dropped, we are left with an under-determined set of equations. The problem is highlighted in the meshing scheme illustrated in figure 2: on such a grid we can resolve only the directions of flow between nodes of known elevation through simple links representing channel reaches. It is our suggestion that the unresolvable channel properties can be aggregated into representative link properties through a sub-grid scale parameterization. These link properties permit the formulation of a fully determined, closed set of equations representing transport and erosion processes consistent with those used in existing geomorphic models.

We define a single channelization index $T$ as the lumped parameter that reflects the ensemble behavior of all the geometric properties of a channel reach that mediate the flow of sediment passing through it; more sophisticated parameterizations may be necessary in the future. This dimensionless index is intended to encompass all the non-linear flow-limiting properties of a channel reach. It quantifies the degree of disequilibrium of the channel geometry with respect to the hillslope and channel sediment fluxes (Montgomery and Dietrich, 1988, 1992). Its properties are discussed in detail below.

A geomorphic model incorporating channelization.—We present a simple geomorphic model that uses the channelization index as a way of solving the channel resolution problem of landscape evolution simulated on a numerical grid. The governing equations of this model are derived from first principles in such a way that the requisite “stream power law” empirical constraints are obtained under certain model conditions. For example, a power law relationship between sediment discharge and channel slope is obtained under stable channelization, which is the condition of local channel equilibrium.

By far the biggest simplification of this model is that it simulates only the flux of sediment: the flux of water is not explicitly modeled. This simplification may seem ironic in the light of our emphasis here on the need to model the details of surface fluxes and channel dynamics. However, a single flux model is very instructive, for a number of reasons. First, it demonstrates how easy it is to reproduce the morphology of mountain landscapes by simply ensuring that the model equations enforce (1) flow convergence and (2) a channel slope inverse-dependence on upstream area. This serves as a salutary reminder of how difficult it is to use topographic (DEM) properties of slope and upstream area to constrain the physics of erosion and sediment transport. Second, the sediment flux model demonstrates that “tools” based models may be valid
after all, that is, that the morphology and scaling of landscapes are consistent with a model where the rate of bedrock channel erosion is determined by the rate of wear by sediment abrasion and related processes. Third, by keeping the mathematical complexity to a minimum, it allows ready comparison with related models of channel network physics (Banavar and others, 2000).

The basic equations.—The starting point for our model is a relationship between the flux of sediment $q$ through a link of channelization $T$ between nodes with an elevation difference $S$

$$ q = \kappa(T)S. \quad (11) $$

It is important to realize that all these measures are resolved at the pixel resolution $\sigma$ of the numerical grid used to perform the geomorphic simulation. In general, the grid spacing $L$ and the pixel resolution $\sigma$ are equal, $L \sim \sigma$. The flux of sediment $q$ is not the flux over the channel surface; rather, it is the discharge of sediment per unit pixel width (equivalent to contour length), $q \equiv Q/L$. The link slope is also grid resolution dependent — it is the mean potential gradient along the principal channel segment between the nodes, $S \equiv \nabla h$.

The rate at which sediment can pass through the channels parameterized by $T$ is described by the rate parameter $\kappa(T)$. The channelization index is defined through its exponential dependence on this rate parameter,

$$ \kappa(T) = \kappa_0 \exp(\alpha T). \quad (12) $$

These measures are resolution dependent: if the pixel size were to change, both the channelization indices and the corresponding rate parameters would have to be recalibrated. Our model description of the physics of erosion and sediment transport is contained almost entirely in the formulation of the channelization index. Remember that the channelization $T$ represents the ease with which sediment can flux through a channel reach. We assume that the dynamics of the channel reach properties can be represented through a simple differential equation describing the increase and decrease of the channelization index. We further assume that (1) the energy supply to the channel reach increases the degree of channelization through channel deepening and widening; (2) the channel reach degrades, and sediment flux is inhibited, by bank collapse and other related processes. In reality the dynamics of channelization is far more complex, but this formulation suffices for the present study.

The differential equation for channelization becomes a balance between positive and negative feedback processes,

$$ \frac{dT}{dt} = \mu qS - \eta \exp(\beta T). \quad (13) $$

Channelization $T$ grows at a rate proportional to the local rate of work done $qS$ and is balanced by a dissipation term that itself grows exponentially with $T$. This specific form for the channelization dynamics is chosen to make the asymptotic, equilibrium behavior of the geomorphic model consistent with empirical relations between such measures as sediment flux, channel slope, and upstream area. It is also chosen so that the rescaleable DEM measures of stream power and stream ratio, explored in the topographic scaling analysis, can be related to the basic behavior of the geomorphic model,

$$ \frac{dT}{dt} = \mu qS - \eta \left(\frac{q}{\kappa_0 S}\right)^{\beta/\alpha}. \quad (14) $$

A channelization model of landscape evolution
since at steady-state, for $A \sim q$, we have (sediment) stream power $\varphi \sim qS$ and stream ratio $\varphi \sim q/S$.

