THE ANGLE BETWEEN THE a-AXIS AND THE TRACE OF THE RHOMBIC SECTION ON THE \{010\}-PINACOID IN THE PLAGIOCLASES

GEORGE TUNELL

ABSTRACT. The angle $\sigma$ between the $a$-axis and the trace of the rhombic section on the \{010\}-pinacoid is given in terms of $\gamma$, the angle between the $a$-axis and the $b$-axis, and the angle between the \{001\}-pinacoid and the \{010\}-pinacoid by the formula

$$
cot \sigma = \frac{\cos (001 \wedge 010)}{\cot \gamma}.
$$

Values of $\sigma$ for a number of plagioclases from pure albite to pure anorthite were calculated from the interaxial angles of Wülfling by means of this formula. The values of $\sigma$ for these plagioclases calculated from the interaxial angles of Wülfling previously by Rosenbusch and Mühge were computed by means of an incorrect formula.\(^1\)

The position of the rhombic section of any plagioclase can readily be calculated from the crystallographic interaxial angles, $\alpha, \beta, \gamma$. Rosenbusch and Mühge (1927, p. 744) calculated the values of the angle $\sigma$ (the angle between the $a$-axis and the trace of the rhombic section on the \{010\}-pinacoid) for plagioclases of the following compositions: $\text{Ab}_{100}\text{An}_0$, $\text{Ab}_{95}\text{An}_{15}$, $\text{Ab}_{70}\text{An}_{30}$, $\text{Ab}_{63}\text{An}_{37}$, $\text{Ab}_{50}\text{An}_{50}$, $\text{Ab}_{30}\text{An}_{70}$, $\text{Ab}_{25}\text{An}_{75}$, $\text{Ab}_{30}\text{An}_{70}$, $\text{Ab}_{60}\text{An}_{40}$, making use of the crystallographic axial angles of Wülfling. Unfortunately, however, their calculated values were obtained by the use of an incorrect formula.\(^1\)

The rhombic section is defined as the plane through the $b$-axis that intersects the \{010\}-pinacoid in a line perpendicular-

\(^1\) The incorrect formula of Rosenbusch and Mühge (1927, p. 744) is

$$
cot \sigma = \cot \beta - \frac{\alpha}{\cos \alpha} \frac{\cos a}{\cos \gamma \sin \beta}.
$$

If one solves the oblique spherical triangle formed by the poles of the pinacoids 100, 010, and 001, for the value of the angle between 010 and 001 as a function of the interaxial angles $\alpha, \beta,$ and $\gamma$, and substitutes this value in equation 1 of the present paper, one obtains

$$
cot \sigma = \cot \beta - \frac{\cos a}{\cos \gamma \sin \beta}.
$$

The latter formula is in accord with the statement of Rosenbusch and Mühge that “massgebend für die Lage des rhombischen Schnittes ist lediglich die Lage von \{010\} zu \{010\}” (op. cit., p. 747), whereas the formula of Rosenbusch and Mühge is not.
lar to the b-axis. The position of the rhombic section can be found by the following construction. In figure 1 ac'a' is the great circle through the poles of 100, and 001, and 100, and xx' is the trace of a plane through the center of the reference sphere parallel to 010. The intersection u of ac'a' and xx' is a point lying in a plane parallel to the rhombic section through the center of the sphere, since the radius of the sphere ou lies in the plane parallel to 010 through the center of the sphere and also lies in a plane through the center of the sphere perpendicular to the b-axis (the plane of the zone-circle containing the poles of 001 and 100 being perpendicular to the b-axis). The pole r of the rhombic section must then lie on the zone-circle containing the poles of 001 and 100 and at a

Fig. 1. Construction of the pole of the rhombic section from the poles of the faces 100, 010, and 001.

