

NEW RHEOLOGICAL AND POROSITY EQUATIONS FOR STEADY-STATE COMPACTION

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ABSTRACT. Diagenetic theory, as it is often stated, is formally incomplete in the sense that it contains more dependent variables than the number of equations in the theory. Heretofore, this situation has been resolved ordinarily by introducing an empirical equation for porosity, φ , or equivalently the solid volume fraction, $\varphi_s = 1 - \varphi$, as a function of depth. In contrast, the theory of compaction, that is combined momentum and stress balances, leads to a differential equation that governs the behavior of φ_s , thus completing standard diagenetic theory.

Based on recently acquired in situ data, we advance that the steady state change in solid volume fraction, $d\varphi_s$, during compaction is well described by a function of the change in effective stress on the solids, $d\sigma'$; specifically,

$$d\varphi_s = A \exp(-b\sigma') d\sigma'$$

where A and b are parameters that specify the initial compressibility and the attenuation of compressibility, respectively. From this rheology and the justifiable assumption that the Darcian contribution to the stress can be neglected, the steady-state distribution of φ_s is governed by the equation

$$\varphi_s = \frac{(\varphi_s)_o (\varphi_s)_\infty}{(\varphi_s)_o + ((\varphi_s)_\infty - (\varphi_s)_o) \exp(-\beta x)}$$

where x is depth, β is a depth-attenuation constant, and the subscripts o and ∞ indicate values at the sediment-water interface and the asymptotic value at great depth, respectively. Fits of φ_s data with this new equation are similar in quality to those obtained with the classical exponential and power-law functions.

INTRODUCTION

Porosity decreases with depth are ubiquitous in the upper meter of fine-grained aqueous sediments. The primary cause of this change in porosity, φ , with depth is compaction (consolidation) during burial, but both chemical reactions and bioturbation can be contributory processes (Berner, 1980; Boudreau, 1986; Bennett and others, 1996). Such porosity gradients have significant effects on the geochemistry and early diagenesis of sediments, for example, Jørgensen (1978), Murray, Grundmanis, and Smethie (1978), Jahnke and others (1986), Rabouille and Gaillard (1991a,b), Martin and others (1991), and Mulsow, Boudreau, and Smith (1998). Porosity, or equivalently the void ratio, $e = \varphi/(1 - \varphi)$, is also the main variable measured in studies of sediment compaction, for example, Smith (1971), Chilingarian and Rieke (1976), and Hamilton (1976), and for correlating geotechnical and geomechanical properties of sediments, for example, permeability (Bryant, Hottman, and Trabant, 1975; Silva, Hetherman, and Calnan, 1981; Bennett and others, 1989).

The pervasive role of porosity in diagenesis is reflected in its nearly ubiquitous presence as a variable in properly formulated diagenetic models. However, a curious situation has developed in regard to our theoretical understanding of its depth distribution. To appreciate this problem, one needs to consider the equations that describe the steady-state conservation of total porewater and total solids (Berner, 1980; Lerman, 1977; Boudreau, 1997). Specifically, when the sediments are not mixed or mixing

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doesn't effect porosity (that is, intraphase mixing, Boudreau, 1986), then

$$\frac{dv_w \varphi}{dx} = 0 \quad (1)$$

and

$$\frac{dv_s \varphi_s}{dx} = 0 \quad (2)$$

or, if mixing affects porosity (that is, interphase mixing, Boudreau, 1986), then

$$\frac{d}{dx} \left(D_B \frac{d\varphi}{dx} - v^* \varphi \right) = 0 \quad (3)$$

and

$$\frac{d}{dx} \left(D_B \frac{d\varphi_s}{dx} - v^* \varphi_s \right) = 0 \quad (4)$$

where x is depth relative to the sediment-water interface (positive x pointing downward), φ is the porosity, v_w is the actual velocity of the porewater phase relative to the sediment-water interface, v_s is the velocity of the solids, v^* is the volume averaged velocity of the solids and fluid phases (Boudreau, 1986) which is a function of v_w and v_s , and D_B is an appropriate mixing coefficient. Five assumptions are implicit to these equations: (1) a steady state is operative, (2) the density of both the porewater, ρ_w , and the total solids, ρ_s , are relatively constant with depth, and neither is compressible, (3) reactions do not cause appreciable changes in the porosity or the solid volume fraction, (4) the porosity changes are strictly vertical, and (5) mixing is diffusive. These assumptions are reasonable for a wide variety of sediments.