Finally, mass conservation requires that the surface sediment flux is the balance of material supply rate $U(x; t)$ and the erosion rate at each node

$$\nabla \cdot \mathbf{q} = U - \rho \frac{\partial h}{\partial t}. \quad (15)$$

### Sediment supply: weathering versus transport limits.

In this geomorphic model, the sediment supply from hillslopes and from channels is explicitly separated: hillside erosion and sediment yield is quantified at each node; channel erosion and sediment yield are encapsulated in the channelization index. The assumption behind this is that the rate of sediment supply from channel erosion is negligibly different from the rate of hillslope erosion. This is not inconsistent with channel disequilibrium so long as the fraction of sediment supply from channel bed erosion is very small compared to that from the hillslopes. So, the geomorphic model cannot simulate deeply incised gorges or landscapes where the channels are very strongly out of equilibrium with the hillslopes.

The mass balance equation (15) also requires that the supply of sediment at each node, that is, on the hillslopes, is not weathering-limited. Therefore the geomorphic model cannot simulate bedrock hillslopes where the development of regolith and soil, on the pixel scale, is a limiting factor on the rate of erosion. Nevertheless, a broad range of mountain landscapes is within the scope of the model. For example, despite the limitations discussed here, the model can simulate hillside erosion by shallow soil and bedrock landsliding. The parameterized modeling of specific geomorphic processes is determined by the choice of exponents $\alpha$ and $\beta$ and is discussed in more detail below.

### Local channel-hillslope equilibrium.

When the channelization $T$ stabilizes and no longer varies with time,

$$\frac{\partial T}{\partial t} = 0. \quad (16)$$

This local equilibrium represents a balance between hillslope erosion and channel erosion and sedimentation, where the channel geometry below the pixel (link) scale has adapted to the fluxes across it. By applying this condition to eq (13) we obtain a power-law relationship between sediment flux and channel slope,

$$q^{\beta-a} = \kappa_0^{\beta} \left( \frac{\mu}{\eta} \right)^\alpha S^{\beta-a} \quad (17)$$

which can be simplified into

$$q = \kappa S^n \quad (18)$$

where

$$n = \left( \frac{\beta + \alpha}{\beta - \alpha} \right) \text{ and } \kappa = \left[ \kappa_0 \left( \frac{\mu}{\eta} \right)^\alpha \right]^{\frac{1}{\beta-a}}. \quad (19)$$

This power-law relation $q \sim S^n$, which is of the form of the so-called “stream power law”, is obtained if and only if the hillslopes and channels are in equilibrium: that is, when channel geometries have reached a stable, asymptotic form, and the rate of erosion of channels equals that of the coupled hillslopes. It will produce landscapes with
slope-area relations of the corresponding power-law form of $S \sim A^n$, with a negative or positive exponent $n$ depending on the scaling of the dominant transport process within each link, parameterized by $\alpha$ and $\beta$.

Fluctuations away from channel equilibium, which will certainly occur when boundary conditions change, will cause exponential changes in the channel stream power. Therefore the broad scatter typically observed in DEM-derived slope-area data (for example, fig. 3) may reflect widespread channel disequilibrium in addition to substrate heterogeneity.

Channel flow resistance.—Another power law is also obtained when a channel link is at equilibrium: between inverse channel resistance $\kappa$ and sediment flux $q$,

$$\kappa(q) = \kappa_q q^r$$

where

$$r = \left(\frac{2\alpha}{\alpha + \beta}\right) \quad \text{and} \quad \kappa_q = \left[\kappa_0 \left(\frac{\mu}{\eta}\right)^{\alpha + \beta}\right]^{-\frac{1}{\alpha + \beta}}.$$

This power-law dependence $\kappa \sim q^r$ of the link rate parameter on the sediment flux demonstrates clearly how the parameters $\alpha$ and $\beta$ determine the type of transport process across a link. The issue then becomes: how to represent particular transport processes and under what ranges of values of channelization $T$.

A natural threshold arises in our model where the channelization $T$ crosses zero. In our numerical solutions of this geomorphic model we have used this critical value of channelization to define a crossover between hillslope processes such as rain-splash and creep and mass-wasting and channel processes.