2 If the crystal were ideally developed so that the faces, 110, 110, 110, 110 had equal intercepts on the a-axis, then the intersection of the plane of the rhombic section with these faces would be a rhombus, one diagonal of which would be the b-axis.
distance of 90° from \( u \). The spherical triangle \( crb \) is a right spherical triangle: \( \angle crb = 90° \) since the axis of the zone-circle \( brb' \) is the trace of the rhombic section on the plane 010 and the axis of the zone-circle \( arcua' \) is the \( b \)-axis. The angle \( \sigma \) between the \( a \)-axis and the trace of the rhombic section on 010 can be obtained by solving the right spherical triangle \( crb \). In the triangle \( crb \), the side \( cb \) is the angle between the \( \{001\} \)-pinacoid and the \( \{010\} \)-pinacoid, and the angle \( rcb \) is the interaxial angle \( \gamma \). Hence, applying Napier's rules, we obtain

\[
\cot \sigma = \frac{\cos(cb)}{\cot \gamma},
\]

and

\[
\cot \sigma = \frac{\cos(001 \wedge 010)}{\cot \gamma}. \tag{1}
\]

According to the convention used by Schmidt (1919), by Wülffing (1915), and by Rosenbusch and Mügge (1927), the sign of \( \sigma \) is positive when the pole of \( r \) lies between the pole of \( T00 \) and the pole of \( 001 \), and is negative when the pole of \( r \) lies between the pole of \( 100 \) and the pole of \( 001 \).

Recalculated values of \( \sigma \) computed by means of this formula are given in table 1 together with the incorrect values calculated by Rosenbusch and Mügge.

The angle \( s \) between the rhombic section and the plane 001 differs only slightly from the angle \( \sigma \), as is readily proved. By applying Napier's rules to the right spherical triangle \( crb \), we obtain

\[
\cot s = \frac{\cot(001 \wedge 010)}{\cos \gamma}. \tag{2}
\]

The values of \( s \) computed by the use of this formula are also given in table 1 for comparison with the values of \( \sigma \).

\footnote{This equation leads to the same absolute values of \( \sigma \) as does the equation of E. Schmidt (1919, p. 401). According to Schmidt}

\[\cot \gamma = \pm \tan \sigma \cos MP\]

wo das obere Vorzeichen für die Verhältnisse in Fig. 7, das untere für die in Fig. 8 gilt." Schmidt’s figure 7 represents the angular relations of albite and his figure 8 represents those of anorthite (1919, pp. 392-93). Moreover in Schmidt's table 27 (1919, p. 400) he gives for albite (\( \text{Ab}_{100}\text{An}_{90} \)) \( \text{MP} = 86°26'0'' \), \( \sigma = +40°8' \), \( \gamma = 87°0'7' \), and for anorthite (\( \text{Ab}_{90}\text{An}_{100} \)) \( \text{MP} = 85°48'0'' \), \( \sigma = -20°52' \), \( \gamma = 91°34'1' \). Schmidt's statement that the negative sign in his equation is to be applied in the case of anorthite thus conflicts with his own figures and with his statement "Bei Albit haben wir also einen positiven, bei Anorthit einen negativen \( \sigma \)-Wert" (1919, p. 392).
<table>
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<th>24</th>
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*The values of α, β, and γ in this table are those used by Rosenbusch and Mügge (1927, table 2, p. 744); they were based on the axial angle values of Wülfing.
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The importance of the rhombic section in petrography arises from the fact that in ideal pericline twins it is the composition plane. That the composition plane of pericline twins in the plagioclases in many cases does not coincide with the rhombic section is, however, indicated by observations of Reinhard (1931, p. 87), of Mügge and of Barth (1928). Thus Reinhard stated that "Wir haben an sehr albitreichen Plagioklasen an Hand der Spaltrisse (001) feststellen können, dass die Periklinverwachsungsfläche bei demselben Anorthitgehalt sehr verschiedene Lagen einnehmen kann. Es scheinen alle Übergänge vorzukommen, zwischen (001) = Aklín und dem von Willie angegebenen Wert des rhombischen Schnittes von Albit + 37°. Mügge hat kürzlich ähnliches vom Anorthit mitgeteilt, wo wir es übrigens ebenfalls festgestellt haben."

References


Department of Geology
University of California
Los Angeles, California