These two sets of two equations each contain three (unknown) dependent variables, that is φ , v_w , and v_s (included in v^*); consequently, either set constitutes a mathematically under-determined system. With practicality in mind, many geoscientists have resolved this problem by introducing empirical formulas for φ , or equivalently φ_s , as a function of depth, of which the most popular (Murray, Grundmanis and Smethie, 1978; Martin and others, 1991; Rabouille and Gaillard, 1991a,b; Soetaert, Herman, and Middelburg, 1996) is

$$\varphi = (\varphi_o - \varphi_\infty) \exp(-\alpha x) + \varphi_\infty \quad (5)$$

where φ_o is the porosity at the sediment-water interface ($x = 0$), φ_∞ is the apparently constant porosity reached at some depth, and α is an empirical attenuation constant. Use of eq (5) is illustrated in figure 1 as fits to $(1 - \varphi)$ data (Mulsow, Boudreau, and Smith, 1998), and the goodness of fit, R^2 , is listed in table 1 for each site. Formulations similar to eq (5) have also been employed by engineers to describe near surface porosity changes in a variety of materials—see references in Hsu and Cheng (1991). A theoretical basis for eq (5) has been partially explored by Fukumori, Christensen and Klein (1992), as noted below.

Engineers and some scientists, who have investigated the geotechnical problem of compaction, have taken different approaches to the indeterminacy of eqs (1) through (4). The most common course is to introduce equations for the conservation of momentum, for example, Gibson, England, and Hussey (1967), Sharp (1976), Lehner (1984), Garg (1985), McVay, Townsend, and Bloomquist (1986), Landman, White, and Buscall (1988), and Toorman (1996). When suitable momentum equations are coupled to either