Diffusive hillslope transport.—Rain-splash and soil creep are modeled as diffusive transport mechanisms at the link scale. Using eqs 11 and 20 we can see that a linear, diffusive sediment transport equation will arise when $\kappa(q)$ is constant and independent of $q$, that is,

$$r = \left(\frac{2\alpha}{\alpha + \beta}\right) = 0 \Leftrightarrow \alpha = 0$$

which in our numerical model occurs when $T < 0$. As a result,

$$\kappa = \kappa_q = \kappa_c = \kappa_0 \Leftrightarrow q = \kappa_0 S.$$  \hspace{1cm} (23)

Hillslope mass wasting.—An alternative link transport regime can be simulated for $T < 0$. Using eq 17 we can see that if

$$\alpha = \beta$$

then the hillslope gradient is independent of sediment flux and at equilibrium (eq 16) takes the constant value of

$$S = \left[\frac{1}{\kappa_0} \left(\frac{\eta}{\mu}\right)^{\alpha + \beta}\right]^{-\frac{1}{\alpha + \beta}} \rightarrow S_0 = \left(\frac{\eta}{\kappa_0 \mu}\right)^{1/2}$$

where the parameter $\kappa_0$ is a measure of erodibility.

This is an interesting result, because it states that once an equilibrium between hillslope and channel is achieved, the flux of sediment from the hillslope is determined entirely by the rate of channel incision. The rate of hillslope erosion does not determine equilibrium angle of the hillslope; instead it is determined entirely by the material properties of the soil and bedrock parameterized by $\{\eta, \mu, \kappa_0\}$. Therefore in
this model there is an asymptotic hillslope gradient, but there is no threshold hillslope gradient (Schmidt and Montgomery, 1995). It is not possible to distinguish the two alternative models morphologically, either in the field or in DEM analysis.

We infer that this parameterized transport process describes pixels dominated by mass-wasting. Since the gradients tend toward an asymptotic value, model solutions of
this equation will yield rectilinear slopes. This property is illustrated in the numerical model examples demonstrated in figure 9A, B. This figure also demonstrate that model solutions that include the behavior $\alpha = \beta$ are computationally stable.

**Channel transport.**—Channel dominated pixels are described by a non-linear relation between channel conductance and surface flow (eq 20). Under such conditions, a perturbation increase of the flow will cause an amplified perturbation increase in the conductance of the channel, through enhanced channel incision. This in turn brings about increased flow into the channel and further amplification of channel conductance. This positive feedback is a process of non-linear flow weakening and flow focusing, and it illustrates how channels initiate and grow following small, local fluctuations in surface flux (Smith and Bretherton, 1972).

In the simple version of our model presented here a channelized regime arises for pixels with $T > 0$. In eq (20), non-linear amplification of channel conductance occurs for

$$r = \frac{2\alpha}{\alpha + \beta} > 1 \Rightarrow \alpha > \beta.$$  

Eq 20 indicates that when this is the case, flow focusing will occur, and channels will develop. This inference is borne out by numerical experiments. We can constrain possible values of $\alpha$ and $\beta$ using slope-area results from topographic (DEM) analyses. From eq 19 we can derive

$$\frac{\alpha}{\beta} = \frac{n - 1}{n + 1}$$  

where $n$ is the exponent in the power-law slope-area relation illustrated in figure 3A. In general, DEMs indicate that

$$-3 \leq n \leq -2$$

for channelized links. Thus

$$2 \leq \frac{\alpha}{\beta} \leq 3$$

are reasonable bounds on the ratio of $\alpha$ to $\beta$ for modeling channels.

**Numerical solution: resistor-capacitor networks**

We take a novel approach to solving the coupled system equations that describe the channelization model. Rather than using finite difference grids or adaptive finite element meshes, we employ a network of non-linear resistors and capacitors. The state variables and model equations are converted into equivalent electrical analogs, and the evolution of the system is solved using an iterative, approximate, non-linear conjugate gradient scheme (Press and others, 1992). The correspondences between the geomorphic model parameters and their electrical equivalents are listed in table 1. Here are some highlights of this numerical scheme:

1. Each node is tied to ground through a (linear) capacitor.
2. Topographic elevation is mapped to node potential, equal to the potential difference across each nodal capacitor.
3. The rate of erosion is mapped to the rate of discharge of the capacitor.
4. The rate of tectonic mass input is given by the rate of recharge of a capacitor.
5. The flow of sediment down a hillslope or channel is equivalent to the flow of current through a resistor.
6. A square grid of resistors spans the model landscape, and the nodes joining each set of four resistors represents a pixel.
The potential difference across each resistor represents the slope (channel or hillslope), so it represents the driving parameter for flow.