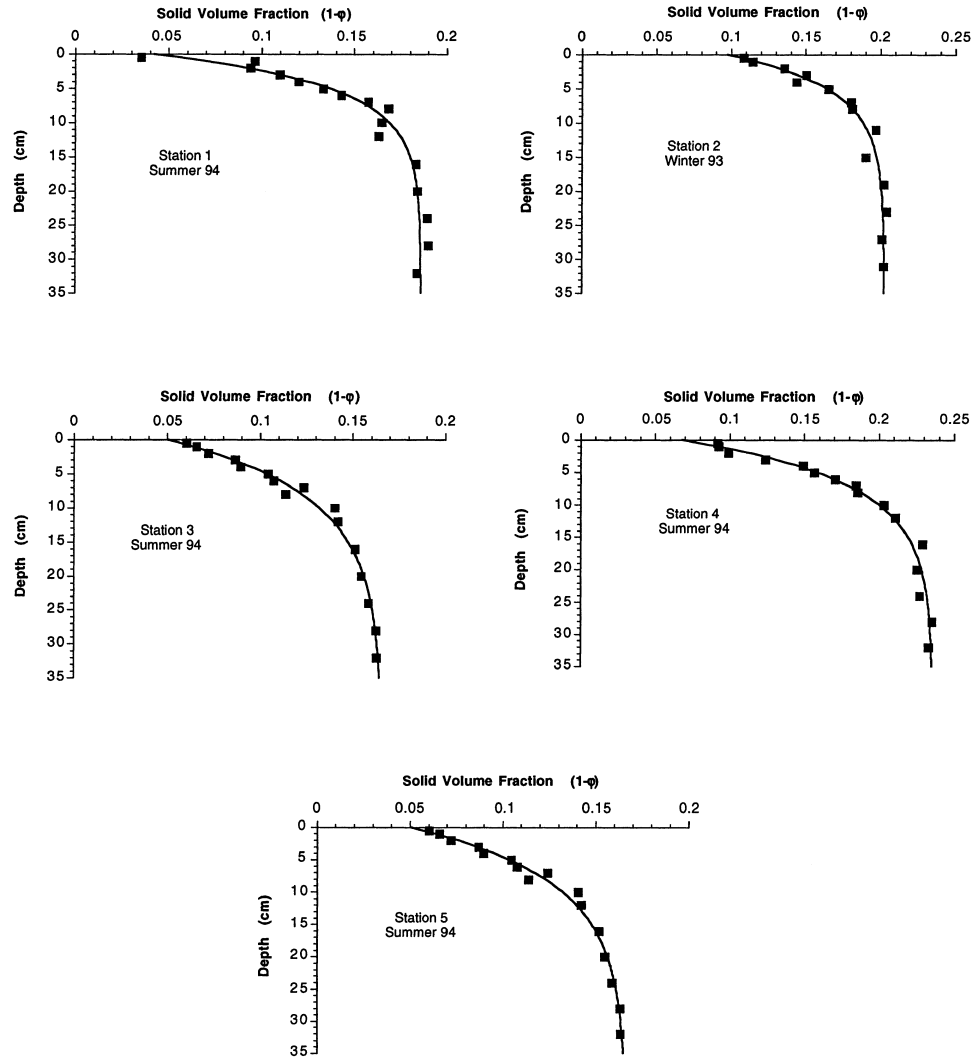


Fig. 1. Plots (profiles) of porosity versus depth at five sites on the Eastern Canadian Margin—see Mulsow, Boudreau, and Smith (1998) for details of the station locations and the measurement method. The curve fits are for eq (5) after conversion to ϕ_s form. The values for the coefficient of determination, R^2 , for the fit in each case, which represents the proportion of the variance explained by the fitting equation, can be found in table 1.

eqs (1) and (2) or eqs (3) and (4), a complete (determined) theory emerges; unfortunately, two aspects of this theory have perhaps discouraged its use amongst investigators of early diagenesis. First, the resulting equations are highly nonlinear and apparently cumbersome (Fukumori, Christensen, and Klein, 1992). Secondly, to make matters worse, the equations are normally stated in a Lagrangian frame of reference that follows a deforming sediment layer and not relative to the Eulerian frame of the sediment-water interface. Fortunately, Toorman (1996) has recently clarified the Lagrangian-Eulerian transformation, and that problem is largely eliminated. However, the end-result of these limitations has been that no explicit mathematical expression, that describes/predicts the variation of ϕ or ϕ_s as a function of x for steady-state diagenesis has arisen from this theory.

TABLE 1
Values of the coefficient of determination, R^2 , for fits with eqs (5) and (22), as applied to data from Mulrow, Boudreau, and Smith (1998)

Station	Eq (5)	Eq (22)
1	0.948	0.934
2	0.980	0.979
3	0.986	0.988
4	0.988	0.991
5	0.986	0.988

Alternatively, Fukumori, Christensen, and Klein (1992) have employed the theory of elasticity from soil mechanics, that is, the Young's modulus approach, to resolve the problem. Though perhaps less utilized, this elasticity approach does lead to an explicit expression for bulk (wet) density ($\rho = \phi \rho_w + \phi_s \rho_s$, where ρ_w = density of porewater and ρ_s = density of solid sediment) as a function of depth. The resulting ρ -equation is generally in the form of a power function in x , but a form like eq (5) results for ρ when Young's modulus is proportional to the square of the bulk (wet) density. While the elastic medium approach is certainly useful and consistent with the data, Fukumori, Christensen and Klein (1992) do suggest that solution to the momentum equation is the first approach that logically should be investigated.

Scientists and engineers, who are interested in compaction and its effects on early diagenesis, need to confront the nonlinear momentum-based compaction theory if we wish to have a non-ad hoc approach to porosity changes. This paper examines the question of the solution to that nonlinear theory, under some reasonable restrictions, and the practicality of that solution to diagenetic modelling. In addition, this paper briefly investigates the relationship between the momentum-based theory and eq (5).

EQUATION OF MOTION FOR THE POREWATER

Compaction-driven flows in slowly-accumulating, natural, fine-grained sediments are characterized by values of the porewater Reynolds number, Re_p , that are far less than one (Bear 1972),

$$Re_p \equiv \frac{\phi v_w d_s}{\nu} \leq 1 \quad (6)$$

where d_s is the average grain diameter, and ν is the kinematic viscosity of water. Typical values of these parameters are $\phi = 0.85$, $v_w = 10^{-12} - 10^{-8} \text{ cm s}^{-1}$ (that is $3 \times 10^{-4} - 3 \times 10^{-1} \text{ cm yr}^{-1}$), $d_s = 10^{-4} - 10^{-3} \text{ cm}$, and $\nu = 10^{-2} \text{ cm}^2 \text{ s}^{-1}$; substitution of these values into eq (6) produces a left-hand side no greater than 10^{-8} , which is indeed small. Under such conditions, the one-dimensional, steady-state equation of momentum conservation for the porewaters is simply (Slattery, 1981; Toorman, 1996)

$$\rho_w g - \frac{dp}{dx} + f_w = 0 \quad (7)$$

where ρ_w is the porewater density, g is the acceleration due to gravity which is positive in the normal diagenetic reference frame, p is total pressure in the fluid phase, and f_w is the frictional dissipation between the fluid and the solids.

The fluid pressure, p , has two components. The first component is the hydrostatic pressure, p_h , due to the weight of water above a given point. The second component results from the fact that the solid sediment does not support all its own weight; that is why it compacts, and this excess pressure, p' , is transferred to the porewaters.

The term f_w is the momentum dissipated by any flow (velocity) of the fluid relative to the solids, that is the Darcy flow. This term is usually assumed to have the form (Bear, 1972; Toorman, 1996)

$$f_w \equiv -\frac{\rho_w g \varphi}{K} (v_w - v_s) = -\frac{\mu \varphi}{k} (v_w - v_s) \quad (8)$$

where, following hydrological usage, K is the hydraulic conductivity or coefficient of permeability (Freeze and Cherry, 1979, p. 27), k is the specific or intrinsic permeability (Freeze and Cherry, 1979, p. 27), and μ is the dynamic viscosity. The parameter K has units of length per unit time, that is a velocity, and k has units of squared length. Both K and k are considered to be functions of both φ and φ_s .

Substituting the definitions for p and f_w into eq (7) reduces this former equation to Darcy's Law,

$$\frac{dp'}{dx} = -\frac{\rho_w g \varphi}{K} (v_w - v_s) \quad (9)$$

which is the final relevant form.

STRESS (LOAD) BALANCE FOR THE SOLIDS

The momentum conservation for the total sediment is stated as a load balance (Terzaghi, 1943; Gibson, 1958):

$$\sigma = p_h + p' + \sigma' \quad (10)$$

where σ is the total weight of the solid sediment plus the porewater, σ' is the load carried by the solids, p_h is the hydrostatic pressure and p' is the excess pore pressure, as noted above (see fig. 2 in Smith, 1971). The total load is evaluated as

$$\sigma = \sigma_o + g \int_0^x (\varphi(x') \rho_w + \varphi_s(x') \rho_s) dx' \quad (11)$$

where σ_o is a reference load, that plays no further role in our development.

Differentiating eqs (10) and (11) and combining produces (Toorman, 1996)

$$-\frac{dp'}{dx} = g(\rho_w - \rho_s) \varphi_s + \frac{d\sigma'}{dx} \quad (12)$$

and further substitution of this last equation into eq (9) gives that

$$\frac{d\sigma'}{dx} = g(\rho_s - \rho_w) \varphi_s + \frac{\rho_w g \varphi}{K} (v_w - v_s) \quad (13)$$

which is the saturated-soil stress balance. Without the Darcy term, eq (13) is equivalent to eqs (18) and (19) of Fukumori, Christensen, and Klein (1992).

SIMPLIFICATION OF THE POREWATER STRESS BALANCE

Eq (13) is the starting point in our goal of deriving an equation for porosity. Unfortunately it is implicitly nonlinear in φ or φ_s , because both K and σ' are nonlinear functions of these two variables. The nonlinearity associated with σ' is not problematical, as shown below, and that of K can be circumvented. Specifically, the term containing K may not be as important as other terms in eq (13), and as such, its neglect is possible in a steady state. The coefficient $g(\rho_s - \rho_w)$ has a value of about $1500 \text{ g cm}^{-2} \text{ s}^{-2}$ for a density difference on the order of 1.5 g cm^{-3} between solids and fluid; in comparison, the

parameter grouping $g\rho_w(v_w - v_s)/K$ has the same units, but a magnitude smaller by a factor of between 10^2 and 10^5 for $(v_w - v_s) = 10^{-12} - 10^{-8} \text{ cm s}^{-1}$ and $K \approx 10^{-7} - 10^{-5} \text{ cm s}^{-1}$, as extrapolated for high porosities (≥ 70 percent) from the data in Bryant, Hottman, and Trabant (1975) and Shephard and others (1987). Allowing for a tenfold potential difference between ϕ and ϕ_s , the second term on the right-hand side of eq (13) is still some 10^{-4} to 10^{-1} times the first term on the right-hand side. This particular balance holds only for slow compaction as a result of the load from self-weight and neglect of the Darcian term in eq (13) is unlikely to be valid for situations with fast compaction and/or added external loads.

Neglect of the Darcian term produces the simplified load equation

$$\frac{d\sigma'}{dx} = \Delta\gamma_s\phi_s \quad (14)$$

where $\Delta\gamma_s \equiv g(\rho_s - \rho_w)$. Eq (14) states that the effective stress on the solids increases with depth due to the cumulative load of sediment, corrected for buoyancy. For equilibrium conditions (no porewater flow), eq (14) is exact, whereas it is only approximately true for actual field conditions, because equilibrium is never truly reached. If ϕ_s was a constant, eq (14) would predict a linear increase of σ' with depth, analogous to the increase in hydrostatic pressure.