The resistors are pseudo-linear in the sense that they obey a linear voltage-current relation, representing the linear transport law given in eq 11.

The resistors are non-linear in the sense that their resistance is temperature dependent.

The resistor temperature is the analog of the channelization $T$ given in eq (12).

The flow of current through a resistor dissipates energy through the power $VI$, which is equivalent to the stream power $qS$.

This energy dissipation generates heat, which drives up the temperature of the resistor.

This heat is lost by radiation at a rate controlled by the resistor temperature.

The thermal balance equation is equivalent to the channelization evolution eq (13).

The hotter the resistor, the lower its resistance, which promotes further inflow of current and drives a positive feedback mechanism producing localization of current flow on the network (this is the opposite of real electrical resistor behavior). This process is equivalent to the formation of stream channels. The radiation of heat, which is analogous to the degradation of a channel, stabilizes the positive feedback and prevents thermal runaway.

Thermal equilibrium represents channel equilibrium and an asymptotically constant channelization (eq 16).
There are a number of advantages of this numerical scheme. First, strong disorder can be imposed on the initial topography (node potentials), the channel conductances (inverse resistances), bedrock densities (node capacitances), or other parameters. In this way, correlated, quenched noises such as self-affine, fractional Brownian motion can be used to simulate spatial heterogeneity in the initial landscape and in the erodibility of the soil and bedrock (Stark, 1994). In the example simulations illustrated here (figs. 9A and B and 10), a weak brown noise disorder is imposed on the reference conductances \( k_0 \) and densities \( \rho \). This spatially correlated disorder is an important factor in the formation of drainage network structure.

A complementary feature of the resistor-capacitor network method is the ability to handle discontinuities. Transitions such as that from a hillslope pixel (node) to a channel pixel may impose an abrupt change in slope. Such discontinuities (singularities) are often very difficult to handle numerically using conventional methods, and more sophisticated techniques like multigrid may be required. In our approach, some of the discontinuous behavior at the hillslope-channel transition is described by abrupt changes in the channelization index; these changes model the inception of channels and their developing morphology across a single pixel. The remaining singular behavior is handled in a moderately robust fashion by the resistor-capacitor combination, because this pairing has an built-in dampening effect. The need for numerical diffusion or dampening is obviated. Care must nevertheless be taken to avoid very large positive feedbacks in flow and conductance; these can lead to increases in channelization that are too fast to resolve for the chosen numerical time step.

The differential equation (13) that describes the approach of each channel pixel towards dynamic equilibrium is equivalent in the electrical analog to a local heat equation for each resistor. This equation is non-linear, but in our solution scheme we linearize it by a perturbation expansion. We employ a Taylor series approximation of the non-linearity and take only the first term of the series. The resistor-capacitor network then requires a linear set of simultaneous equations to be solved in order to find the node potential at each successive time step. The beauty of the conjugate gradient method then becomes apparent, because the solution for the previous time step can be used as an approximate solution for the next time step. The quality of the initial guess determines how fast a conjugate gradient scheme converges to the true solution, which in this case is a set of balanced flow equations. For small time steps, the structure of the flows and potentials does not change markedly, and the guessed flow solution for the next time step is always very good. Grids of the order of \( 10^4 \) nodes are numerically tractable.

**Boundary conditions.**—Another advantage of the resistor-capacitor network method is the ease with which boundary conditions can be adapted. A constant current input at each node is equivalent to either (A) constant uplift, or (B) constant subsidence. Pinning a row of nodes (a bus bar) at a constant voltage is the geomorphic equivalent of maintaining a boundary at a constant base level. Floating boundary conditions at the edges of a network, where nodes are connected by only two or three resistors, are stable. Such boundary conditions are preferable to the periodic edge conditions often applied in numerical geomorphic models, because they do not impose extra symmetry on the sets of simultaneous node equations. Networks with either pinned or floating boundary conditions yield strictly band-diagonal matrices of node equations and are less cumbersome to solve.