At the end of the previous section it was stated that eq (14) is equivalent to the equations derived by Fukumori, Christensen, and Klein (1992), based on an elastic model. However, the Fukumori, Christensen, and Klein (1992) equations are the result of an a priori assumption regarding the dominance of grain-to-grain contacts in carrying the load, whereas eq (14) is derived from a quantitative evaluation of the preeminence of terms in a more complete description; no effective difference perhaps but arguably more theoretically satisfying.

A NEW RHEOLOGICAL/CONSTITUTIVE EQUATION OF STATE

The derivation of a ϕ_s versus depth, x , relationship from eq (14) can be accomplished by introducing a rheological or constitutive equation of state that defines the relationship between ϕ_s and σ' , explicitly. The existence, let alone the precise form of such an equation, is in debate (Toorman and Huysentruyt, 1997). These latter authors argue that such a relationship involves not only ϕ_s and σ' but probably also derivatives of these quantities and is, in general, time dependent. However, the use of simple rheological models is widespread, and they are useful descriptions for slow, steady-state compaction of normally accumulating sediments from self-weight.

The literature on compaction contains many constitutive equations. Most of these relations deal with the void ratio, $e = \phi/\phi_s$, and not ϕ_s . This group includes linear relations (for example, Terzaghi, 1943; Lee and Sills, 1981; Been and Sills, 1981), the famed e -log (σ') equations (for example, Skempton, 1943; Poskitt, 1969; Imboden, 1975; Sharp, 1976), and some power laws (for example, Janbu, 1963; Koppula and Morgenstern, 1982; McVay, Townsend, and Bloomquist, 1986), all of which are purely empirical. All these equations of state lead to significant problems, either conceptually or mathematically in the treatment of our data (see below). For example, the linear equations do not reflect the fact that sediments become increasingly resistant to compaction as the degree of compaction increases. This behavior reflects the increasing effects of electrostatic repulsion between clay particles as they move closer together and the mechanical friction between other types of particles as more contact points occur. The other cited relations do account for this behavior, but they also lead to highly nonlinear equations that do not appear to possess simple analytical (closed-form) solutions.

Faced with these limitations, a new rheological equation is needed that overcomes both difficulties, specifically phenomenological correctness and mathematical tractabil-

ity. In this search, we have recourse to two guides. The first is a unique set of data (fig. 2), acquired by Bennett and others (1999), that provides, for the first time, simultaneous, in situ values of ϕ_s and σ' with depth in surficial sediments off the coast of Mendocino, California. Previous $\phi_s - \sigma'$ data sets are either laboratory-based measurements or deal with situations not relevant to diagenesis. Details of the sample locations, retrieval and measurement methods, and data reduction are provided in Bennett and others (1999) and are not repeated here.

Each plot in figure 2 shows that, overall, ϕ_s decreases systematically with increasing σ' , reaching an apparent asymptotic value at depth. The variability in these data is due to changes in sediment composition and grain size and the deposition/compaction history. In addition, dissolution of minerals, such as opaline silica, can cause changes in solid

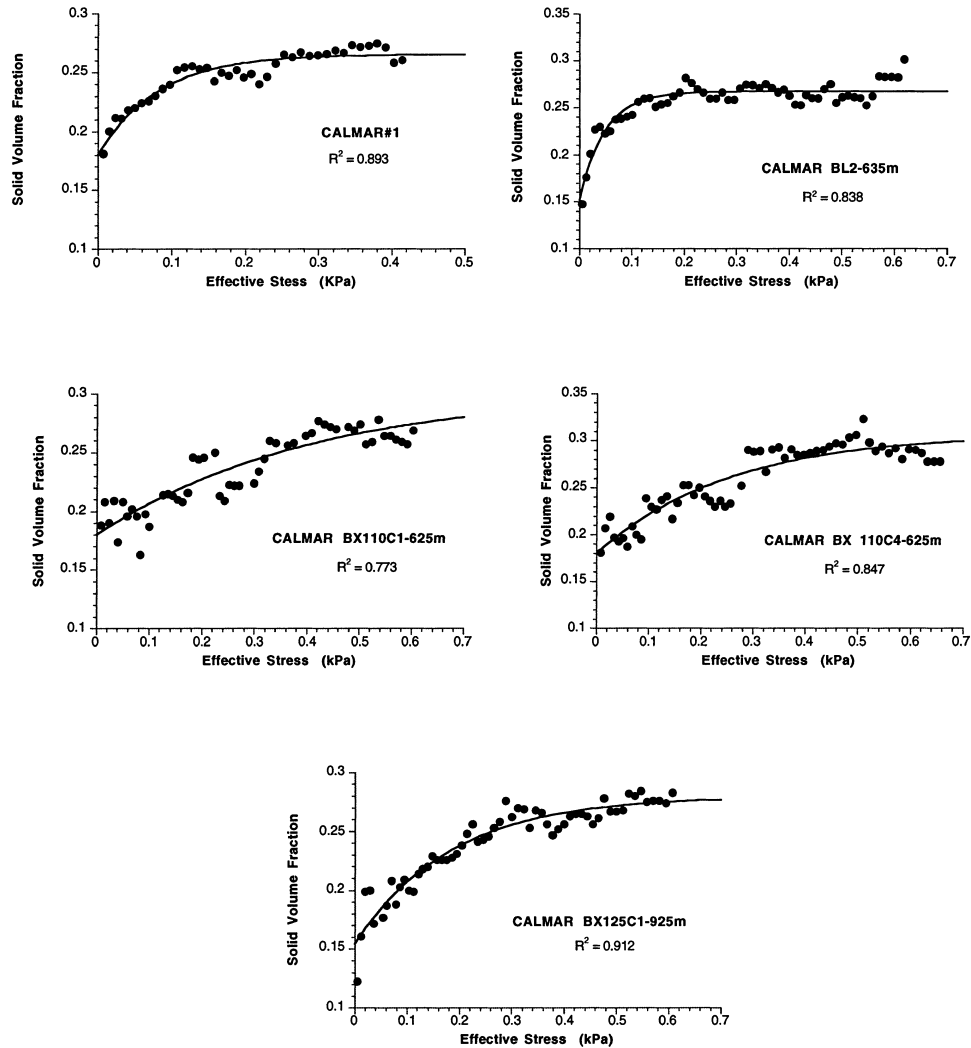


Fig. 2. Plots (profiles) of solid volume fraction, ϕ_s , versus effective stress at five sites on the California Margin off Medocino—see Bennett and others (1999) for details of the station locations and the measurement method. The curve fits are for eq (16), the new rheological equation of state.

volume fraction. Redeposition of minerals at particle contacts can strengthen the microfabric and inhibit consolidation under normal loads. These geochemical and sedimentological processes result in the “noise” in the data; nevertheless, the general trends in figure 2 remain obvious and must be captured by our rheological equation.

The other tool to guide our search is Occam’s Razor, a dictate that is interpreted as requiring that the simplest of competing theories be preferred to the more complex. This simplicity of theory applies not only to the initial formulation of the model but also to its solution. Thus, we seek the simplest rheological equation that also produces a useful and explicit $\varphi_s - x$ equation, specifically one that is expressed in terms of elementary functions. In this search we also assume that σ' is the independent variable that governs the observed changes in φ_s and not vice versa. This latter supposition admittedly contains an element of arbitrariness, but a certain justifiable logic exists in the belief that the application of a force leads to a deformation, rather than the other way around.

To obtain the observed asymptotic behavior (fig. 2), the derivative of φ_s with respect to σ' must decrease precipitously with depth. After considerable trial-and-error testing of various possibilities, the simplest functionality that conformed to the data and produced a workable solution was, in differential form,

$$d\varphi_s = A \exp(-b\sigma') d\sigma' \quad (15)$$

where A is a parameter that relates the initial compressibility and b is a stress attenuation-coefficient that characterizes the increasing difficulty in compressing the sediment as compaction progresses by adding load. Eq (15) is readily integrated to obtain

$$\varphi_s = \frac{A}{b} [1 - \exp(-b\sigma')] + (\varphi_s)_o \quad (16)$$

where $(\varphi_s)_\infty$ is the asymptotical solid volume fraction at great stress. Note that in a steady state, the ratio A/b must be equal to $(\varphi_s)_\infty - (\varphi_s)_o$, where $(\varphi_s)_o$ is the initial solid volume fraction. Figure 2 illustrates the fit of eq (16) to our $\varphi_s - \sigma'$ data sets. The fits are more than satisfactory; however, the parameter values derived in this manner should not be accepted uncritically. The nonlinear regression used in these figures ignores the random errors in σ' , assigning all variability to φ_s . Nonlinear regression codes that account for errors in both variables are not that common, and one was not used here.