**EXAMPLE SOLUTIONS: STEADY-STATE LANDSCAPES**

The task that now remains is to demonstrate that the incorporation of channel disequilibrium into a geomorphic model has significant consequences for the evolution of model landscapes. We will base this demonstration around the geomorphic evolution of a fault block subject to a uniform uplift rate and driven toward a steady
Fig. 10. Response of a steady-state landscape to an increase in uplift rate. A wave of increased erosion rate propagates up the stream network and dissipates slowly on the hillslopes.
state, uniform erosion rate. First, we show that the channelization model and its numerical implementation are stable and simulate realistic landscapes given these initial and boundary conditions (fig. 9). Second, we demonstrate the effect of perturbing the steady state by abruptly increasing the rate of tectonic mass input. We show that the geomorphic response to this change in tectonic boundary condition is critically dependent (fig. 10) on the spatio-temporal behavior of channelization, that is, the degree of channel disequilibrium. This numerical experiment serves as a good example of how the incorporation of such dynamics in landscape evolution models might facilitate progress in our theoretical understanding of steady state mountain belts.

**Fault block.**—The model response to different boundary conditions is shown in figure 9. In both experiments a mesh of 64 × 64 nodes was created with uniform current input across all but one row at one edge. On this row, the polarity of the current input was reversed, and set to balance the total current input across the rest of the grid. This geometry was chosen in order to simulate a fault block undergoing uniform vertical uplift, bounded at one edge by a normal fault and an associated sediment sink. A constant sediment (current) sink is a more stable boundary condition than a constant elevation (voltage) boundary.

Both experiments show the model topography at steady-state, and all the process parameters were held the same. The hillslope process was again chosen to be mass-wasting ($\alpha = \beta$). The difference between the two experiments lies in the rate of tectonic mass input or nodal current input: the uplift rate in figure 9B is twice that in figure 9A. As a result, the erosion rate is forced to increase, the channel density is higher, and the terrain is more rugged.

However, water and sediment fluxes are tied together in this single flux model. So the increase in node current input is also equivalent to an increase in the frequency or intensity of peak storms. We would expect greater drainage density and higher rates of mass-wasting following such a change, and the model results bear this out.

**Response to changing tectonic input.**—The concept of steady-state landscape is meaningless unless the changes in boundary conditions are reflected rapidly in the rates of erosion. If the rate of tectonic supply accelerates, the system will remain out of steady-state until this change has propagated throughout the landscape and until the erosion rate has increased to match the new tectonic boundary condition. The rate of propagation of this change determines the minimum time and length scale at which the landscape can be described as at steady-state.

A first attempt at addressing this issue is illustrated in figure 10. The sequence of nine images shows the transition from the low erosion rate, steady-state landscape of figure 9A, to the high erosion rate, steady-state landscape of figure 9B. The color attribute in these images is the rate of erosion.

In figure 10A, the landscape is in steady-state, and the spatial variation in erosion rate is very small. The range of color variation is normalized in each image so that even these weak variations are visible. Blue indicates a higher erosion rate, white indicates a lower erosion rate.

In (B), the rate of tectonic input is abruptly doubled. The response of the landscape is immediate, and the rate of erosion in the main channels increases in concert with the tectonic change. This wave of enhanced erosion rate propagates rapidly up the channel networks in (C) and (D).

The response of the hillslopes is much slower. In (D) to (E) the rate of hillslope erosion increases as a spreading wave away from the main channels. In (F) and (G) the hillslopes are still adjusting to the increased rate of erosion in the channels, which the channels have already equilibrated with the fault boundary condition.
In (H) and (I), the system has returned to steady-state: the variability of the erosion rate is again weak, although the normalized color range exaggerates the range. The landscape in (I) is identical to that shown in figure 9B.

CONCLUSIONS

We have looked closely at how the spatial resolution of geomorphic models and topographic data limit our ability to study landscape evolution. We have suggested that sub-grid scale parameterization schemes are a way to address this resolution problem, and we have focussed on the particular issue of how to parameterize for the properties of channels hidden with a pixel. Through a scaling analysis of digital topographic data, we demonstrated that measures based on channel slope and upstream area can be corrected for pixel resolution, at least in a distributional sense. We then developed a sub-pixel parameterization scheme based loosely on these scaling results in an attempt to define a rescaleable measure of the disequilibrium of channels within each pixel. This channelization measure was then built into a prototype geomorphic model designed to illustrate some of the consequences of treating channel disequilibrium explicitly in a simulation of landscape evolution. Simple example solutions of this model demonstrate that changes in tectonic boundary conditions and the time that such changes take to propagate across a landscape are critically dependent on the spatio-temporal behavior of the channelization measure. The time scale on which mountain landscapes change from one steady-state to another as tectonic and climatic driving forces change is likely to be partly a function of the time scale on which channels move in and out of equilibrium.

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