A NEW SOLID VOLUME/POROSITY EQUATION

Eq (16) now allows us to derive a new theoretically-based equation for φ_s and therefore φ as a function of depth x . If eq (16) is substituted into eq (14), an equation for stress as a function of depth results,

$$\frac{d\sigma'}{dx} = \Delta\gamma_s \left[\frac{A}{b} (1 - \exp[-b\sigma']) + (\varphi_s)_o \right] \quad (17)$$

Integrating this equation by parts (see Beyer, 1978, p. 368, no. 526) leads to

$$\sigma' + \frac{1}{b} \ln \left[\frac{(\varphi_s)_\infty - \frac{A}{b} \exp(-b\sigma')}{(\varphi_s)_o} \right] = (\varphi_s)_\infty \Delta\gamma_s x \quad (18)$$

or, upon substitution of eq (16),

$$\sigma' = (\varphi_s)_\infty \Delta\gamma_s x - \frac{1}{b} \ln \left[\frac{\varphi_s}{(\varphi_s)_o} \right] \quad (19)$$

The first term in eq (19) represents the increase in effective stress in the absence of compaction, while the second term accounts for the correction to that stress caused by compaction of the sediment matrix and expulsion of fluid (E. Toorman, personal communication).

Finally, eq (19) is substituted back into eq (16),

$$\varphi_s = ((\varphi_s)_\infty - (\varphi_s)_o) \left(1 - \exp \left(-b \left\langle (\varphi_s)_\infty \Delta\gamma_s x - \frac{1}{b} \ln \left[\frac{\varphi_s}{(\varphi_s)_o} \right] \right\rangle \right) \right) + (\varphi_s)_o \quad (20)$$

Using the identity $\exp(\ln[y]) = y$, eq (20) becomes,

$$\varphi_s = ((\varphi_s)_\infty - (\varphi_s)_o) \left(1 - \frac{\varphi_s \exp(-b(\varphi_s)_\infty \Delta\gamma_s x)}{(\varphi_s)_o} \right) + (\varphi_s)_o \quad (21)$$

or, re-arranging to be an explicit function of φ_s ,

$$\varphi_s = \frac{(\varphi_s)_o (\varphi_s)_\infty}{(\varphi_s)_o + ((\varphi_s)_\infty - (\varphi_s)_o) \exp(-\beta x)} \quad (22)$$

or, in terms of the porosity,

$$\varphi = 1 - \frac{(\varphi_s)_o (\varphi_s)_\infty}{(\varphi_s)_o + ((\varphi_s)_\infty - (\varphi_s)_o) \exp(-\beta x)} \quad (23)$$

where $\beta \equiv b (\varphi_s)_\infty \Delta\gamma_s$ as is a depth attenuation-coefficient. We believe that eq (22) constitutes the simplest theoretically-based equation for the solid volume-fraction change during steady-state diagenesis that is consistent with the available $\varphi_s - \sigma'$ data in figure 2, the application of Occam's Razor, and our premise that φ_s be expressible as an explicit function of σ' .

To illustrate the use of eq (22), the data in figure 1 are replotted in figure 3 along with the best fits with this equation. Goodness-of-fit parameter values, R^2 , are listed in table 1. The comparison shows that eq (22) provides fits that are essentially equal in quality to those of eq (5). By comparison, a power-law equation was also tested but was found to produce appreciably lower R^2 values than either eq (5) or eq (22) with our particular data.

One further noteworthy point is that eq (22) was stated without explicit reference to the presence or absence of bioturbation. In fact eq (22) is the valid form in either situation, at least when the Darcian terms in eq (13) can be neglected. Bioturbation does not explicitly influence the form of eq (22); instead, its effects are confined to modification of the parameter values in eq (15) or (16), that is, the equation of state, which should not be interpreted to mean that bioturbation has a minor role to play. Quite to the contrary, we believe that a sediment with and without bioturbation, but similar in all other respects, would have significantly different values of A and β . If bioturbation increases porosity (for example, Rhoads and Boyer, 1982), then the effect would be to decrease β and probably increase A .

THEORETICAL BASIS FOR THE EXPONENTIAL EQUATION

As correctly indicated to us by the reviewers of this paper, eq (5) also satisfies eq (14). The question then becomes, what rheological model is embodied in eq (5)? By working backward from eqs (5) and (14), we found that eq (5) implies a constitutive model of the form

$$\sigma' = \frac{\Delta\gamma_s}{\alpha} \left[(\varphi_s)_o - \varphi_s - (\varphi_s)_\infty \ln \left(\frac{\varphi_s - (\varphi_s)_\infty}{(\varphi_s)_o - (\varphi_s)_\infty} \right) \right] \quad (24)$$

or in differential form,

$$\frac{d\sigma'}{d\varphi_s} = \frac{\Delta\gamma_s}{\alpha} \left[\frac{\varphi_s}{(\varphi_s)_\infty - \varphi_s} \right] \quad (25)$$

While this is a perfectly valid rheological equation, like eq (16), in the context of the present paper it suffers from a vital flaw; that is to say, apparently eq (16) cannot be rewritten so as to express φ_s as a function of σ' , as was our stated goal. This fact is not a cause for rejection of eq (5) for use in diagenetic studies, but we argue that our new eq (22) has a theoretical lineage that is simpler, more elegant, and consequently, more satisfying; however, we are prepared to admit that beauty, mathematical or otherwise, is in the eye of the beholder.

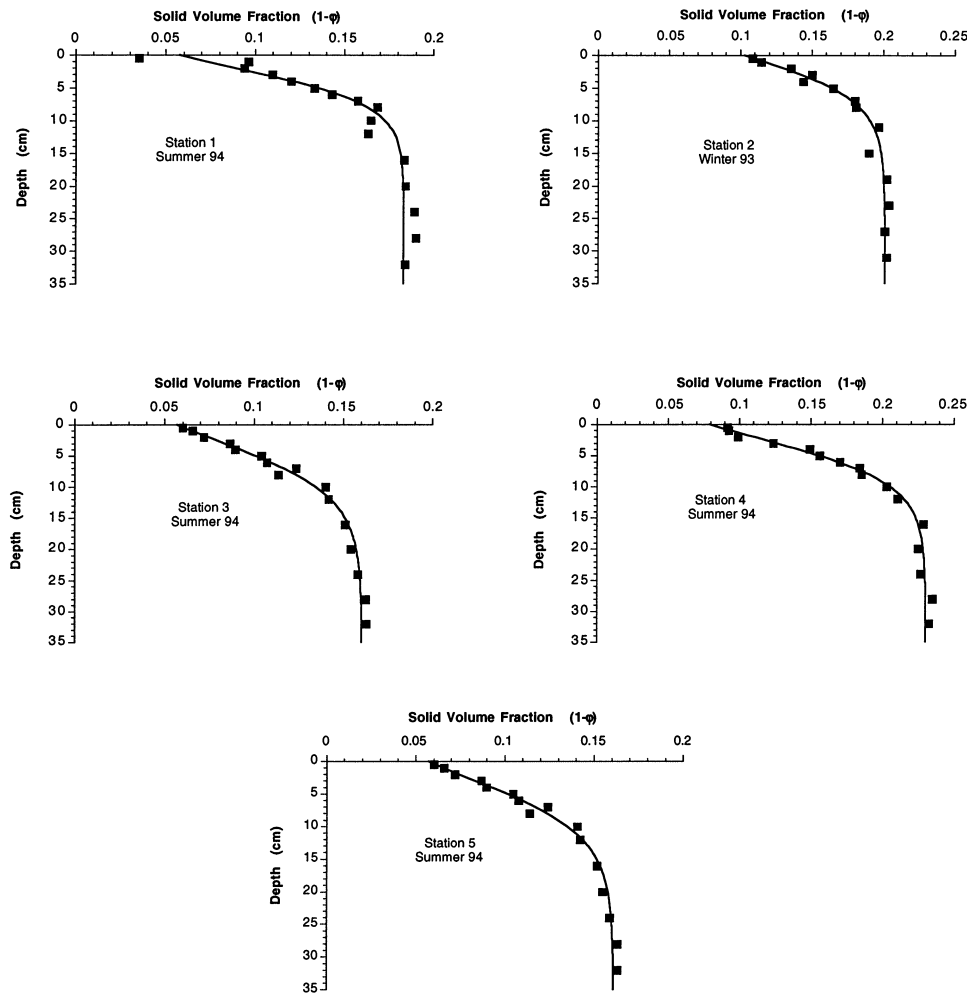


Fig. 3. Same data as in figure 1, but curve fits provided by eq (22). The values for the coefficient of determination, R^2 , for these fits can be found in table 1.

CONCLUSIONS

Diagenetic theory should be considered formally incomplete without the addition of momentum-stress balance equations (for example, Toorman, 1996). Their inclusion allows for the derivation of an equation governing the diagenetic behavior of porosity. Given a rheological equation of state in the form of eq (15), which explains all the data for natural compaction available in figure 2, the theory developed here leads to a steady-state solid-volume equation of the form

$$\varphi_s = \frac{(\varphi_s)_o (\varphi_s)_\infty}{(\varphi_s)_o + ((\varphi_s)_\infty - (\varphi_s)_o) \exp(-\beta x)} \quad (22, \text{repeated})$$

where φ_s is the solid volume fraction, that is $1 - \varphi$ where φ is the porosity, x is depth, and β is a theoretically-based depth attenuation-constant. Eq (22) can provide excellent fits to porosity changes during early diagenesis.

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